Fundamentals of Gas Dynamics
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Week - 04
Lecture – 14
Variable Area Adiabatic flow

In this lecture, we are going to see how the stagnation properties relate, and the Mach number relates to the area change, and how it is affecting the flow in a compressible scenario. So, what I am going to discuss is the variable area changes that is going to affect the density and the velocity, the adiabatic flow.

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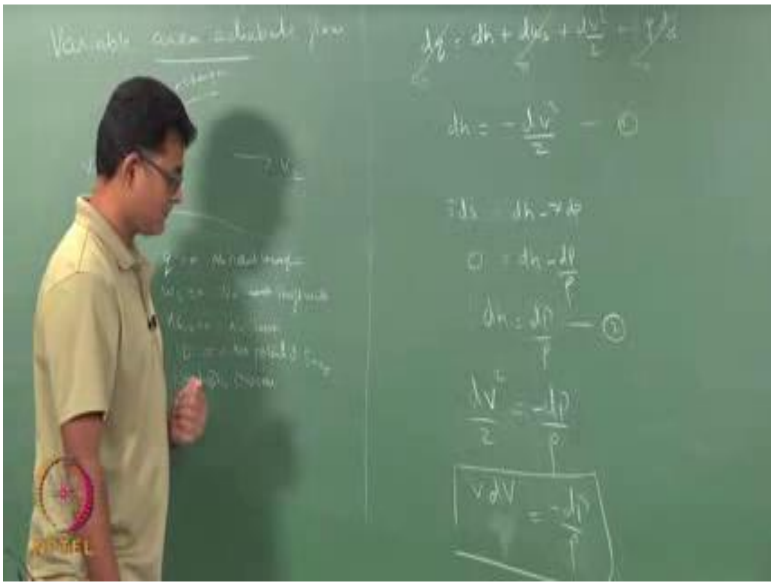

So, I have a situation, where the flow is through some duct, where the area is changing. So, I have a velocity v 1 I have a velocity v 2 and what are the changes that is going to happen here, in terms of Mach number and density. I assume q is 0; so, no heat transfer; w s is 0; so, no work done; no shaft work; no losses. Essentially, I am trying to do a 1 D flow. So, this is an isentropic process. So, what I am trying to do here is an isentropic flow through a variable area duct.

So, I write the energy equation in the differential form; this would be d q equals d h plus d w s plus d v square by 2 plus d g d z. I also assume no potential. So, my heat transfer is 0; shaft work is 0; potential energy is 0. So, I am left with d h equals minus d v square by 2; that is my equation 1. From Gibbs relation, T d s equals d h minus v d p; d s is 0. So, 0, d h minus d p by rho, this is my equation 2, from which I will get d v square by 2 equals, d p by rho minus, or v d v by 2, 2 would go; v d v equals d p by rho.

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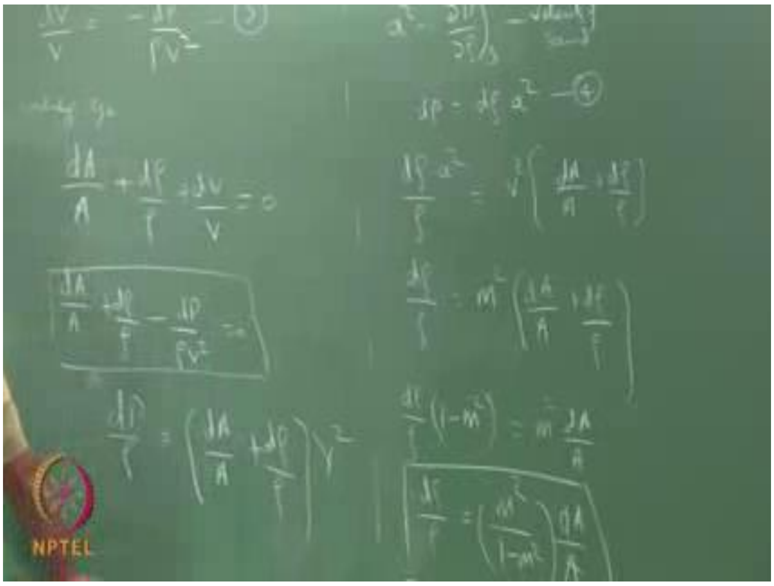

This equation is now substituted in the continuity equation; continuity equation, which we have already seen in this form. So, I will substitute d v by v here. So, d A by A, plus d rho by rho, minus d p by rho v square equals 0. So, I have replaced d v, the changes in velocity, in this form. So, my aim is to represent this in terms of Mach number. d p by rho equals d A by A, plus d rho by rho, multiplied by v square. I know A square is d p by d rho, which is at constant velocity. So, that is the velocity of sound; again, we have seen this before.

So, I use that relation here, in this, and obtain these quantities, in terms of Mach number. I would change my d p as d rho into A square. So, I substitute, this is equation 4, the equation four, and here. So, your d p is now d rho into A square by rho, equals v square into d A by A, plus d rho by rho, which is m square into d A by A plus d rho by rho. Now, I take d rho on this side. So, that would be, d rho by rho into 1 minus m square, equals m square by d A by A.

I would end up with a relation d rho by rho, equals m square by 1 minus m square into d A by A. So, the changes in density related to changes in area, in terms of Mach number. So, I will rewrite this equation here, and we will collect all those equations, and we will try to discuss some physics; but these equations are the end of the lecture.

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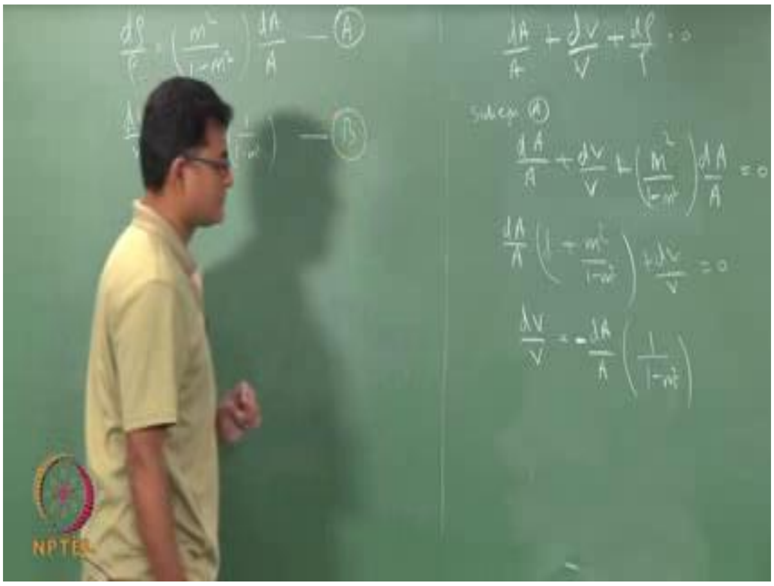

So, we have the first equation d rho by rho equals m square by 1 minus m square into d A by A. Now, I substitute that, this equation in the continuity equation, in that form.

So, I go back to the continuity equation, d A by A, plus d v by v, plus d rho by rho equals to 0. d rho by rho is now replaced by this equation, which now I call it as A. Substitute equation A; I would get d A by A, plus d v by v, equals plus m square by 1 minus m square into d A by A, equals 0. So, I take d A by A outside; 1 plus m square by 1 minus m square, plus d v by v, equals to 0. So, I can write my d v by v in terms of Mach number and area change; d v by v equals d A by A, by 1 minus m square. So, I (Refer Time: 10:27) take this equation and write it down here, for discussions later.

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That is a minus sign.

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Yes, OK.

So, this is my equation B. We have also seen this equation d p by rho, equals d A by A, plus d rho by rho, into v square, where I substitute now, my d v by, d rho by rho from this equation here.

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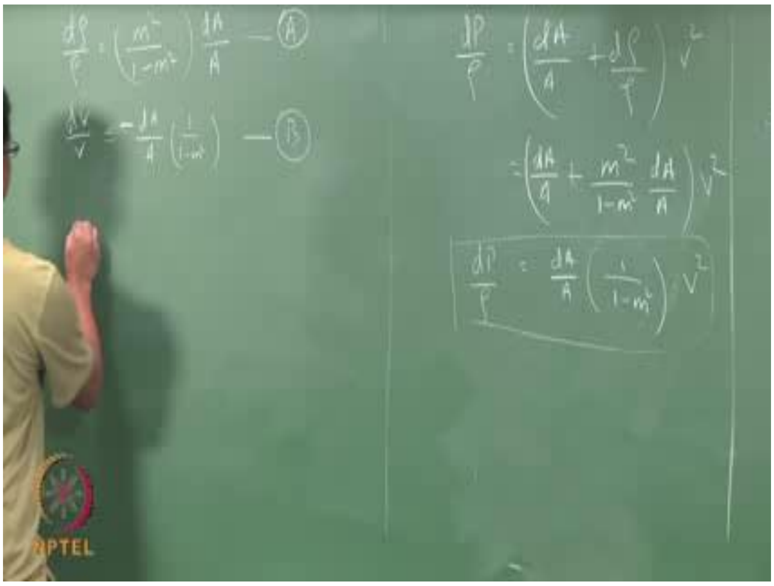

So, d A by A, plus d rho by rho, plus d rho by rho, is m square by 1 minus m square, into d A by A, into v square. So, this is d A by A; there is again 1 by 1 minus m square, into v square; d p by rho; that is the equation for pressure change, which again, I will write it down here. From equation A and B, from equation A and B, I can eliminate my d A.

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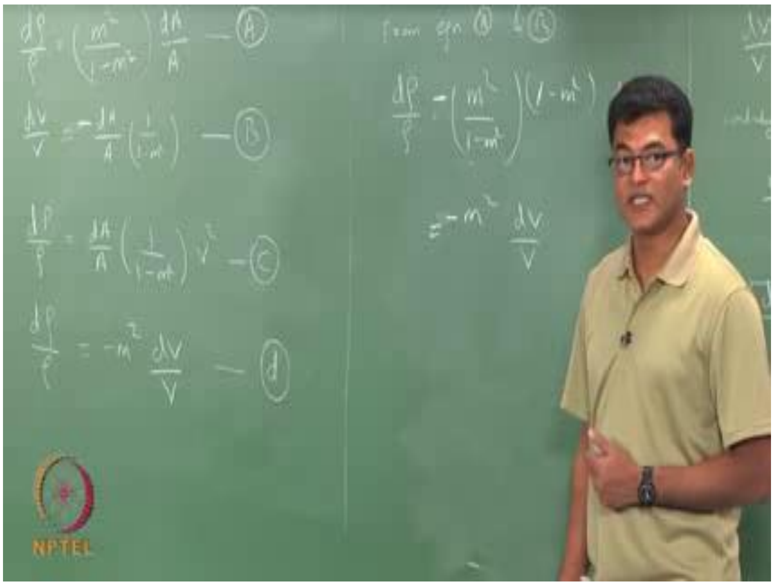

So, d rho by rho equals m square by 1 minus m square, is 1 minus m square into d v by v with a minus sign. So, this is nothing, but m square, minus m square d v by v. So, I take that equation here, equation C. So, all we have done here is modified the continuity equation; we take an isentropic flow, and we have modified those equation with the area change and the velocity changes, along with the Mach number relation. Now, we will try to see what does this physically mean. So, let us take first, the equation B. So, d v by v equals minus d A by A, 1 minus m square.

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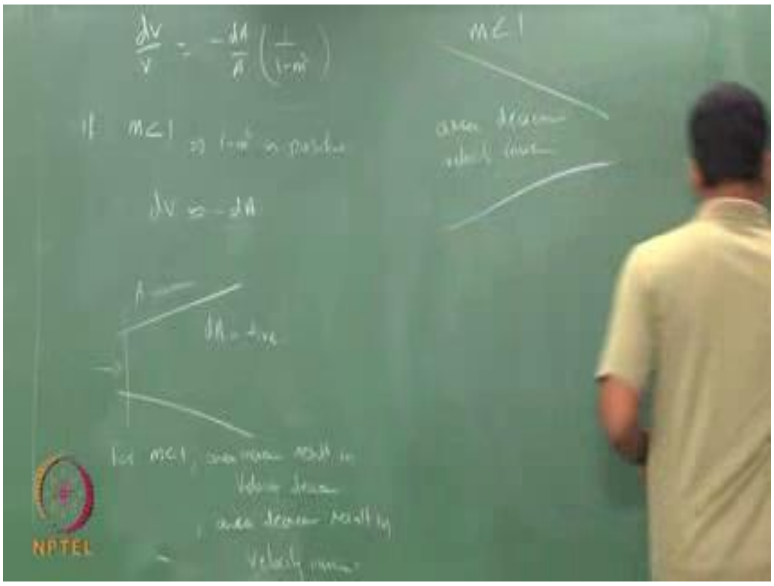

If m is less than 1, if m is less than 1, this quantity is positive; 1 minus m square is positive, which means, the d v is minus, goes as minus d A. So, if there is a positive change in d v, there should be a negative change in d A. So, what is d A? If I have a diverging area like this, my area is increasing; area A is increasing along with the flow, which means my d A is positive. So, if d A is positive, then, my d v is negative. So, for a condition where m is less than 1, my area increases, means my velocity should decrease. For m less than 1, area increases, would give, would result in velocity decrease, and vice versa. So, if for m equals 1, if area decreases, result in velocity increase; which means a scenario like this; my area is decreasing, my velocity increases, if m is less than 1.

Now, this is something which we have already learned in our incompressible flow. When you have water, or you use your continuity equation, and you find that, the velocity is changing according to this. So, this would be a diffuser that would be a nozzle. Now, if m is greater than 1, m is greater than 1, what happens if m, 1 minus m square is negative. So, m is like, greater than 1, this quantity is negative.

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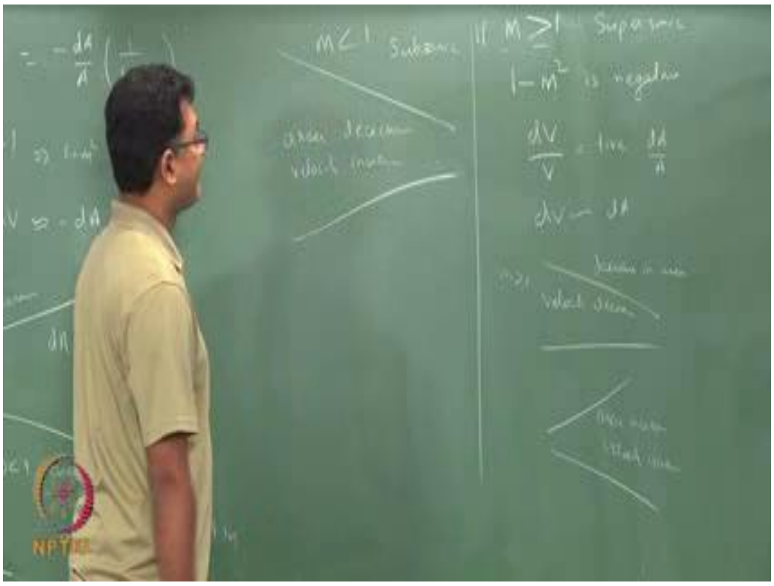

So, your total on the right hand side is positive. So, your d v by v is now positive quantity, with d A by A. So, your d v goes as d A. The sign of d v is the same as the sign of d A. So, if there is an increase in area, there is an increase in velocity; if there is a decrease in area, there is a decrease in velocity. So, that is different from what you have seen in your incompressible flow, or your subsonic flow, in this case. So, this is subsonic; this is supersonic.

So, I have, this is decrease in area; velocity will also decrease, if my m is greater than 1; and this, area increasing, the velocity also increases; that is from equation B. So, these inferences, we have deducted from equation B. Likewise, you can think of the equation A, where the area changes are related to your density changes. Before going to equation A, look at equation D.

So, in equation D, if your Mach number is much less than 1, it is very, very small, your m square is also going to be very, very small.

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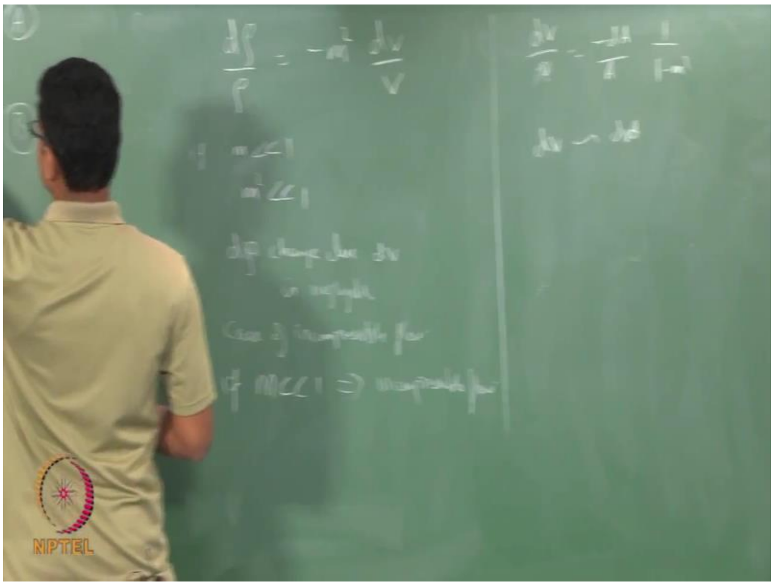

So, your d rho changes, due to d v were nearly small; it is now very, very small, if the Mach number is very small. This is the case of incompressible flow, where we assume, the density changes due to your velocity change is very small. So, in such a case, we can infer that, if m is less than, much less than 1, this will approximate to incompressible flow. So, if your density change is very small, for m equals a very small quantity, if I go back, and look at equation B, equation B is d v by v equals minus d A by A, into 1 minus m square. Now, m square is very, very small. So, your balance is now between velocity and area.

So, at m equals very small case, your density changes are very small, the balance is between velocity and area. Now, if you look at the other way around, if m is large, or let us take m equals 1. If m equals 1, let us take the first equation; m equals 1. Your 1 minus m square into d rho by rho, equals m square d A; or, if I take the next equation, 1 minus m square equals d v by v, equals minus d A by A.

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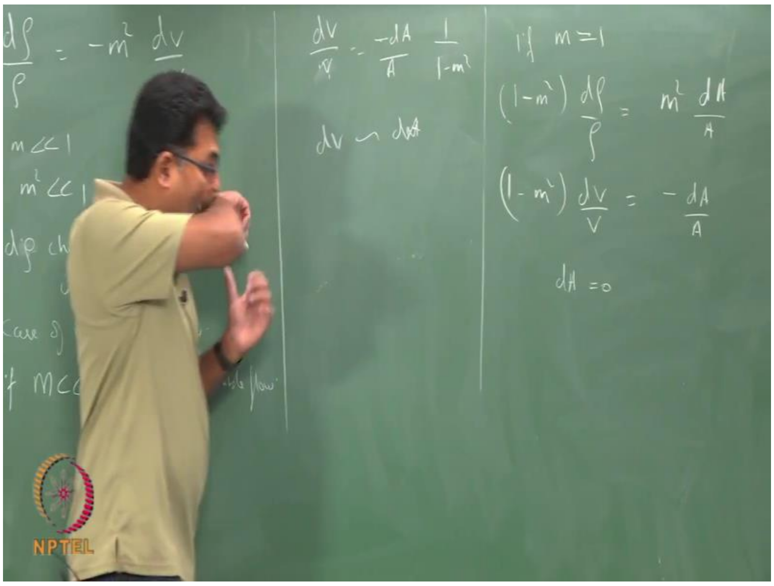

So, this is equation A, and this is equation B. So, if m equals 1, it just means that, your d A is 0. So, that is what we call as a throat condition, which we will discuss it in a later lecture. So, here, the area change is small. So, the adjustments are all between velocity and density.

So, at very small Mach number, we see, the balance is between velocity and area; and, in the large Mach number, or at Mach number 1, the balance is between density and v. For all other conditions, it is a balance between density, velocity and area. So, that is the difference between incompressible flow and the compressible flow. The concept has to be slightly modified, and these are all valid, if the process is isentropic. So, all these have been derived from the continuity equation alone, with the assumption of isentropicity.

If it is not isentropic, there will be small changes here. So, that is the message that you get from these equations. So, essentially, what we have done here is, we have taken the continuity equation; we have used the isentropic condition, isentropic condition into the continuity equation; you derived these 4 equations, where you can find the changes in pressure, density, velocity, and area, all related to Mach number. That is the discussion on variable area adiabatic flow.

Next class, we will try to see how this affects the flow through a nozzle, or diverging and converging nozzles.