Fundamentals of gas dynamics
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Week - 03
Lecture - 8.1
Discussion on velocity of sound and Mach number

What we had seen in the previous lecture is the Definition of Velocity of Sound. How a pressure change is going to give you a wave front that is moving with a velocity which we had seen to be the velocity of sound. So, we have equated the changes in pressure with density at constant entropy of P to be the velocity of sound, which is also equated to gamma R T, for an isentropic process.

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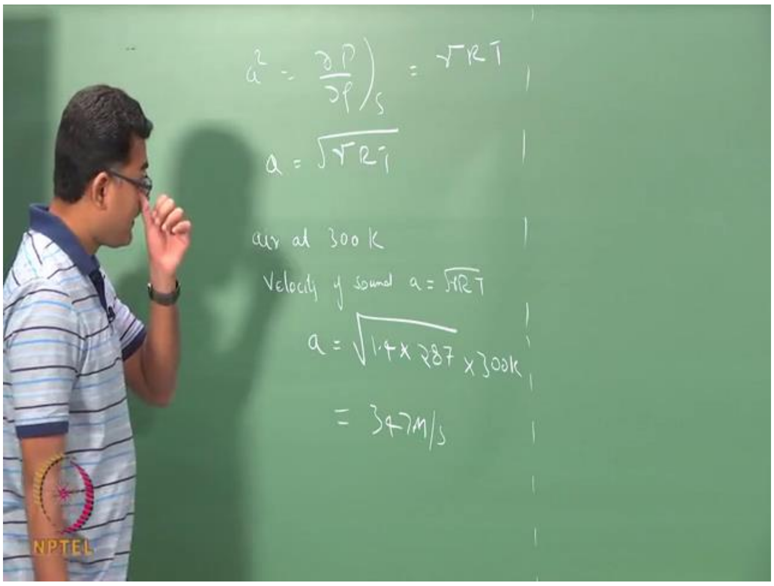

We will see that how this is different in different fluids. Say for example, if I have air at 300 Kelvin. What would be the velocity of sound? Meaning if there is a small change in pressure, this is going to create a wave front and that wave front is going to generate a sound of the order of this. So, velocity of sound is root of gamma is 1.4; R is 287 multiplied by 300 Kelvin, so this is around 347 meters per second.

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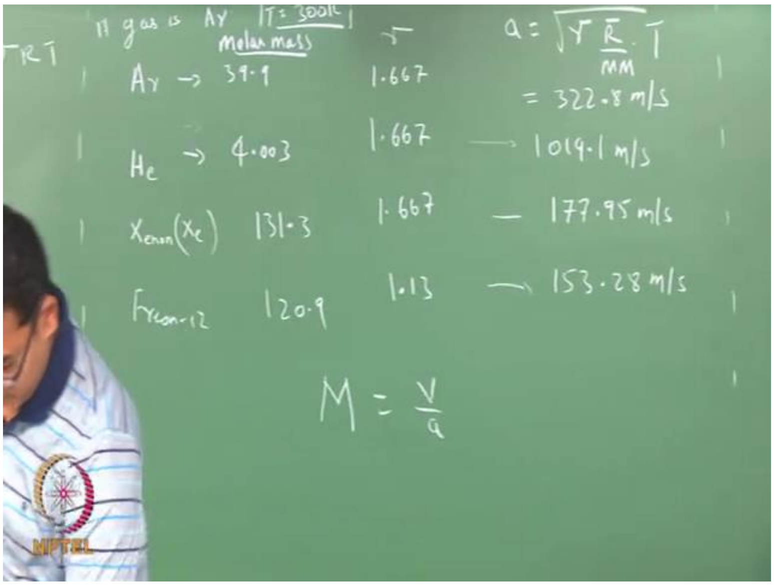

Now, if the gas is something else, the gas is Argon. Argon has molar mass of, argon has the molar mass of 39.9 and the gamma as 1.667. In this case, a is root of gamma into R universal bar by molar mass into temperature. So, this R I have taken here is the specific gas constant, this is the universal gas constant. So, R bar by mm is the specific gas constant for argon.

So, this is going to be around 322.8 meters per second. Let us look at few other gases, again the temperature is 300 Kelvin, at 300 Kelvin we are comparing the velocity of sound for different gases. So, helium the molar masses are 4.003 and gamma is again 1.667. So, your a is with this value substituted in this equation, the value is 1019.1 meters per second. If it is a gas called xenon, xenon you typically use X e. The molar mass is 131.1 and gamma is again 1.667, this would give me a speed of 177.95 meters per second.

If I have Freon-12 gas the molar mass is 120.9 and this speed ratio of specific heat is 1.13, gives me a speed of sound to be 153.28 meters per second. So, this exercise is just to give you an impression on how the velocity of sound varies in different gases. So, associated with these gases you are depending on the speed of the gas, your mach number would be hugely different.

If I define my mach number to be v by a, if I use small value of a for the same value v - I would get a large mach number. So, if I want to generate high mach number and if I can use Freon-12 then I would get a much larger value as instead of a air where the velocity is around 350, here it is 150. So, that is a small exercise on velocity of sound we have few more exercises. Now, I am going to see, on what conditions where the velocity of fluid can reach mach numbers of the order of 1 and m power. So, I have a Propeller, Aircraft propeller working at 20 degree Celsius, diameter of the propeller is 2.5 meters 2.5, we are taken the value to be 2.5 meters.

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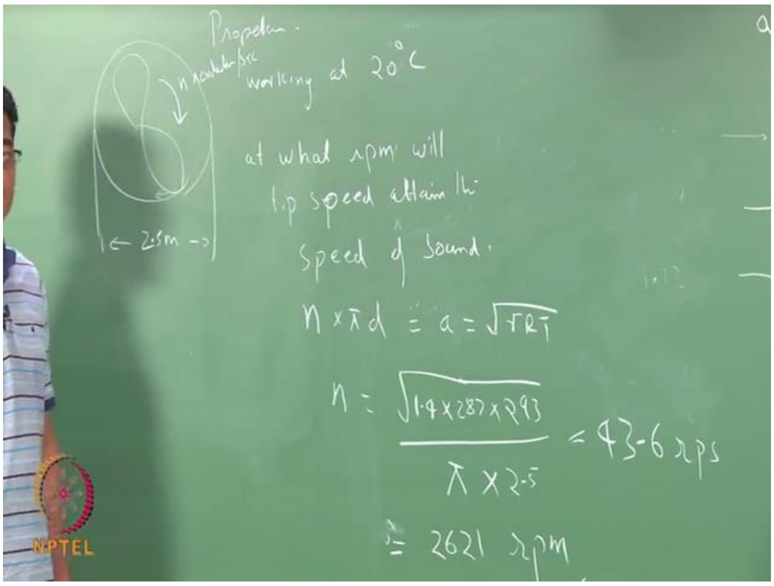

The question is - this is rotating at some rpm. The question is at what rpm will, the tip speed attain the speed of sound. So, I will explain the question I have a propeller here that is rotating at some rpm and at what speed or at what rpm will the tip speed reaches the velocity of sound at that particular temperature. So, what is the tip speed if it is n evolution per second? So, if it is n evolution per second tip speed is nothing but I into d.

If the speed reaches, the velocity of sound reaches gamma R T. The question is what is the value of n that is a very simple substitution; just that you have to understand the question appropriately. So, in this 1.4 into 287 into 20 degrees Celsius which is 293 Kelvin divided by pi into 2.5 meters. So, this would be your RPS which is around 43.6

evolutions per second. So, your rpm is 2621 rpm. So, when the propeller reaches this rpm, the tip velocity reaches velocity of sound or the tip velocity reaches mach number 1. So, your tip mach number is v by a is 1, if n is 2621 rpm.

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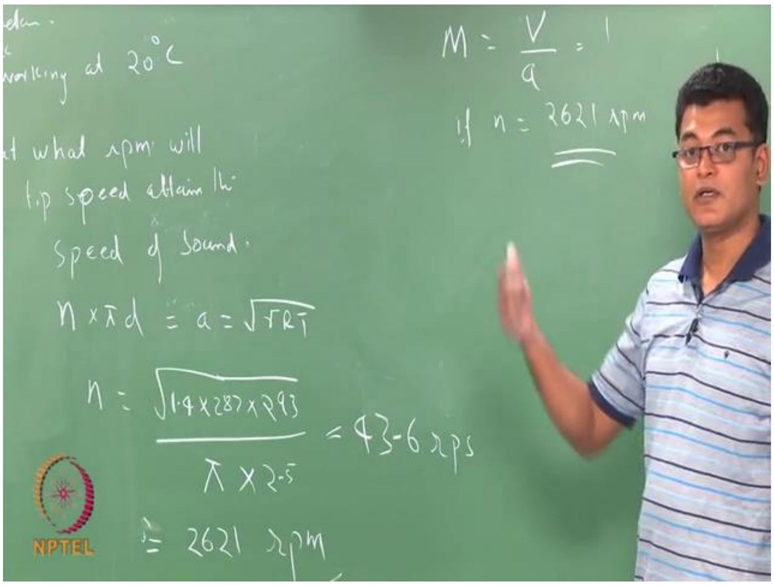

Now, we have seen the speed of sound changes with temperature which brings in an interesting scenario in atmosphere where as we move up in the altitude the temperature goes down. So, your velocity of sound at that particular location would also be different. If the temperature varies with some linear dependence on the altitude then your velocity of sound should also vary accordingly, so from the sea level when you go up the t decreases.

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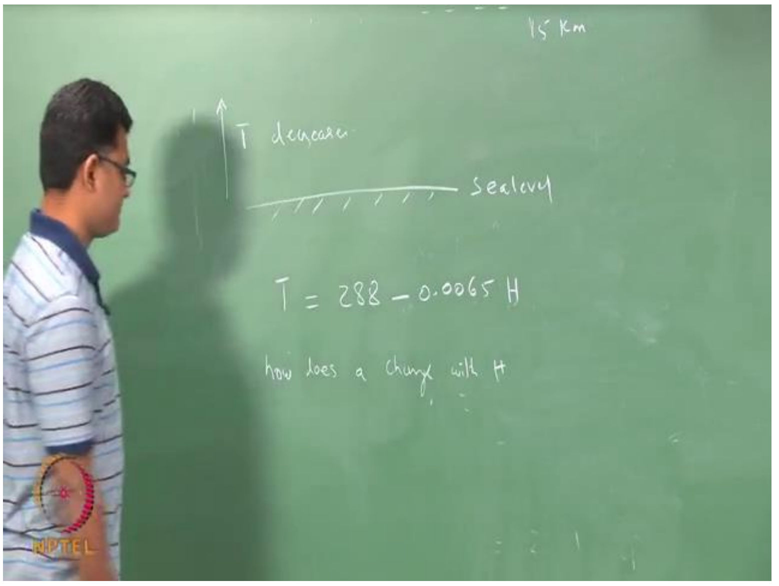

So, in the lower atmosphere which is up to around 15 kilometers, you can assume your temperature changes as this function. So, depending on the value of h from the sea level here temperature changes according to this equation.

So, the question is how does a change with h, a is root of gamma R T, it is root of gamma R 288 minus 0.0065 H.

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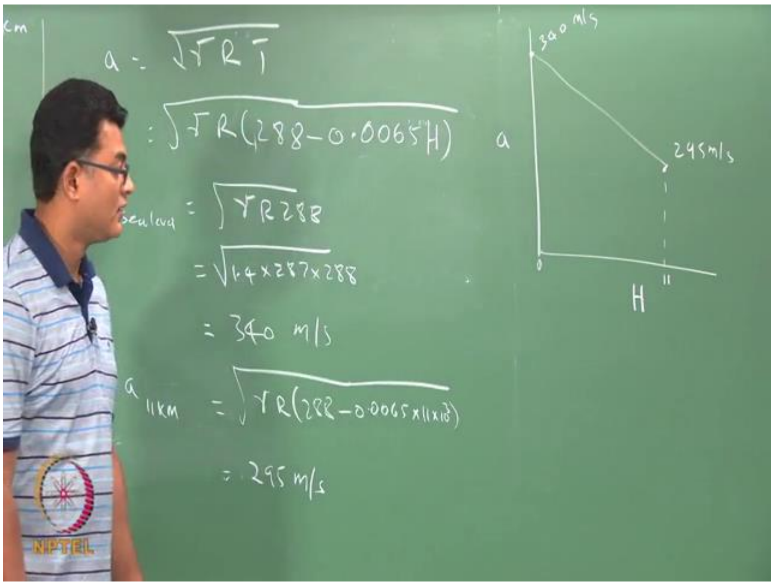

So, a at sea level is nothing, but gamma R 288 which is 1.4 into 287 into 288 is around 340 meters per second. But, a at 11 kilometers is root of gamma R into 288 minus 0.0065 into 11 kilometers which is 11 into 10 power 3, bracket close. So, this is approximately 294 point, approximately 295 meters per second. So, if I draw the variation of a with H at 0 H which is at sea level, my value of a is sound velocity is something around 340 and it drops like this to a value of 295 meters per second which is meters per second, this is 11 kilometers, this is 0 kilometers.

So, as you go up the velocity of sound also goes down because your temperature goes down. So, for the same velocity an aircraft flies at a higher mach number if it is at a larger altitude; which brings in the next question.

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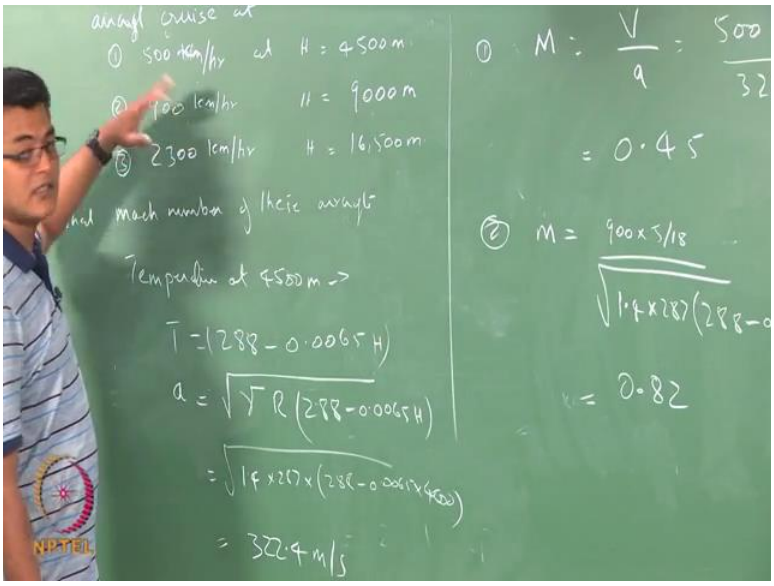

So, all these changes are due to temperature change. An aircraft flies at 500 meters per hour at a height of, OK. And 900 kilometers per hour H equals 9000 meters. Another aircraft flies at 2300 kilometers per hour at H equals 16,500 meters. What is the mach number? What is Mach number of these aircraft? Aircraft fly at cruise at. So, aircraft is cruising at a constant velocity of 500 kilometer per hour at the height of 4500 meters, what is the mach number? So, for that we should know the temperature at 4500. So, we will use the same relation where T is 288 minus 0.0065 H. So, the temperature at 4500 would give me value with which I have to compute my sound velocity. My sound velocity a is root of gamma R 288 minus 0.0065 times H, H is 4500 here so gamma is 1.4 into 267 into 288 minus 0.0065 into H is 4500, so the a is 322 meters per second.

Mach number is velocity by a, velocity is 500 kilometer per hour is converted to meters per second divided by velocity of sound which is 322 meters per second. So, this is mach number of around 0.45. So, the first aircraft is flying at a mach number of 0.45 at a height 4500, the speed is 500 kilometers per hour. Likewise, you can find the mach number of the second aircraft v is 900 into 5 by 18 divided by root of 1.4 into 287 into 288 minus 0.0065 into H is 9000 meters. So, this would give me approximately the mach number 0.82.

The second aircraft flies at a mach number of 0.82 at a height 9000 meters, if it flies at 900 kilometers per hour. So, the third one m equals 2300 kilometer per hour is converted to meters per second divided by root of 1.4 into 287 into 288 minus 0.0065 into H is 16,500; 16,500, this is approximately 2.3.

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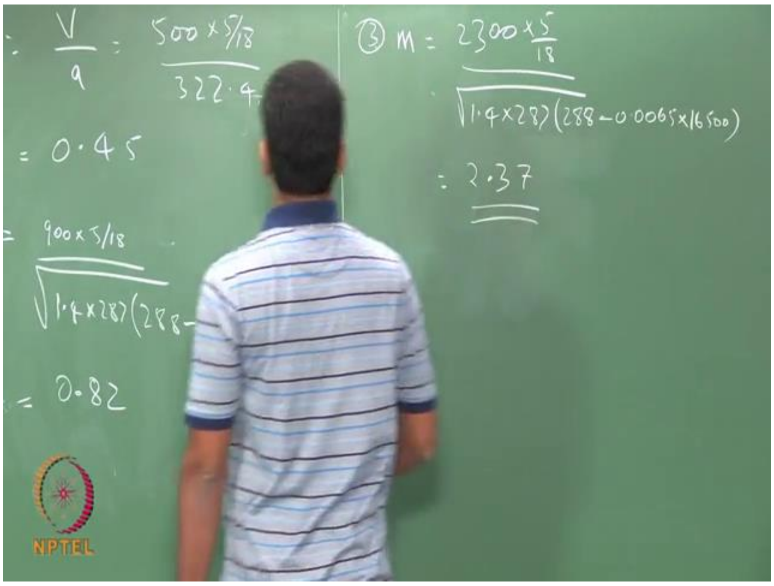

So, the third aircraft is cruising at a supersonic speed of 2.37 at a height of 16,500 if the speed is 2300. The aim of the exercise is to show the dependence of temperature on velocity of sound, and as we have seen the depending on the height of the atmosphere or the location where you are measuring the velocity of sound the temperature has a role to play and if at very high altitude with a temperature is very small and hence the velocity of sound is very small, but for a small cruise velocity you are going to have a large mach number. And also depending on the gases that we use the velocity of sound is different. Hence, the mach number that you can attain will also be different with different gases for a given speed of gas.

That actually ends the third week lectures and tutorials. So, the assignments should be based on the problems which we have discussed.

Thank you.