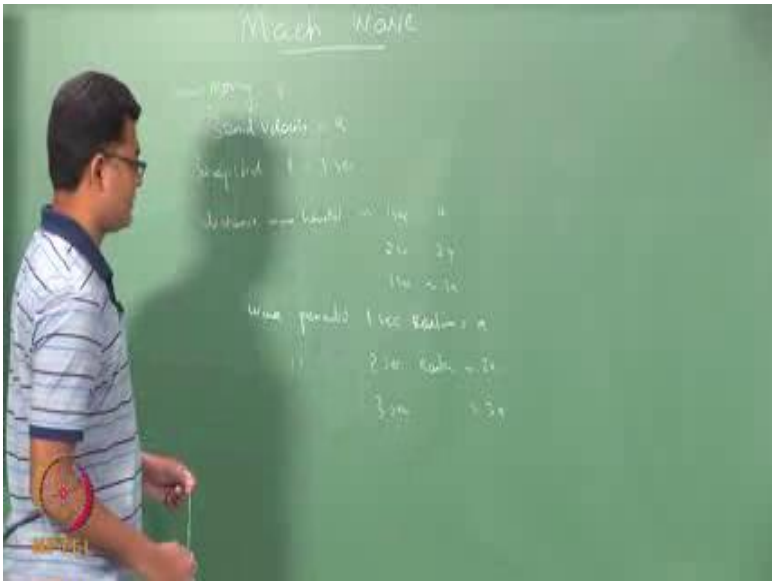
Fundamentals of Gas Dynamics
Dr. A. Sameen
Department of Aerospace Engineering
Indian Institution of Technology, Madras

Week - 04
Lecture – 12
Mach wave

In the last lecture, we have seen how the wave propagates when there is a disturbance in the flow. So, if I have moving projectile or any bluest body that is moving the air this is going to disturb the flow. So, the projectile or the body is going to push the fluid outside the surface, which is going to propagate as pressure wave. The propagation of the pressure wave is what we see as sound wave propagation and we have seen how the wave propagates when the fluid is incompressible, they were the pressure ways of very small you have a symmetric concentric circles which we had shown to be the propagation of the wave.

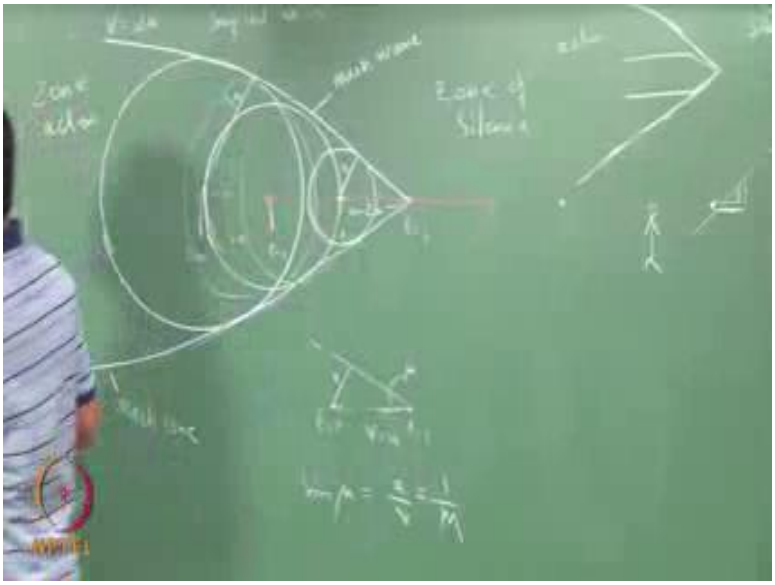
In subsonic flow, you have wave front that is moving a heard of the body and in sonic case where the mach number is 1 you had seen the project the pressure wave as well as the projectile reaches any given location at the same time, there is no information that is given to the in measure point ahead of the projectile reaching that point. We will discuss something more on the supersonic case. I will write to redraw whatever we had drawn in the other lecture. So, far the discussion today is on mach wave. We have assumed the disturbance is created by a point source.

(Refer Slide Time: 01:44)



So, the point source is moving with the velocity v and the sound velocity; sound speed is a, and we take a snapshot at t equal to 3 second. Now, we tried to find what is the evaluation of these sound waves that is generated few second earlier to the snapshot point? So, the distance to the wave travelled in 1 second is a, in 2 second is 2 a, in 3 second is 3 a. So, we will see what happens to the wave, that is the wave generated 1 second earlier would have travelled distance and wave generated 2 seconds earlier could have travelled 2 wave distance and wave generated 3 seconds earlier would have traveled 3 a. So, we will see how the how this has been evolved? How we have arrived at something called mach cone in the last class?

(Refer Slide Time: 03:51)



So, I have a point here snapshot at 3 and velocity is 2 a. If that is 2 a, the velocity are the wave that as generated 3 seconds earlier would have reached 3 a distance, the wave that is generated 3 seconds earlier is 3 a distance and the location of the wave after 1 more second is 2 a. So, we have 2 a, this source has moved a distance of 2 a, this is 2 a, this is t equal to 0 this is a t equals 1. So, this is the location where the source would produce a wave which is 2 seconds earlier than the snapshot point. So, the wave that is generated 2 seconds earlier would have traveled 2 a distance, and in the next 1 second the source would have moved another 2 a distance which is somewhere here, and this is t equal to 2 which is 1 second earlier than our snapshot point. So, that would traveled distance of a.

So, the wave that is generated 1 second earlier would have traveled a distance a, so that is this and in the next 1 second the source would have moved a distance 2 a again. So, that is at t equal 3, the sources moving along this line. So, the wave that is generated at t equals 3 that is also our snapshot point, this will stay there at the source itself, it does not move out because that is exactly the point and which we are taking the snapshot.

So, we have infinite wave that is generated in between and we are considering only these 3 at present. So, the source is much ahead of your wave, we have also drawn the tangent of all those waves that is generated and we obtain something called a mach cone. So, at

this tangent whatever wave that is generated in between the infinite wave that is generated in between would come and hit this particular chord. So, there are infinite wave that is generated as you move from t equals 1 to t equals 3. So, this particular location or they edge of this cone is the collection of all this waves.

This edge of this cone is typically called as your mach wave and if you look at this geometry, this is 2 a distance in 1 second. So, last 2 points and the radius of the circle is a, and this 2 a is your v. So, I redraw that this is a t equals 2, this is t equals 3, this distance is v which is equals 2 a and it touches the mach cone at a distance a, and if I call this angle as mu then I can write my sin mu as 1 by m or I will write this as a by v or 1 by m.

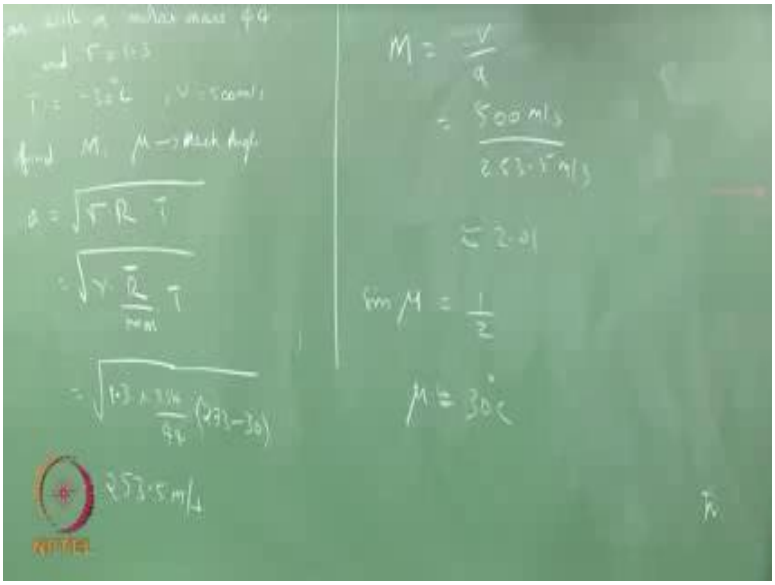
So, if I have projectile that is moving with the some mach number, I would have a cone that is generated like this which is the collection of all the waves that is coming and hitting. You have also seen that this inside the cone as the so called zone of action outside the corners zone of silence, meaning any point here would not know about the approaching projectile. The point will know about or hear about the projectile after the projectile as passed, only if only when the point inside the mach cone you would have you the point would measure something or point would hear something.

So, this edge where you call it has mach cone is actually the collection of all these. So, if I have a bullet that is passing, bullet that is moving in the air at supersonic speed, this would generate a cone like this, where outside cone I have silence zone of silence and inside I have zone of action and this edge of the cone is where all these waves might have collides. So, a person is standing here or measuring probe that is kept here will pick from signal only where at this goes into this.

If I assume that there is no movement here with the signal here is 0, this probe will pick some signal only when there is wave moves in here and this signal would be maximum when it crosses the mach cone because that is the collection of all your waves. So, when it reaches somewhere here, when the wave when projectile moves here. This is the inside the mach cone and hence they will hear or at the change in pressure at this particular point.

So, there are 3 things to understand, 1 the projectile moves ahead of your pressure wave or the projectile moves, the projectile velocity is larger than your velocity sound. So, the pressure differences will not be felt ahead of your body at this point 1. Point 2 is that outside the cone its known silent there, you will not know about the upcoming projectile till you are inside the cone. Third; the signal or the effect of pressure variation would be maximum at the edge of the cone because that is where all your mach, all your signal collies and the edges the so called mach wave. At the mach wave you will have maximum variation. So, that is essentially what we need to talk about the mach wave. Now, that brings in some interesting phenomenon which we will discuss in the form of few problems.

(Refer Slide Time: 13:02)



Question I have is; a gas with a molar mass 44 and gamma equals 1.3 is at temperature minus 30 degree Celsius and velocity 500 meter per second. Find your mach number which is quite straight forward and once you have the mach number you can find the cone angle which is also called as your mach angle? So, this is your mach angle. So, the question is straight forward. So, how do you find velocity of sound? Velocity of sound is gamma r t gamma into r bar by m into t. So, this is 1.3 into 8314 divided by 44 into t is minus 300. So, that is 273 minus 30, this is 253.5 meter per second.

So, that is the velocity at which the gases, the velocity of sound at this particular situation, the state t equals minus 30 and the velocity is 500 meter per second and in that state you have velocity to be 253 meter per second. If that is the case, what is the mach number? Mach number is v by a, which is 500 meter per second divided by 253.5 meter per second, which is approximately 2.

Student: (Refer Time: 16:15).

Approximately 2, if that is a case, your mu is sin mu is 1 by 2. So, your mu is approximately 2. So, this would be approximately to 2. There is no projectile here, but we have defined a mach angle associated with it. So, if I have a gas that is flowing at minus 30 degree Celsius at a velocity 500 meter per second, the associated with this there is a sound velocity because we have defined sound velocity and there is a velocity of gas that is defined already or given t already, there is a mach number.

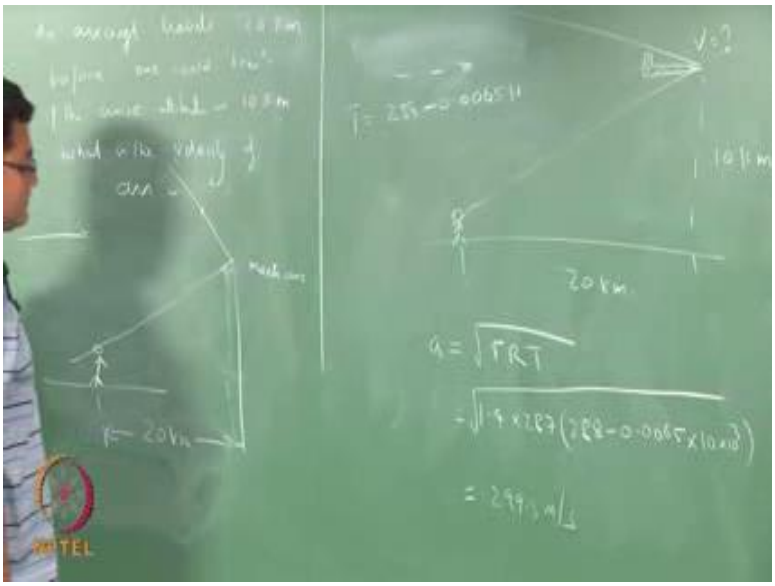
(Refer Slide Time: 17:07)

So, with every mach number you can define a mach angle. So, even though there is a no there are no projectiles here, we have defined the mach angle because all these particles or all these walls of the tubes would give you a pressure variation because it coming and heating this would change the pressure. There is a pressure gradient and that pressure air

is the sound velocity compare to that sound velocity there is a mach angle. So, for everywhere there is a mach angle. So, it is always that even though there is no projectile. So, the impression that you need a projectile to a mach angle is wrong that is the phenomenon that is going to be there in the supersonic flow.

Now, we will do another question related to this, travels 20 kilometer before, if the gross altitude is 10 kilometers, what is the velocity at which of air craft?

(Refer Slide Time: 18:30)



When air craft flies and it travels 20 kilometers before I could hear, what is the velocity of aircraft? I am standing here, aircraft is flown 20 kilometers passed me, but this point I could here when aircraft has flown 20 kilometers after me I could hear the sound. So, the mach cone that is generated has just reached me, this is my mach cone. So, mach cone just reached me when it is 20 kilometers. So, it is the vertex should be here, this is my 20 kilometers I will redraw this here.

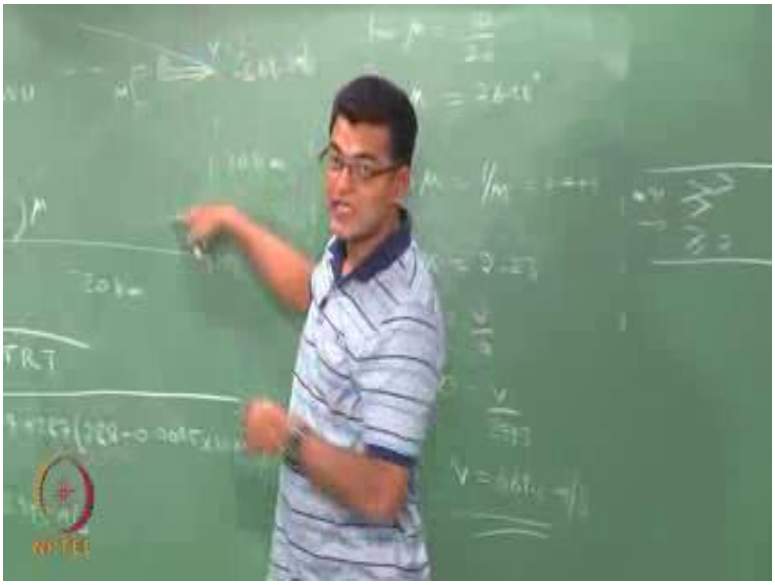
This is my aircraft. So, aircraft has travelled the 20 kilometers before I could hear. So, the height at which aircraft flies 10 kilometers. So, what is the velocity of the aircraft? First let us find the velocity of sound? Velocity of sound is gamma r t that could 1.4 into 287, t is 288 minus 0.0065 into the altitude; altitude is 10 kilometers, 10 into 10 power 3.

So, we have used the temperature to be of the form this 288 minus 0.0065 multiplied by the altitude. So, the temperature here is different from what you have at 3 levels; this is an example which you have seen in the earlier lecture. So, use that relation to find the temperature and get your velocity of sound, this is.

Student: 299.3.

299.3 meter per second. So, if this is the velocity of sound.

(Refer Slide Time: 23:21)



Now, I can find this angle mu which is this angle mu. I know tan mu is opposite's side by edges inside which is 10 by 20, mu is sin mu is 1 by m.

Student: (Refer Time: 23:59).

26.56 degree.

Student: (Refer Time: 24:10).

0.447, and m is the inverse or that is around 1.

Student: (Refer Time: 24:22).

2.23. Now, we have velocity of sound, we have the mach angle, from the mach angle you can find the mach number and hence the velocity of aircraft is 2.23 in v by 2993 meter per second. So, v is.

Student: (Refer Time: 24:53).

6.

Student: 669.2.

669.2 meter per second. So, this aircraft is moving at a velocity of 669.2 meters per second at a height of 10 kilometer. So, likewise you can find, if you know that time differences between these 2 points, you can also find the velocity of aircraft. So, the aim of this exercise is to understand mach wave and mach angle, how it is useful in computing the quantities which I have just shown, and the presence of projectile is not necessary to have a mach angle. If there is a supersonic flow you would definitely have a mach angle.

So, in a supersonic internal there are mach angles at all points. So, if I have a cross section, if this is m greater than 1 there would be mach angles all the ways, everywhere there would be all those points would create a phenomenon that which we are just say, but in this case its stagnant air and there is a projectile that is coming. So, this is creating the supersonic wave and hence you have 1 mach wave. So, we have seen how the propagation of pressure wave generates new phenomenon in a compressible flow, where you have a pressure variation which is given as the velocity of sound which generates the phenomenon of mach waves and mach angles.

Thank you.