Fundamentals of Gas Dynamics

Dr. A. Sameen
Department of Aerospace Engineering
Indian Institution of Technology, Madras

Week – 04
Lecture – 13
Mach number relations

In the last class, we have looked at the velocity of sound, the mach waves, how to define a mach number, and we have also seen something called the stagnation quantities.

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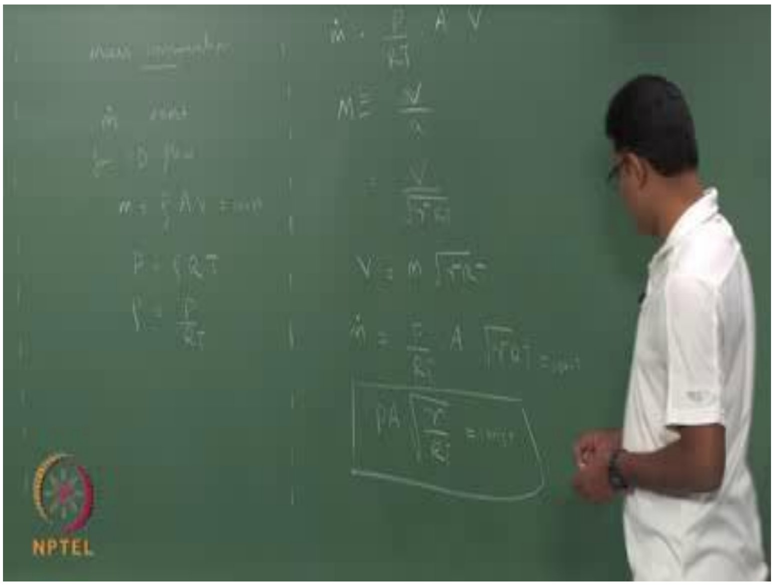

Now we will see how stagnation properties are related to Mach number. For that we will start with the mass conservation. Aim is to write mass conservation in terms of Mach number. So, m dot is constant for 1-D flow equals constant. We know P equals rho R T.

So, I can substitute rho as P by R T. So, my m dot is now P by R T into area A into velocity V. I have defined my mach number as velocity of the fluid by velocity of sound, which we now know as root of gamma R T, which is our A as we have seen in our previous lectures. So, I substitute V as m in the root of gamma R T. If I use this relation my m dot is now P by R T into A into root of gamma R T which is a constant or my P A root of gamma by R T is constant. This is another form of mass conservation equation.

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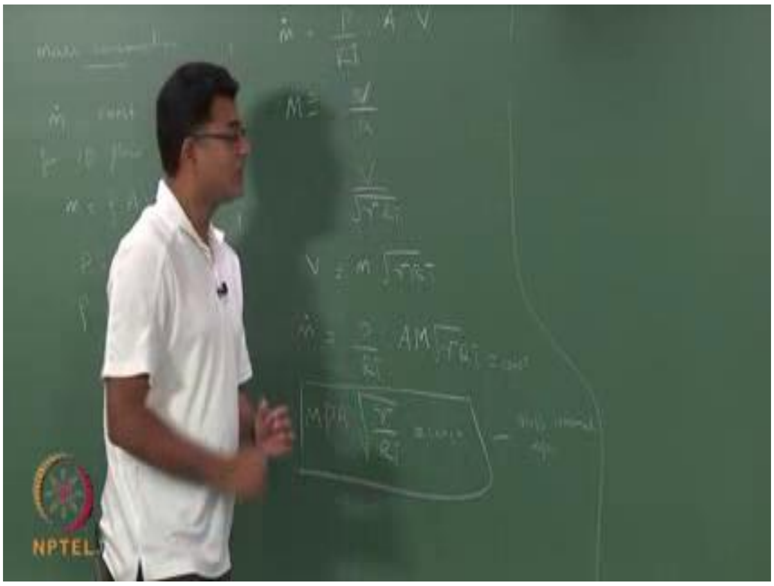

So, that is nothing but conservation equation there is an M missing Mach number M is here. So, this is mass conservation in terms of mach number.

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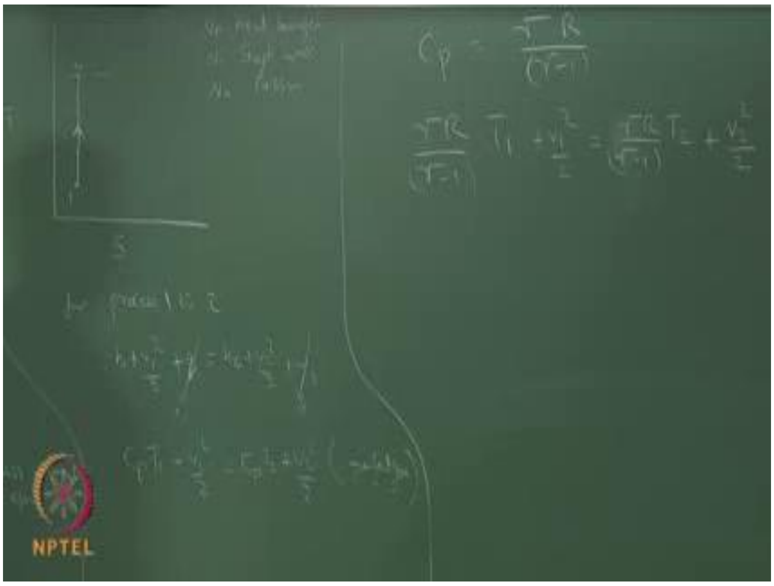

Now let us look at our energy equation. So, in the T S diagram, heat input, heat transfer, no shaft work, no loses. So, in this case between process 1 to 2, my h 1 plus v 1 square by 2 plus q equals h 2 plus v 2 square by 2 plus w s is strike of the quantities which are assumed 0, this is 0, this is 0. C p T 1 plus v 1 square by 2 equals C p T 2 plus v 2 square by 2 for a perfect gas. I also know that my C p is gamma R by gamma minus 1. So, I substitute that in this equation the energy equation. So, it is gamma R by gamma minus 1 into T 1 plus v 1 square by 2 equals C p gamma R by gamma minus 1 into T 2 plus v 2 square by 2.

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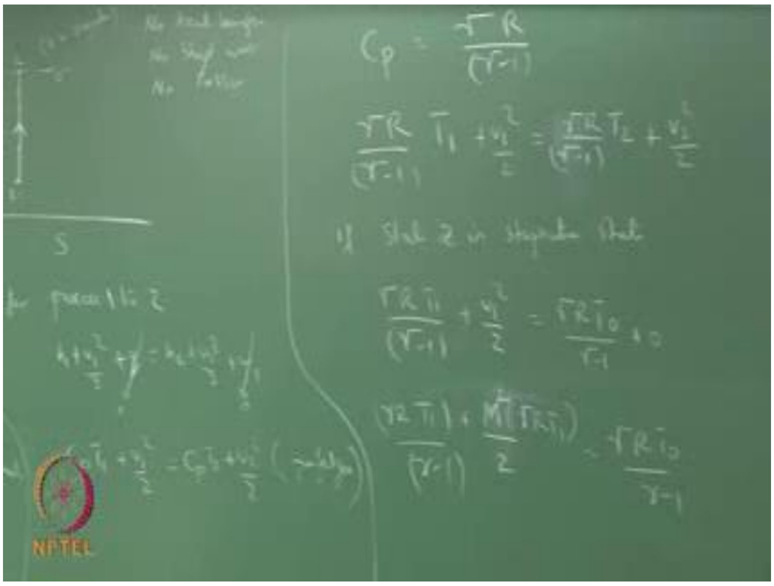

If state 2 is the stagnation state, what do we mean by that I have taken the state 1 to a location where my velocity is now 0. And I have done this assumed these three which means I have taken the fluid from state 1 which has some velocity to allocation where the velocity is 0 isentropically with no heat and no shaft work.

Thus two is stagnation point, which means this is my zero condition. So, if 2 is the stagnation then I can replace the equation as this T 1 by gamma minus 1 plus v 1 square by 2 equals gamma R 0 by gamma minus 1 plus 0 because now v 2 is 0, so gamma R T 1. So, I multiply this equation with gamma minus 1 v 1 square is mach number into gamma R T 1 root of square of that. So, this would be v square is gamma R T 1 M square divided by 2. And I multiply this with gamma minus 1, let it be here for the time being correct. So, I will rub this.

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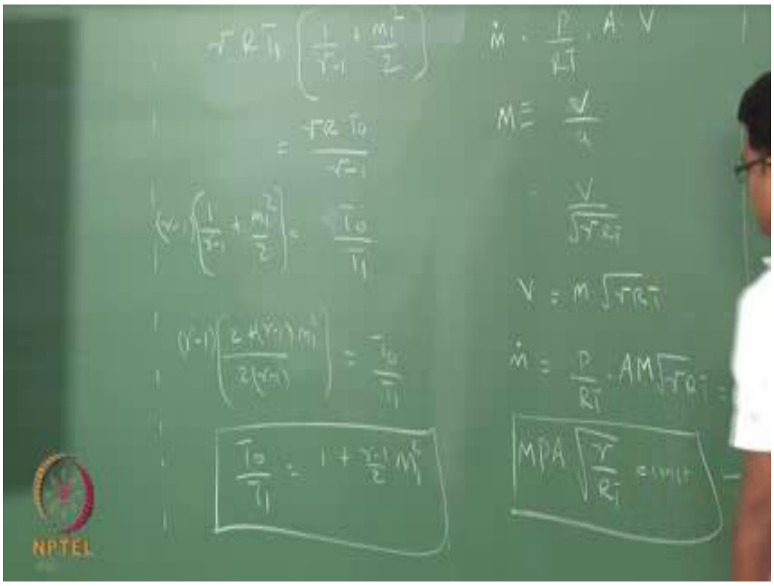

So, I take gamma R T 1 outside. So, this would be 1 by gamma minus 1 plus M 1 square by 2 equals gamma R T 0 by gamma minus 1 which I can rewrite as 1 by gamma minus 1 plus M 1 square by 2 equals multiplied by gamma minus 1 equals T 0 by T 1, which is. We just now 2 plus gamma minus 1 and M 1 square gamma minus 1 equals T 0 by T 1. So, my T 0 by T 1 is nothing but 1 plus gamma minus 1 by 2 into M 1 square.

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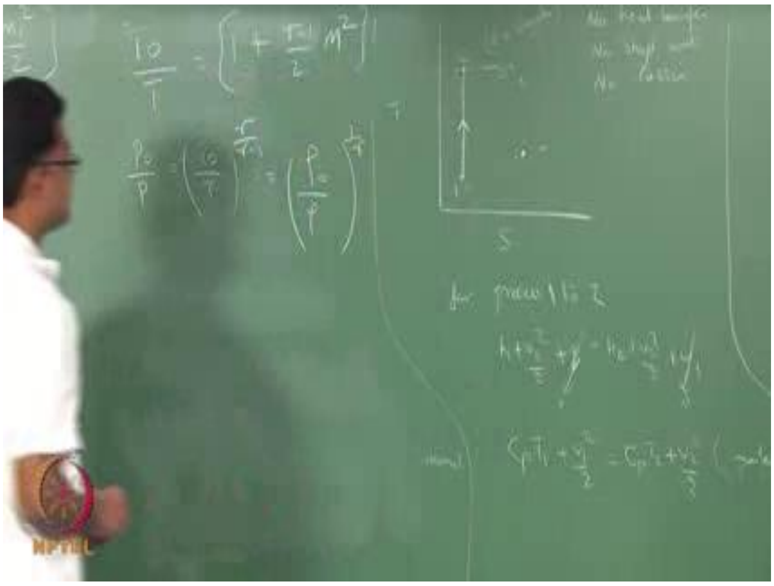

So, from process 1 to 2, if 2 is my stagnation quantity then my static temperature and stagnation temperature is related to the Mach number at state 1 using this equation. So, likewise, for any process, there is an associated stagnation point and if I know the Mach number of the state, I can get the stagnation quantities using the stagnation temperature using this relation.

For any process, I remove the suffix, so stagnation temperature is given by this relation. Now this process is isentropic, which means my P 0 by P is related to T 0 by T as gamma-by-gamma minus 1, which is related to rho 0 by rho as 1 by gamma. So, if I know these ratios, I can get the other ratios from if I know the mach number.

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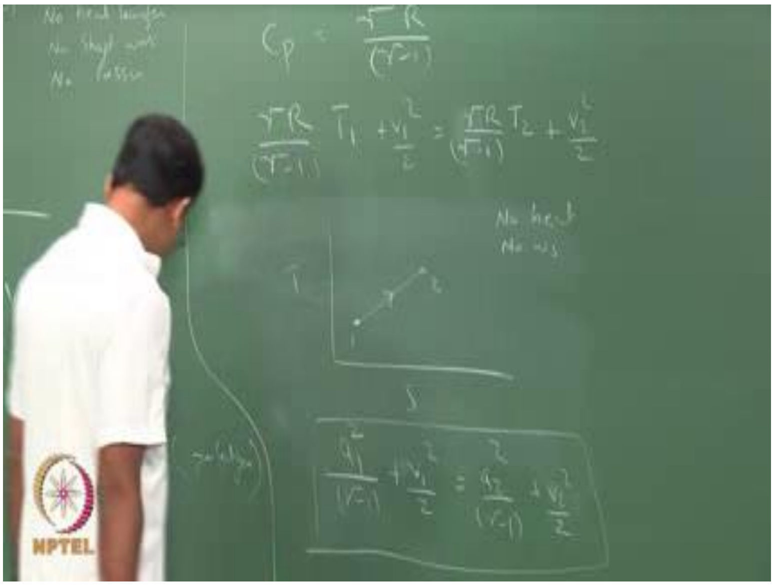

Now, I go back to this equation, which we have written a few minutes earlier, between process 1 to 2, between any process 1 to 2. So, I have T S a process 1 to 2, no heat, no w s, there is some dS change. So, between process 1 to 2, if there is no heat and no shaft work the stagnation enthalpy is same which is what we have written here. And I can reduce it to this form where now gamma R T 1 as my a 1 square by gamma minus 1 plus v 1 square by 2 equals a 2 square by gamma minus 1 plus v 2 square by 2.

So, if I know my sound velocity that is associated with state 1 then I have this relation.

So, it is a kinetic energy of the velocity with the velocity fluid velocity, this is a kinetic energy with the sound velocity, and you can equate that if the stagnation enthalpy is same that happens when there is no heat there is no work.

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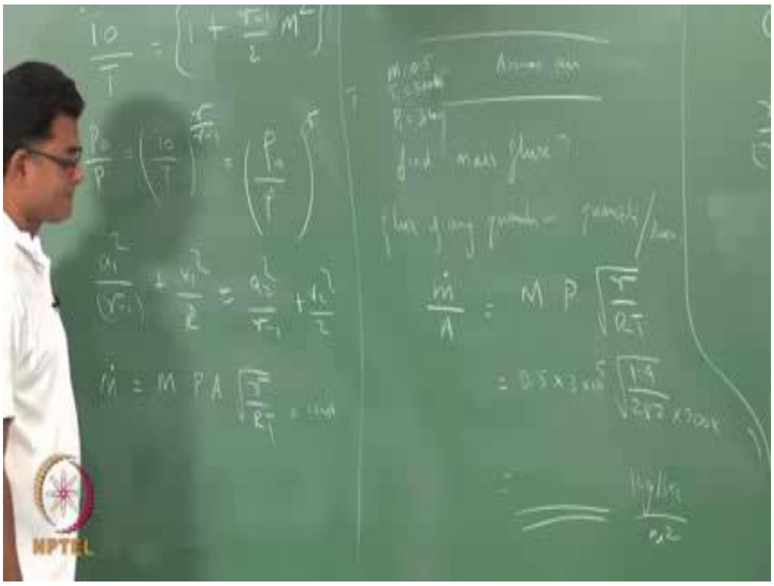

So, the equations we learnt today I will write it down here. It is already here. I will add it along with this a 1 square by gamma minus 1 plus v 1 square by 2 equals a 2 square by gamma minus 1 plus v 2 square by 2. So, there is a correction here, this is run out by rho to the power gamma. So, now to demonstrate the power of these equations, we will do a couple of problems.

The first problem is I have a depth location 1, my mach number is 0.5, static temperature is 500 Kelvin, my static pressure is around 3 bar, mass flux. So, what is mass flux, flux of any quantity is that quantity by area. So, mass flux is my mass flow rate by A. So, the equation we derived the early part of lecture is M dot equals constant is M into P into A by root of gamma by R T, which is constant. So, you use this equation to find your flux.

So, it is a mere substitution you know the mach number, you know the pressure, A has gone now that side. So, this would be root of gamma by R T, so that is 0.5 into 3 into 10 power 5 root of 1.4 divided by 287 if I assume air. So, I assume air into 500 Kelvin that would be your kg per second per meter square. So, it is a mere substitution of the equation which we are seen, but this is what you should understand. I have a constant area duct, I have a mach number, static temperature and pressure, my mass flow rate is now related to these quantities. So, if I measure static properties or derived static properties like this, I can get my mass flow rate.

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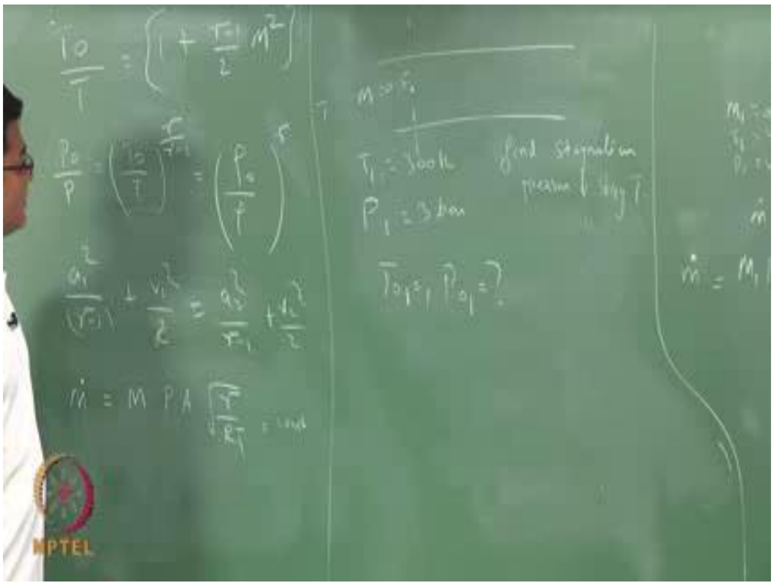

Now the same question if I have a different area, so my area is now section 1 has 20 centimeter square, this has 40 meter square. So, I have M dot equals constant, question may be to find your mach number at section 2. So, you could relate your M 1 P 1 A 1 into root of gamma by R T 1 equals M 2 P 2 A 2 multiplied by gamma by R T 2.

If I know the area at two locations which is given, and if I know my M dot which you have already obtain from this relation if I use the same numbers there then I can easily find my Mach number 2, if I know the T 2. So, if I use the same numbers, so my M 1 is 0.5, T 1 is 500 Kelvin, P 1 is 3 bar then I find my M dot I should find M 2 if P 2 is given, T 2 is given. If my M 2 is M dot by A 2, which is given as 40 centimeter square, it is 40 into 10 power minus 4 divided by P 2, 287 T 2 divided by 1.4 that would be your M 2.

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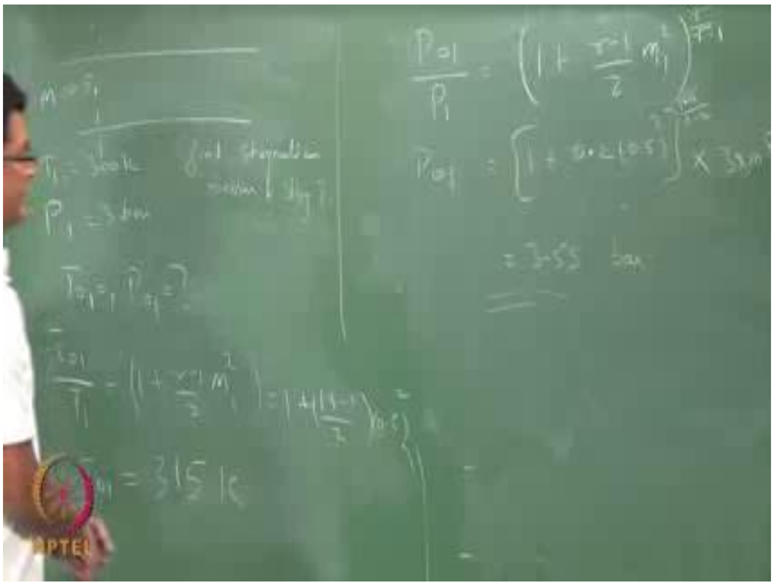

I have a duct at some point here, I have M equals 0.5, T 1 is 300 Kelvin, P 1 is 3 bar, find stagnation pressure and stagnation temperature which essentially means find T 0 1 and P 0 1. So, is the again a mere substitution. We take this relation T 0 1 by T 1 equal 1 plus gamma minus 1 by 2 into M 1 square which is 1 plus 1.4 minus 1 by 2 into 0.5 square, so that would be your T 0 1 which is how much?

Student: 315.

315 Kelvin; so P 0 1 by P 1 is 1 plus gamma minus 1 by 2 into M 1 square to the power gamma by gamma minus 1 which is 0.2 into 0.5 whole square into 1.4 divided by 0.4 into static pressure is 3 bar. So, this would be 3 into 10 power 5 Newton per (Refer Time: 26:09) square, 3.55 bar.

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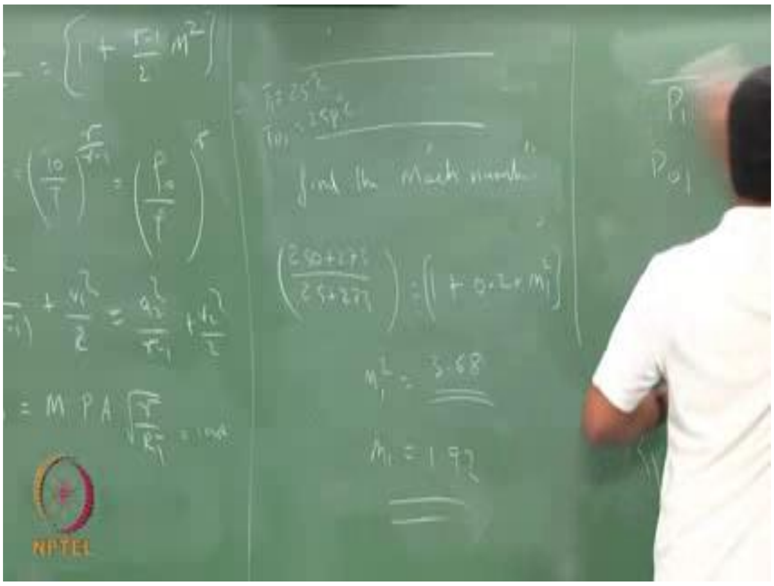

If I reverse this question, the static temperature is 25 degree Celsius and stagnation temperature is 250 degree Celsius, find the Mach number. How do we do this, again use the same relation 250 plus 273 divided by 25 plus 273 equals 1 plus gamma minus 1 by 2, which is 0.2 into M 1 square. So, your M 1 square is now going to be something which is around 3.68 as per the calculation I have done, M 1 is 1.92. If I know these two quantities I can find the mach number. So, in the sense, if fluid is flowing from 1 to 2 and there is no heat, no shaft work my stagnation temperature is going to remain same.

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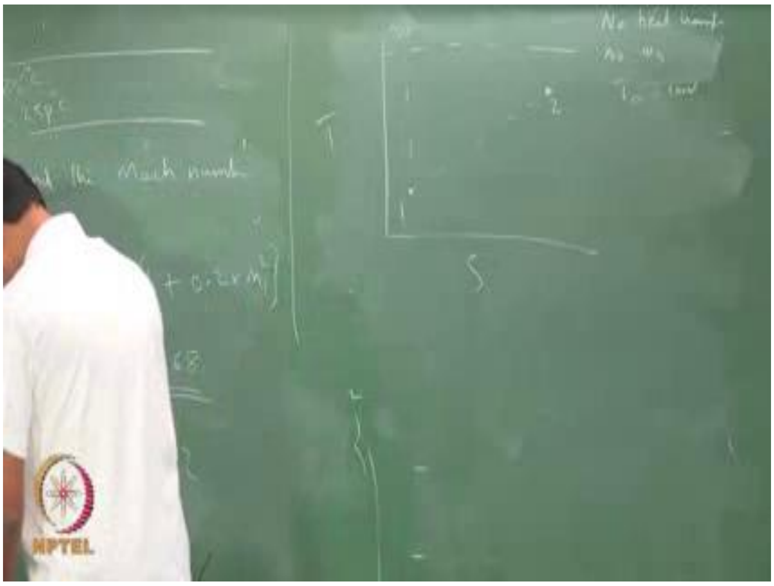

So, if I draw that in my T S diagram, so in this case, so from state 1 to state 2, there is no heat transfer, no shaft work which means my stagnation temperature is constant my stagnation enthalpy is constant. So, for all the state that is associated with this I will have the same T 0 1. Now if I can find my static temperature all along, I can find Mach number variation all along my concentrated duct. The measurement of static temperature is something, which we will discuss later, so that is the meaning of this question.

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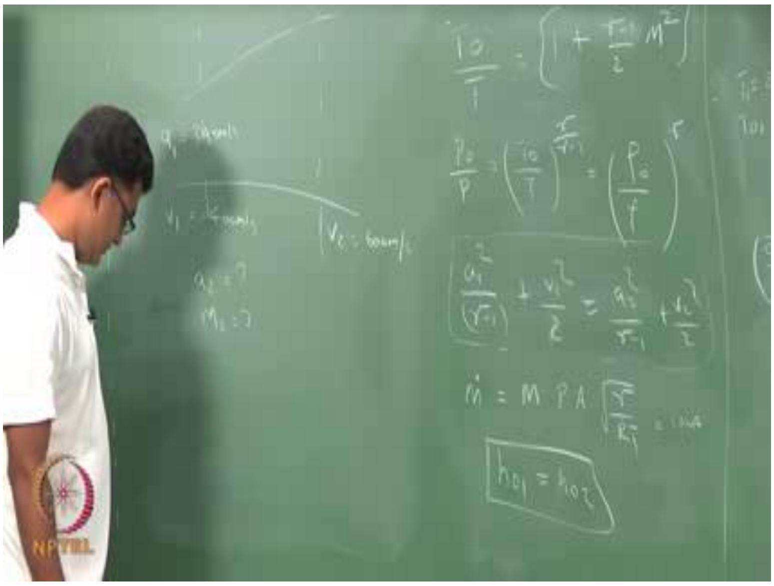

So, put the question in a different way, if I have a 1 to be 340 meters per second, v 1 to be 400 meters per second; and if I measure the velocity at section 2, it is 600 meters per second. So, what is my a 2 what is my M 2. So, it is again this relation, all these relations come from the energy equation where we have assumed no heat, no heat transfer, no shaft work and we have derived we have seen that the stagnation enthalpy is the same at any two sections.

We use this relation which is derived from that equation which is the essential the equation is this, everything comes from your combination of Reynolds Transport equation with the second law of thermodynamics. So, everything has been reduced to this form from there, so that is our parent equation. So, now this is another case where we can use that equation; there is no heat transfer there is no shaft work. So, you could use this relation where we know a 1, where we know v 1 where we also know v 2. So, we can find a 2. So, if we know a 2, we can easily find mach number v 2 by a 2.

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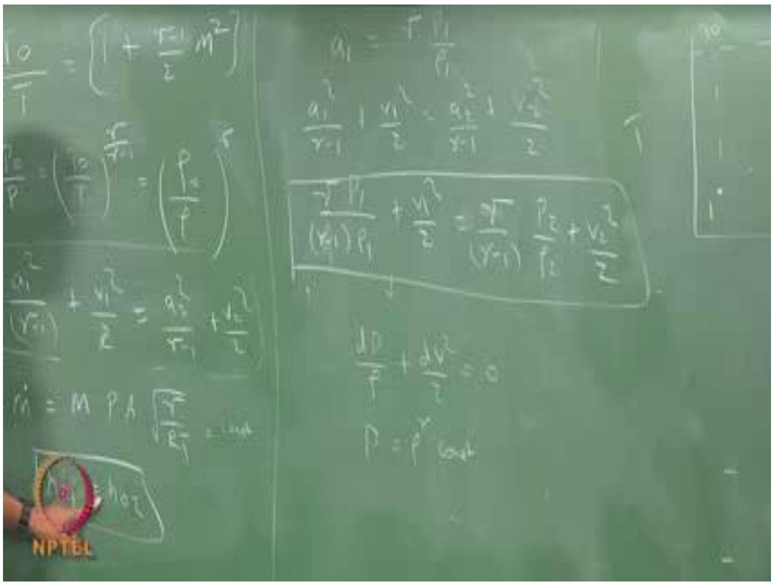

Now we will end this discussion with the different form of this particular equation which is now we are discussing. So, a 1 is gamma P 1 by rho 1 for an isentropic process which we had seen. So, I substitute that here. So, my a 1 I will rewrite that first. So, I substitute this equation. So, it is gamma P 1 by rho 1 plus v 1 square by v 2 equals gamma by gamma minus 1 into P 2 by rho 2 plus v 2 square by 2.

So, this relates your static quantities at section 1 and 2, which is originally derived from our integration of dP by rho plus dv square by 2 equals 0 from this equation we have integrated and obtain this for a isentropic process where P equals rho gamma constant. So, we obtain this equation integration of this to that. Now this is the same thing which we have obtained from this equation where the stagnation enthalpy is same; we obtain this and this. So, they are all connected. So, if density is constant and if it is you can reduce this equation to the Bernoulli equation.

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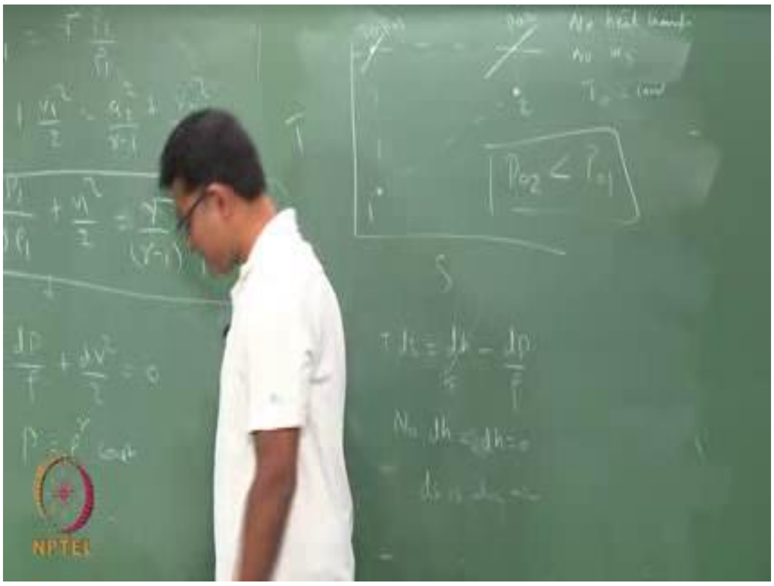

Now, so there is a stagnation pressure associated with these two. So, I have state 1 state 2 under growing what our process we are written no heat, no shaft work and associated with 1 there is a stagnation state, there is a stagnation temperature and at that point you also have a stagnation pressure likewise you have state 2 associated with that there is a stagnation quantity. So, the T the stagnation temperature of 1 is same as stagnation temperature of 2 because there is no heat, no shaft work, but there is a P 0 2, and there is which is different from P 0 1.

Now how do we evaluate these 2? So, if I look at the Gibbs relation my d h or T ds equals dh minus dP by rho no heat, no shaft works. So, we have already seen d h is 0. So, your dh is 0. Your dS should always be positive, so to maintain this the only way you can have is dP should be negative, which means my P 0 2 should be always less than P 0 1. So, this pressure P 0 2 is always less than P 0 1.

With that small note, I will end this lecture which essentially we have combined what are we have learnt in the previous lectures, the concept of stagnation, the mach number relation, the sound velocity all into the continuity equation, the momentum mass consideration equation and the energy equation. And we are done a few problems to demonstrate the power of these equations.

Thank you.