Fundamentals of Gas Dynamics
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Week – 03
Lecture – 07
Velocity of sound

So, this lecture is about the sonic velocity and the relation called Mach number, relation with the Mach number, the importance of mac number in the gas dynamic state, the Velocity of Sound.

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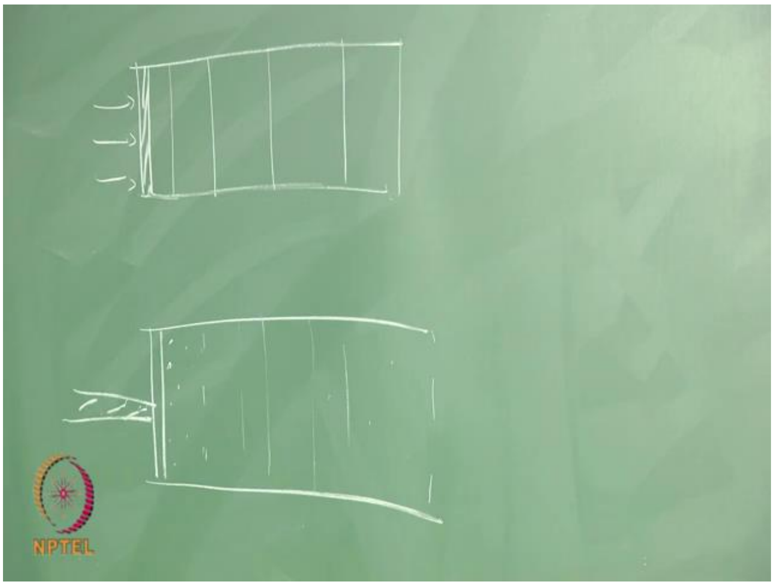

We are going to derive equations velocity of sound. We are going to derive an equation, which relates the velocity of sound to the changes in pressure and density. So, if I have small pressure (Refer Time: 00:57). So, if I clap my hand here there would be molecules; that is getting energized near my hand, which propagates further away from the source. So, this propagation is typically the wave that is forming.

So, if I clap it hard, so it would go fast, so if I clap it slow the waves would be going slow this pressure front this waves front that is moving is actually the velocity with which the waves front moves is actually the sound wave velocity. So, we are trying to derive this set of equations related to this kind of phenomenon. So, if I have a for example, if I have an elastic band and if I try to put a pressure here the region closer to this will be affected first, and then region further away would be affected slowly.

So, the information would pass in stages. So, there is a wave that is moving forward now instead of elastic band you can imagine a cylinder with a piston. So, the piston is slightly moving or pushing the fluid down. So, the molecules near the piston will get accumulated near the piston first, and then the molecules away may not even feel the piston, but if it is continuously being pushed the information passes through this by means of waves the movement of these waves are called the longitudinal waves the velocity of those waves are the velocity of sound through that particular medium. So, we are trying to derive the velocity of that wave which is your sound wave. So, if I have a piston that is moving with a velocity and pushing the fluid.

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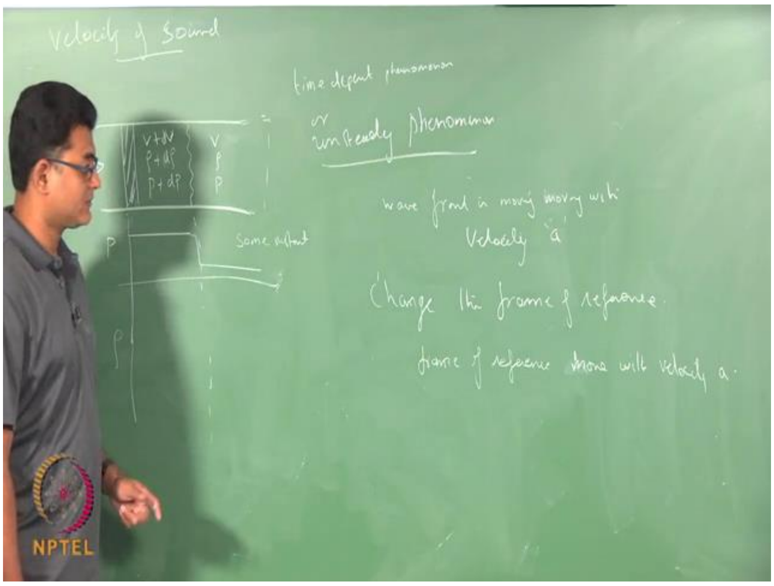

There is a wave that is created. So, if this is moving with velocity v, between the wave front and the piston the velocity of the fluid must have been something like v plus d v density would have been density plus 0 pressure would be p plus d p, and the fluid in front of the pressure wave front will not really know about what is happening before that. So, the here the velocity would be 0. So, the density would be just rho or p would be just p, velocity would be just the velocity which is already there if I assume this to be 0 this would be 0. Now, we are trying to find the velocity of this wave.

So, as it moves these changes. So, the region between the piston and the wave front keep increasing. So, the fluid that is not affected gets affected as time progress. So, if I draw the pressure at this instant the pressure would be higher in this then suddenly it drops to some other pressure. So, this is the pressure in front at the wave front, likewise you can also at this instant it is a snapshot of the process which we are going to discuss same with the case of density. So, density would be large here and then as the pressure front moves the density keeps transmitting there through the region where it is not affected by the piston movement.

Now, this is an unsteady process. So, it varies with time. So, this is a time dependent phenomenon or an unsteady phenomenon. Now, we need to analyze this unsteady phenomenon, one way of doing an unsteady phenomenon is, to if it is traveling with a uniform velocity one way of doing it is to transform the coordinate system. So, here the wave front is moving with a velocity v. So, if I stand (Refer Time: 07:08) if I stand outside and look at the board this is what I see, instead what I am going to do is I will move along with the wave front.

So, I change the frame of reference. So, I have a uniform velocity wave front. Now, what I do is I move along with the wave front and try to measure these quantities. So, that is what I would do. So, if the wave front is moving with the velocity let us put the velocity as a which is different from my d v. So, I would assume velocity a. So, I change my frame of reference, I am moving along with my wave front. If I am moving along with my wave front, what are the changes here? My thermodynamic quantity does not change only the velocity in relative velocity would be changing. So, if I move along with the frame of reference.

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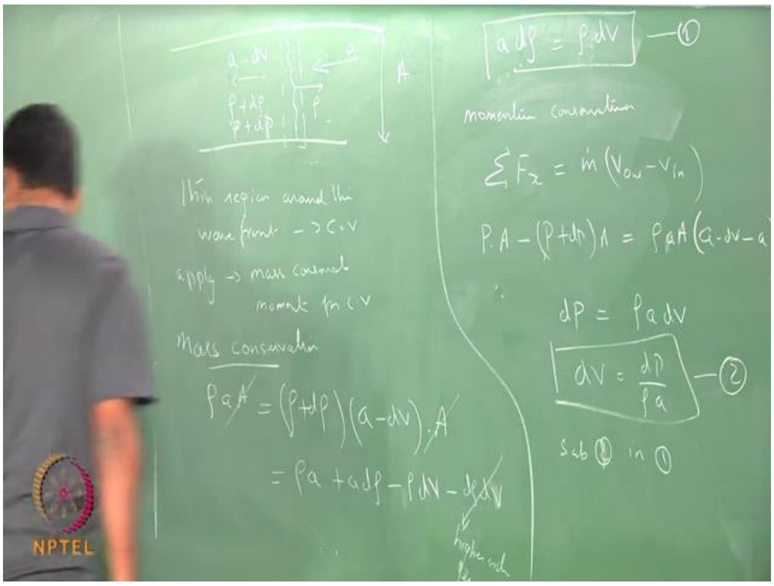

So, this front is moving with this front is the velocity or the way front is a. So, that is moving with velocity a. So, I would see the fluids here approaching v with velocity a. So, this would be my velocity a. What about this, I would see my fluids behind me residing with velocity a minus d v. So, I would face; I would see my fluids coming and hitting me at velocity a and I would see those fluids are those fluid particles are residing me at velocity a minus d v thermodynamic quantities does not change. So, my d plus 0 is same as previously before my change in my reference.

Now, what I do is, I take this as my control volume a thin region around the wave front is my control volume, apply mass conservation and momentum conservation for the control volume. So, if this area is A my mass conservation is rho a A equals rho plus d rho into a minus d v into A which is my area. So, I can strikeout A which is my area. So, it is a constant area control volume, and if I expand my right hand side its equals rho a plus a d rho a d rho minus rho d v minus d rho d v. I will also cancel out my higher order terms rho a and rho a cancels out.

So, I am left with a d rho equals rho d v. now, apply linear momentum conservation across the control volume. So, this is the one d flow. So, sigma f x equals m dot v out minus v in what is sigma f x the pressure force that is acting. So, this would be p into a minus p plus d p into a equals m dot s rho small a into a into v out is a minus d v minus a, p a p a cancels out. So, what is left is d p minus d p equals rho a d v. Now, I substitute this relation there eliminate my v d v. So, d v is d p by rho into a. So, this is my equation one this is my equation two. I substitute, two in one. I would get rho into d p by rho a, which is a square is d p by d rho.

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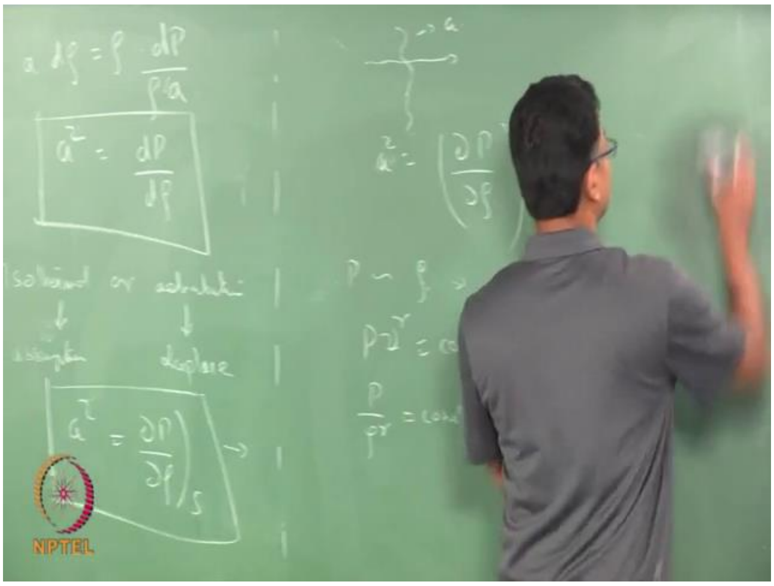

So, your change in pressure with respect to density is your velocity square. Now, this change can be isothermal or adiabatic or any other process. So, if it is isothermal, the assumption of isothermal process; for this was first introduced by newton which was proved to be wrong, but the adiabatic assumption was introduced by Laplace which is now known to be the true case. So, this equation is replaced as d rho by isentropic reversible adiabatic. So, it is an isentropic process which is called as Newton Laplace equation for velocity of sound, your d p by d rho.

So, if the wave front is moving with velocity a, then this a is given by this change, where this change requires a relation between p and rho. So, this relation p and rho if it is isentropic process we know that p (Refer Time: 17:55) p power gamma is a constant for an isentropic process. So, you could find the relation between p and density and get this derivative substitute there.

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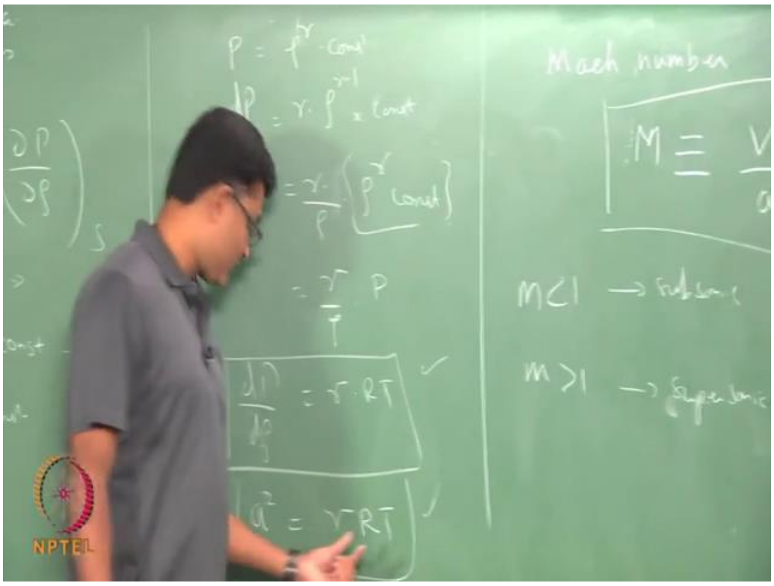

So, what is that? So, d p by d rho is rho minus 1 into some constant. So, I can substitute gamma minus 1 rho power gamma minus 1 is from this relation, I can take rho 1 by rho here. So, there will be rho power gamma constant that is rho power gamma is again p. So, this is p. So, p by rho is r into t. So, my d p by d rho is gamma r t, which means my a square is gamma r t, these relations.

So, if I have a wave front that is moving with velocity a in a gas the pressure changes due to density at isentropic condition is given as a velocity square of the sound velocity or the wave front velocity, which is related to your ideal gas equation r t in this particular form a square minus gamma a square equals gamma r t. So, these are very important relations, which is required for the further investigation of flow of gases, because any pressure change is going to create a sound wave change.

Now, once we are defined your velocity of sound, then how does it relate to the fluid velocity is what is described by something called the mach number m is defined as v by a. So, if m is less than one its sub sonic m is greater than one its supersonic flow. So, if I have a flow velocity at state one associated with that state one I have a static temperature t which is associated with the sound velocity at that particular point. So, for every v the a associated with that v at that particular state is what is given as your mach number.

So, a can be a some reference a or the static a the locally computed temperature or locally measured temperature, and you if and the temperature velocity of sound associated with that locally measured temperature is your mach number use to define your mach number.

In this lecture we are learnt the velocity of sound; how to find the velocity of sound given the pressure difference and the density difference and how does it relate to our ideal gas equations and using those equations, we have come across these two equations which will be used further.