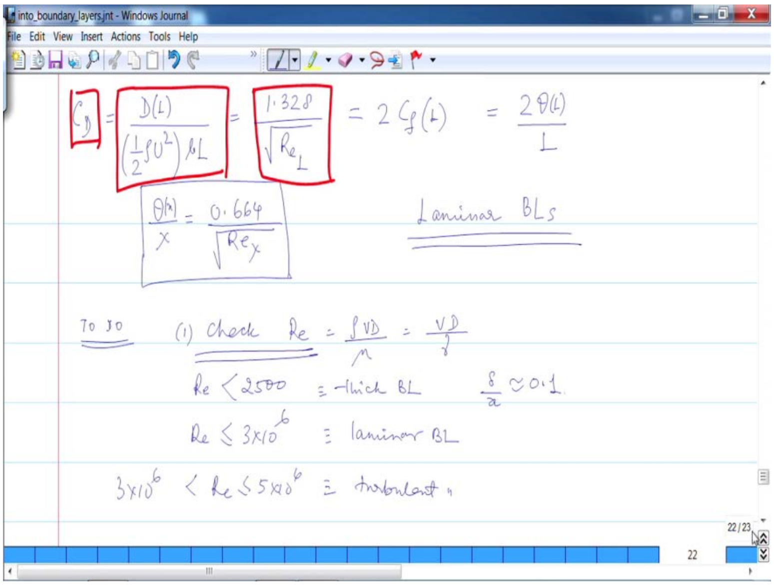
Introduction to Boundary Layers

Dr. Rinku Mukherjee
Department of Applied Mechanics
Indian Institute of Technology, Madras

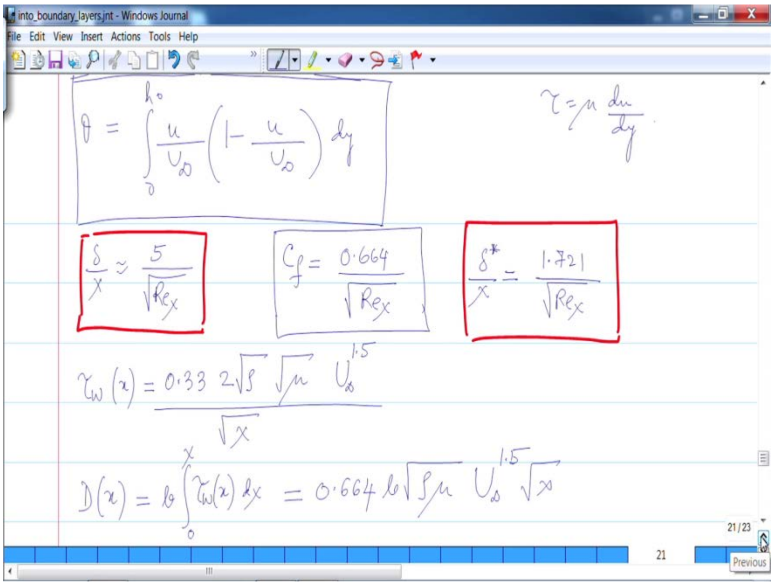
Module - 03
Lecture - 07
Control Volume Approach to derive expressions for over a flat plate

Hi. So, welcome again. We try to understand boundary layer displacement thickness and boundary layer thickness right in terms of; as a concept what exactly do you mean by that and also momentum thickness. We try to understand conceptually, what exactly that means trying to get a feasible feel for it and we also did a control volume analysis to get a quantitative measure in terms of the usual variables or usual parameters that will be available to us. For example, the velocity profile, the size of the plate on which the flow will be moving and so, let us go and revisit the problem that we; what we had started out with regarding the sharp plate. What was the problem?

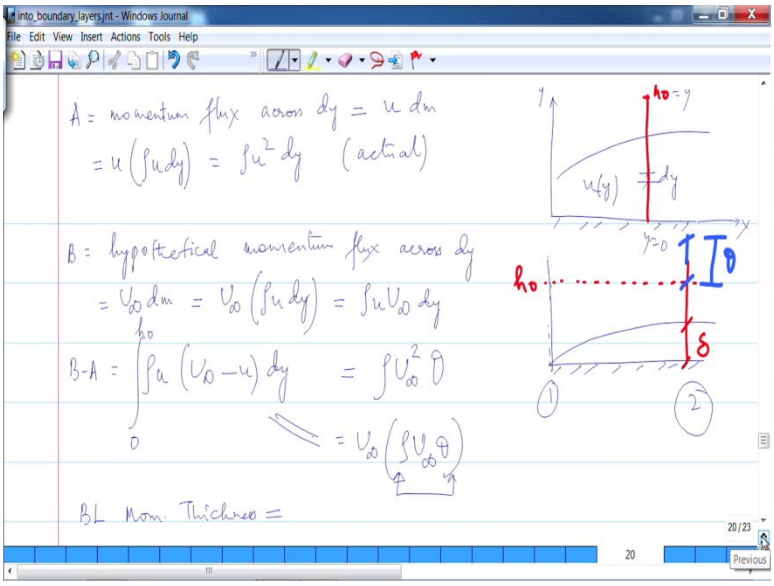
(Refer Slide Time: 01:24)



(Refer Slide Time: 01:28)

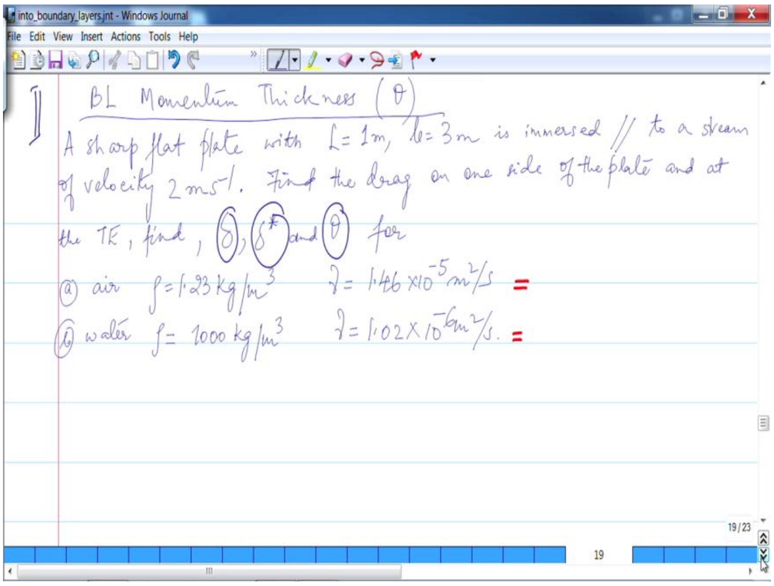


(Refer Slide Time: 01:30)



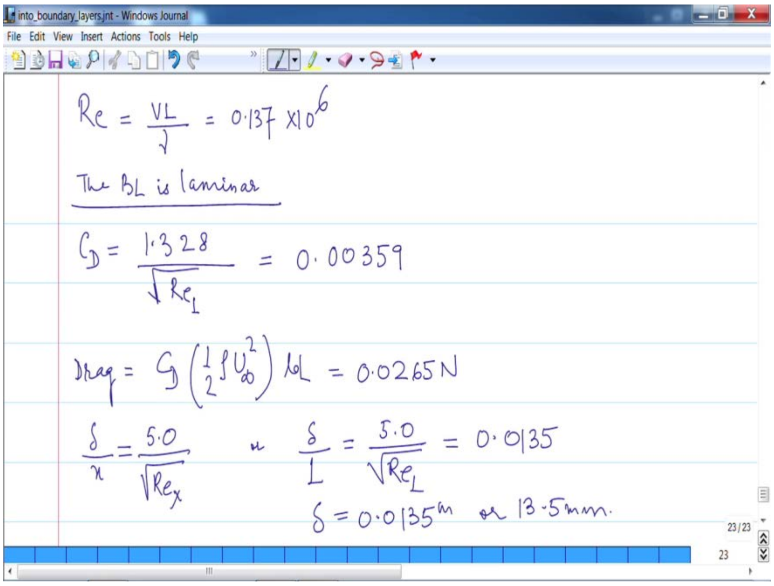
So, let us go back. This is all, what we were doing.

(Refer Slide Time: 01:33)



So, this is what we started out doing. So, we said, a sharp plate with length, 1 meter, b, 3 meters is immersed parallel to a stream of velocity 2 meters per second. Find the drag on one side of the plate and the filling edge? Find the boundary layers thickness, which is this boundary layer displacement thickness, which is this and the boundary layer momentum thickness?

(Refer Slide Time: 02:13)



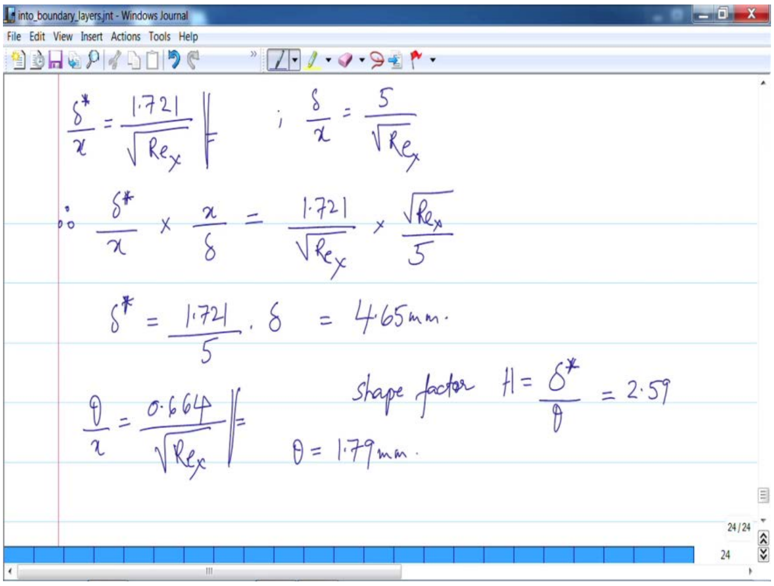
Let us see, how we will set of go about this. So, let me solve this for you, if you have not already done so and if you have, to cross check and if I am making any mistakes, do point it out because there are sometimes typos or error things may be something that I said which is not right. So, make sure you get back to me and say that, there was wrong or maybe you could not prove that. So, your feedback is very important.

Let us find the reynolds of number here, what is reynolds number in this case? This reynolds number in this case, what I get is about 0.14 million, that is what I get here and which means what we were talking about. So, laminar boundary layer is essentially when the reynolds number is below 3 million. Therefore, we can say that in this particular case the boundary layer is laminar. So, I think, it is very important to remember that of course, you will be using formula for both laminar boundary layers and turbulent boundary layers, but you have to remember to check what is the reynolds number, so that you know that in which zone flow does the boundary layer belong to, is it laminar or turbulent? So, that you use correct formulas, that is important.

In this particular case therefore, then CD is something that, where these are things we learn the formula, so, Re. I am going to skip the step and what I get here is this is the value that I get. Therefore, Drag is equal to CD into the dynamic pressure, half row into the area right. So, CD is known, density is known, the velocity is known, free stream velocity is 2 meters per second and the size of the plates, V and L are also known. So, what I get here is newtons.

Then, we also need to know the boundary layer thickness at the edge, basically at the thrilling end. Now, we know this formula, so that is in this case basically, Rex is the total length, we need to traverse from the leading edge of the thrilling edge, which is basically the length of the edge. Therefore, here this comes out to be this or I can basically write delta by L is equal to that by R e L, which we have already calculated and this comes out to be 0.0135 the thickness. Hence, the delta at the end of the plates, so therefore, the delta and L is 1 meter. So, delta comes out to be 13.5 millimeters. So, we will come back and sort of take a look at what do all these things mean.

(Refer Slide Time: 07:20)

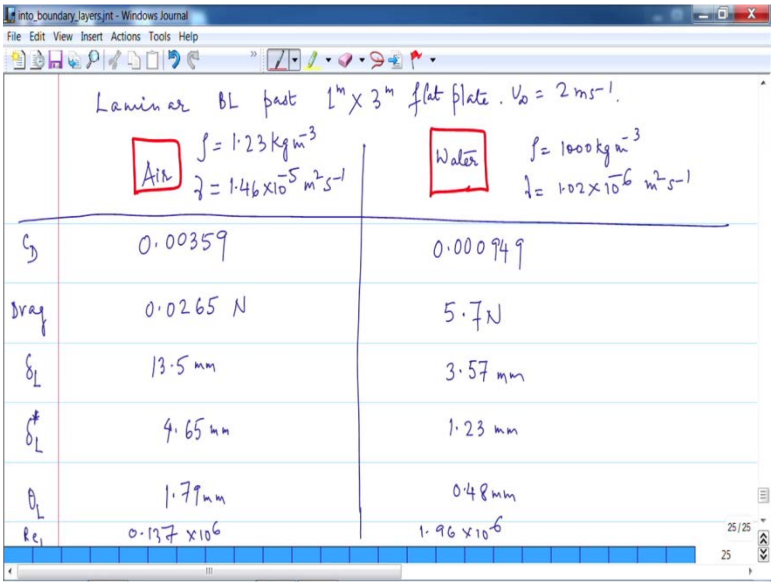


Again, let me go to the next page. So, the displacement thickness is given by 1.721 under root R e x. So well, I mean you can straight away go ahead and find this out. So, there are things you can play around with or so. For example, you can find that out straight away, replace x by L into that. Now, basically 5 by R e x, so therefore delta star by x, I am going to multiply by x by delta, which is equal to 1.721 by R e x into under root R e x by 5. It is just saves you a little bit of math; I mean it is just playing along with the formula. So, if I do that, then all I get is delta star. If I use this, I think what you should do is go and cross check by using this formula and see what you get.

Now, delta star is therefore equal to 1.721 by 5 into delta and what I get as 4.65 millimeters. Again, theta I think, we had developed relationship for theta as well. Let us go back here; we had theta to be this. So, let us write that down, it is 0.664 here. Now, theta again, this is the momentum thickness. So, theta by x, go ahead and use this formula and find out theta by replacing x by L and you could also use something called shape factor, this is not so typically used. Now, these are several parameters which one uses to in terms of understanding the behavior of a boundary layer. So, slowly we have, you will get hang of it.

So, shape factor, H is nothing but the displacement thickness by the momentum thickness and this is actually given to you for flat plate. This value is actually known, it is around 2.6. So, using this you can find out because now, you know delta star and you know theta, you can find that out. Anyway, whether you use this formula or use this what you get here is essentially this. So, theta is 1.79 millimeters, this is what I get. Similarly, this one, this first case was for air, the fluid was air and you need to do the same thing for the second case. So, go ahead and do that.

(Refer Slide Time: 11:19)



Although, I will give you, I will tabulate the values that we get. So, we have laminar boundary layer past 1 meter by width 3 meters, past this is L flat plate, where free stream velocity is 2 meters per second and we have that.

Now, in one case the fluid is air, in other case the fluid is water. Let me just say that. Then, at the end of the boundary layer, no, let us just say first, the total drag. The first case, the CD now, then we have the total drag, then we have delta, I am going to write L because that is the end of the plate, delta L, delta star L and theta L, that is the end of the plate. So, what we get for L, for CD, this is something that we did now. Total drag, we got is 0.0265, that is in Newtons. We got this to be 13.5 millimeters, then we got delta star to be 4.65 millimeters and theta to be 1.79 millimeters and I will give you the value I get for using water, you go cross check this and also what we have calculated here the Reynolds number will change.

In this case, what is the Reynolds number? In this case, it was 0.137 into 10 to the power 6 and in this case the Reynolds number, here is 1.96 into 10 to the power 6. So, 1.96 million and 0.137 millions, basically both are less than 5 million. Therefore, both are laminar boundary layer. These parameters that we developed or we calculated or evaluated, these are all for laminar boundary layer.

Then, CD in this case, in case of water is 0.000949, drag is 5.7 Newtons. Now, delta L is 3.57 millimeters, delta star is 1.23 millimeters and this is 0.48 millimeters, this is very interesting here. Let us see, now for air, whatever given temperature, let me just write down here, the density of course, is 1.23 kg meter cube and kinematic viscosity for air is 1.46 into 10 to the power minus 5 meter square per second and for water, density is 1000 kg meter cube and kinematic viscosity is 1.02 into 10 to the power minus 6 meter square per second.

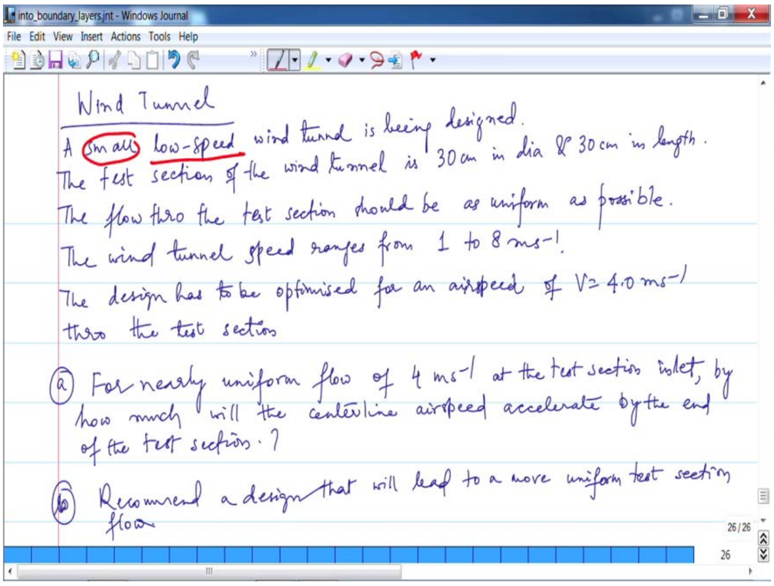
Now, what we see here is that, the same plate, essentially when you are using fluid, which is literally, whose density, is 1000 times more than that of air. So, what you see here is that, if you have water I guess this is not something that you want to surprise, one would expect this. So then, we get a CD which is however much less, is very small and here the drag is 5.7 Newtons. You can see, I mean it is even like more than 100 times, you calculate the amount, how high it is the drag, is a same plate, same initial velocity except that it is water and causes nearly 6 Newtons of drag and this you can say 0.03 Newtons of drag.

Now, the height of the boundary layer, you can see is way less, obviously, or is thicker way dense of fluid. So, it is 13.5 and this is about 4 millimeters or 3.6 millimeters actually. The displacement is around 5 millimeters and this is about just 1 millimeter. So, this was the displacement of air, due to the existence of the boundary layer is 5 times more than the displacement of water of the existence of the water boundary layer and around 1.8 millimeters is the momentum thickness. So, basically that is the height required to counteract the momentum deficit due to the existence of the boundary layer and this is about 0.5 millimeters.

Now, there is something else, I think we have talked about. So, regarding relationship between all these things, we had also said that delta star is about one third of delta L I think we had said that. So, basically delta star is almost one third of delta L. So, these are the kind of things that one would like to quantify. This is how your knowledge and understanding of the boundary layer passed on to a physical problem. I mean, formulae and understanding in control volume is fine, but how do you actually use it or is it useful at all I think that is the question.

So, in keeping with that, let us now do another problem and kind of build on this little more, try and sort of dwell on this, for some more time for another module at least where, we discuss the displacement thickness, what we have understand by that and also try to answer a question which I think I had posed in the couple of initial modules.

(Refer Slide Time: 20:41)



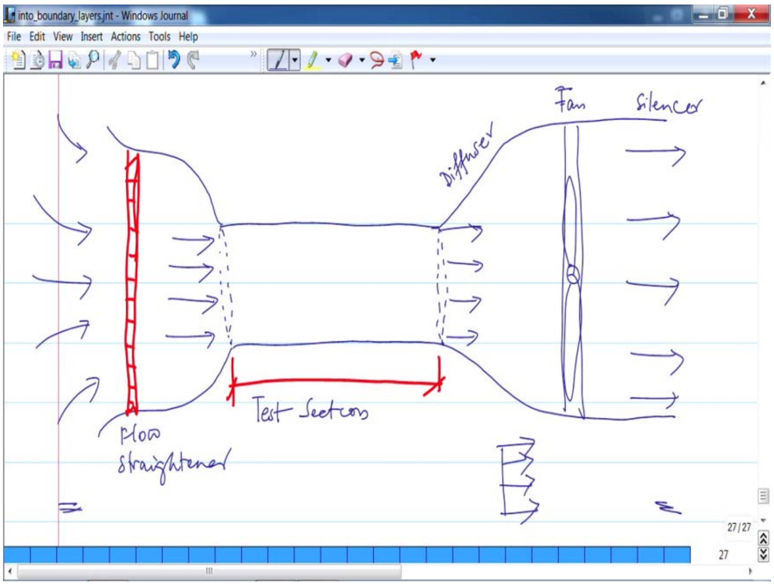
Let us revisit that and see if we are able to answer that now, if things make sense to it. So, let us go and pose this problem. This is a kind of problem, where we are trying to design a wind tunnel, in case you have not seen a wind tunnel. So, this is called, I mean from just the layman's point of view, is the large pipe through which winds moves. That is why this large pipe is basically the tunnel through which wind moves. Now, we use that to do research and to study things, to study the effective things and also these things, for example, this plate that we tried to find out, all the boundary layer thickness and things like that. You could do that in actual wind tunnel and you could go and set up an experiment and do that.

This is the problem here, now a small low speed wind tunnel is being designed. Now, the test section of the wind tunnel is 30 centimeters in diameter and 30 centimeters in length. The flow through the test section should be as uniform as possible. Let me write that. So, the flow through the test section should be as uniform as possible. So, let me finish the question and then we will come back, instead of understand what lot of the terminologies comes up in problems, we do not even understand that, we just go and find out how we can use the formulas, so that we can get a final answer. Let us not do that, let us try to understand or if you do not understand then you will have to ask more questions and figure out, if we do understand it.

The wind tunnel speed ranges from 1 to 8 meters per second. Now, the design has to be optimized for airspeed of 4 meters per second through the test section. The design has to be optimized for airspeed of 4 meters per second through the test section. So, the question is, for nearly uniform flow of 4 meters per second at the test section inlet, by how much, will the centerline airspeed accelerate by the end of the test section? That is the first question and the second question; recommend a design that will lead to a more uniform test section flow? Is that right? So, now we can see that it is like a slightly elongated question. Now, what do we understand? So, let us just go back here, let us look at these things.

Now, we are saying a small. What a small mean? A small basically means that we are talking about test section. So, the things that you will test, they cannot be like really large, it is small. So, that is what you mean by small that the test section itself will be small. Low speed, so that basically will say what? Low speed is now we are talking about is something to do with Reynolds numbers. So, that will boil down to whether your boundary layer is and where is the question of and this is something, this is a question, this is no question of any boundary layer, anything at all but low speed will generally decide, whether your flow is laminar or turbulent. So, that will depend on the wind tunnel low speed, it is kind of intuitive to say that this is, the flow is going to be laminar, test section. So, what I will do is, let me first draw a schematic of this. Let us go and draw a schematic of this.

(Refer Slide Time: 28:55)



This is my wind tunnel sections. Let us say, let us label what path this is. This is basically my wind tunnel. Now, let us do this. So, what I am going to do here is this bit, this is your test section. So, when you see here, this bit is the diffuser, one will have a fan somewhere here. So, that is the fan, these things make a lot of noise, so we will have a silencer somewhere over here. Now, what is happening is, we have got flow which comes in, it just comes in, there is no like uniformly. What you do is usually this is going to be a flow, a straightener. So, usually you pass it through like a sieve or a mesh. So, let us just call this flow, straightener. You see flow is coming in from various directions you want it, when you know you are, use to draw a velocity profiles like this. Clearly you cannot expect that actually, wind is going to come in nice and parallel streams of flow.

That is what we are now talking about. Let us say, this bit is say A and B. So, A is the inlets to the test section, or let us not do that, let us do this and just say that this here, this is the inlet and this is the outlet. So, when I have flow coming in, I would want it to be nice and loose like that. I would really want it to be like that, when comes out of this also, I want it to be as uniform as possible. So, basically it goes out of that. So, what is the fan exactly doing is, kind of helping the flow from this spot into the, towards this end. This is pretty much what the problem is all about. So, we need to find sort of what is coming in, 4 meters per second at the inlet. Now, let us stop here and you look at this problem and let this engage yourself a little bit with this, try to work on this and I will come back and solve this in the next module. So, that is a close for this module.

Thank you.