Introduction to Boundary Layers

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Module – 01
Lecture – 05

Concepts of BL thickness

Hi, welcome back to the Introduction to Boundary Layers class. So, let us; before I begin this module today, let me give you the answer to the question that we posed in the previous two modules. We said there was a long thin plate which is placed parallel to 6 millimeter free stream water at 20 degree centigrade, at what distance x from the leading edge will the boundary layer thickness be? 1 inch and what was given to you was nu water which was 1.1 at 10 to the minus 6 meter square per second. The answer to that is 127.75 meters.

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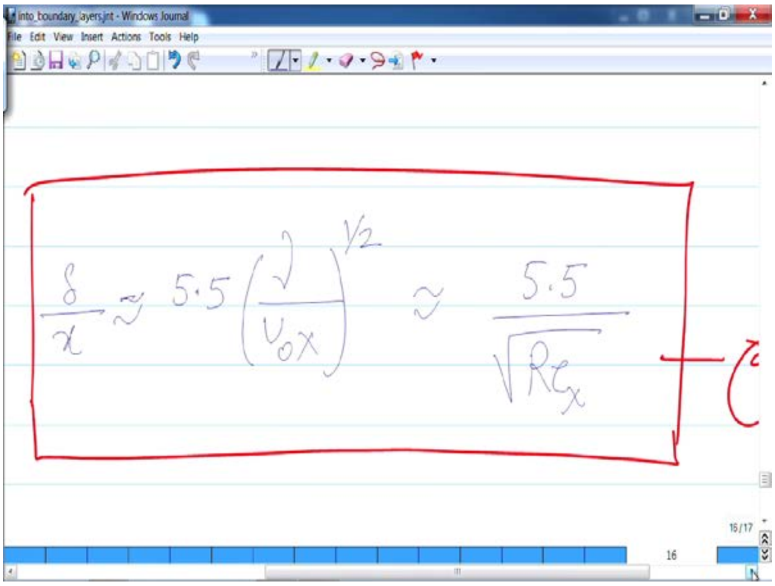

And what we did last time is we went ahead, and what we did last time? Was that, we tried to find out a relationship between the thickness of the boundary layer with respect to the length, with respect to the distance which was this x. So, we did all of that and hopefully you should be able to use all the relations that we developed in the last couples of modules to be to derive at that. So, now the physical implication of this basically means that if there is a flat plate and you have a flow coming on to it, in this case it is 6 meter per second free stream of water at 20 centigrade.

Then, to have the thickness of the boundary layer of 1 inch, you need to travel a distance of nearly 128 meters. So therefore, this zone where the boundary layer exists or this zone where the discuss forces are dominate is kind of small, depends on what you are looking at. So, having talked about the boundary layers thickness, now let us look at some other things which are related to the boundary layers, which we have been talking about. For example, boundary layer displacement thickness. Let us first talk about that, what exactly is that? So, what we will do here is try to kind of understand this, both quantitatively as well as physically. What exactly does this? What is this concept of the boundary layer displacement thickness? And can it be calculated? And what are the theories behind it?

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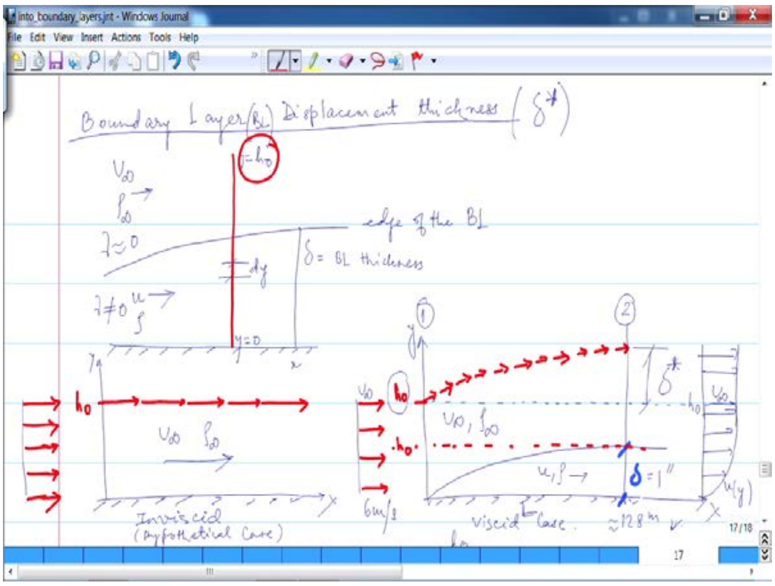

So, what I am talking about here is boundary layer displacement thickness. So, let us see how we shall go about this. Now, let us again go back to the flat plate. So, we got this flat plate; we have got this flat plate here and this is nothing but the edge of the boundary layer. Let us call, use BL as abbreviation for boundary layer. So, this is the edge of the boundary layer. And, so basically here the velocity is basically a profile and the density is this and then here it is the free stream and this is the free stream values, basically.

And of course, nu is not insignificant here and nu is insignificant here. Now, it is important to mention here that when I say that, the viscosity is equal to 0; see viscosity is the property of the fluid itself that cannot become non existence that always exists. But what we are talking about here is, that the effects of the viscosity. The effect of the viscosity is only up to a certain distance from the solid surface on which the fluid is moving. That is all we mean when we say that nu is nearly 0. So, basically what we are saying is the effects of viscosity last, till about in this distance which be a kind of found out relationship and that is what we are calling as delta.

See at this particular x, this is the delta. And this is what we call as boundary layer thickness. So, basically the effect of nu, the effect is nu rather is a dominant till this distance delta, which we call as boundary layer thickness. So, now let us consider a height h naught as such. So, let us take a section somewhere here like this; let us take a section like this so that, here y is 0 and here y is h naught. So, which I can section h naught is which is extending outside of the boundary layer and let us take a small element and call this as d y. So, now let us instead of do this for this little more.

Now, for example, we have this picture here now let us think of two particular cases. One case is when we consider hypothetical case or the inviscid case, where you do not consider the effects of the boundary layer at all; in the sense, if you do not consider the effects of viscosity at all, which means there should be no boundary layer. What is that mean in terms of the flow which is coming on to the flat plate? Which and how would I draw this diagram then? So, the way I would draw that is this, so let me draw. So this same plate how would this look, without taking the effects of viscosity. So, say that and that. So, how would it look like? How would a streamline look like? At say a height h naught, when we consider this to be inviscid, which is a hypothetical case.

So we have got this, say this is the height h naught; this is the same height h naught that we are talking about here, the same height h naught. So, how would a streamline come in, in this case look like? Well, there would be no change; if this was the incoming flow then the way this would look like is just that. That is how it will look like and we do not have a boundary layer because we not considering the effect of the viscosity. So what we have here is, essentially this and the flow. This is pretty much the streamlines as it will look.

In other words, if we draw the velocity profile it would pretty much look like what we have right here at the beginning you know; it pretty much look like this as we move along the plate. So, this is y and this is x. Now, how will this look like when we consider viscous flow? In the sense, we do consider viscosity effects. Then how does this picture change? And what we are looking at, I do not know whether the boundary layer or not or whatever. So, say you were somebody who is coming in; you were the flow which is coming in like that.

In one case, this is the inviscid case there is no viscosity effect so what you do is you come in and move as you know as you came in. However, if there are viscosity effects then how what does it change? Or does it change anything and all? So, what will that be? How will that look like? So, let us see, this diagram again, this is a viscid case. So, what I will do is again this is my flat plate and this is x alright, and you have a flow which comes in and again this is h. So, this point is your h naught and you have free stream which comes in like this.

Now, how does this streamline you know now behave in the existence of a boundary layer? Well, what will happen in this particular case? So therefore, what we now know that this is basically; I do have a boundary layer like that and the height of this is delta; the height of that is delta. So, now what exactly again is happening to this streamline which comes in at h naught? Well, as you can see here that I do have a layer like this which is extending from beyond y is equal to 0 away from the plate, what this will do is shift this streamline away from itself. This is shifting the flow away from itself. So, what will happen to this streamline is something like this, I am exaggerating this. This is essentially the streamline, this is what will happen. So, the streamline which came in you know straight red across here.

Now, here it is going to be displaced, it is going to be displaced from the horizontal which is passing through h naught; which is passing through a height h naught and this horizontal is basically is parallel to the plate. Now then, this displacement from the horizontal is what we denote as, with a star. To denote this, you also call it as delta; we denote it as star. This is nothing but the boundary layer displacement thickness. This is the boundary layer displacement thickness and I say displacement thickness what I mean is this delta. So, you could say this is the section, let say this is section 1 and this is section 2 and you have got. So, this streamline comes in.

So again, essentially what is happening if I again look at the picture here? So, here this is right outside the boundary layer, here is it is free stream? This is free stream, and what you have here is flow is moving this way. So, this is essentially the viscid case. Now, our job right here is to basically then to find out in some expression or quantitatively if we can, some valid for this delta star like we did. So therefore, now the question is we did get some idea, right, or that over a flat plate, if for example, this thing came at 6 meter per second, when at a distance of say section 1. So, at a distance for example L; if this is say about 128 meters, this delta would be 1 inch. So, this is the kind, we kind of got that using that problem.

Now, then what should be the corresponding delta star? I mean, we cannot just say it from here. So, we need to figure out that how we will calculate that you know. Is it red color correlation is delta star also 1 inch? I do not know, may be, let go find out. So, let us do that, I mean how do we find that out? In order to do that let us do this, we will keep this diagram in a view so that we can work with this.

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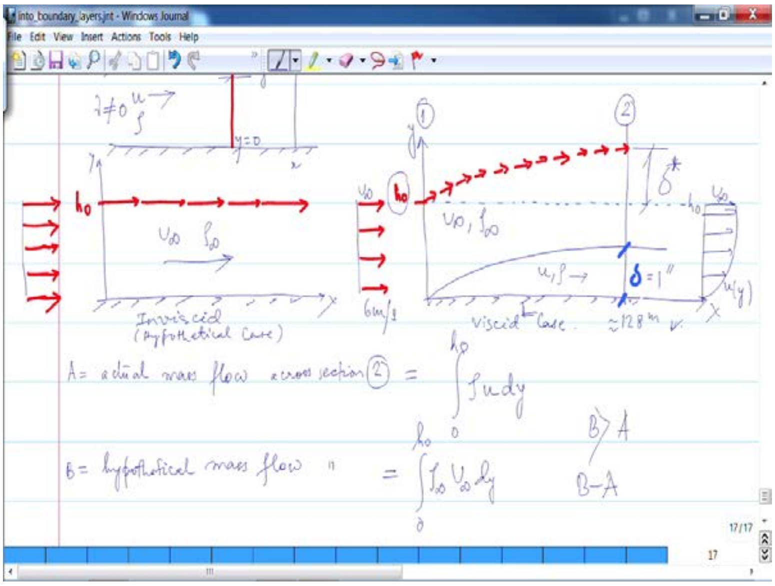

So, let us look at sort of this picture here; this picture here. So, I am going to use a couple of notations here, let say A. Now, this A is equal to actual mass flow which is per unit depths, perpendicular to the page. So, the depth is per unit, but depth per unit depth perpendicular to the page. So, A is the actual mass flow across section 2 and what would that be? The actual mass flow across section 2 should be what? See that should be, so section 2 if I do that, that is 0 to h naught rho u d y. Let me come back and revisit that a little bit. And B, is the hypothetical mass flow and across section 2 and which is basically the inviscid case and that is equal to 0 to h naught which is rho infinity u infinity d y.

So, essentially what we are trying to say here that if you are looking at this picture, if you looking at this viscid case; this case here. So, we are looking at height h naught, we looking at a height h naught and what we trying to find out that what is the flow that enters at location 1? And what is the flow which exists at location 2? So, and the height h naught still remains same, at the same height. So, what comes out of here, the total mass that comes out of here is nothing but rho u d y 0 to h naught, is not it, because this is nothing but u infinity, is not it. And here, as you know I am going to just draw this, this would be your, this is nothing but u y, this is your velocity profile at the exit and this is a velocity profile at the entrance, so therefore, rho u d y up to 0 to h y.

Of course, from here to here this is u infinity; after delta it is u infinity. So therefore, the flow which is coming out of the exit at 2 at height say h naught is this is not it, so which is rho u and d y h naught, but if the hypothetical mass flow in the absence of this would simply be that we would just get this out here. This entrance the velocity profile at the entrance would remain same at the exit in the absence of viscous forces, which in term means in the absence of the boundary layer. So, that it would be nothing but rho infinity u infinity and d y.

So, therefore, we can say that the mass flow which is missing, clearly even this is more than this. The clearly b is, from here you can see this is hypothetical mass flow and this is what we actually get. So, therefore the difference therefore, this B minus A is the difference which is happening because of the existence of the boundary layer. So therefore, let us do the math now.

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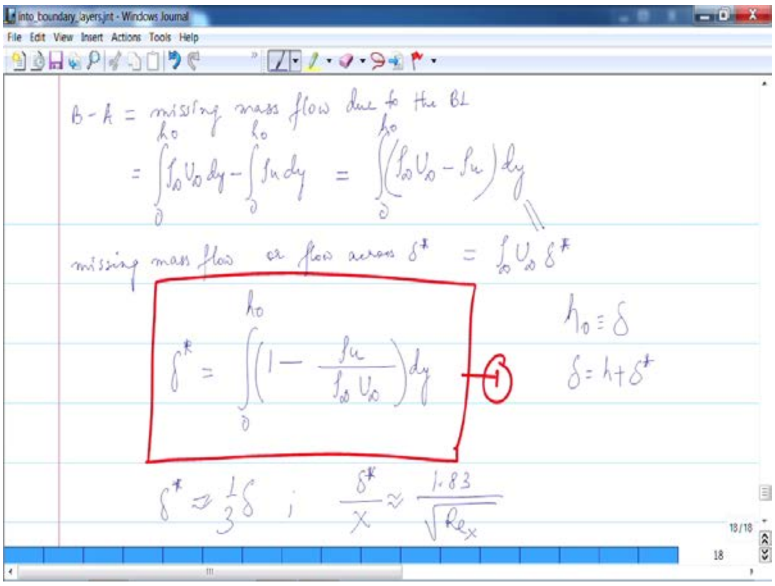

So, what we will say is, that B minus A is the missing mass flow due to the boundary layer which is equals to 0 to h naught rho infinity u infinity d y 0 to h naught rho u d y, which is equal to 0 to h naught rho infinity u infinity minus rho u d y. Now, if we go back just little bit, now we side that the high; what you can see is that whatever mass, the mass which should have come out at the height h naught is pushed away. So therefore, the excess mass is basically is what we get which comes across this height delta star. So, if I do that so therefore, if I write down the missing mass flow in terms of delta star, the missing mass flow or flow across delta star. So, basically whatever mass flow misses out coming out of h naught, it goes across, it is pushed away and it takes another height of delta star to exit.

So therefore, I can write this. This is equal to rho infinity u infinity delta star. So, if I am able to write that so therefore, we will equate these two, if we equate these two, if basically I equate therefore, this to this. In that case, we can write delta star is equal to 0 to h naught 1 minus rho u rho infinity u infinity d y. So, we get as you can see an expression that I am going to call this as what I can. So, this is delta star, this placement thickness and we get an expression for that in terms 0 to h naught. Now, that depends on the amount of height you are looking at, depends on where you looking at and rho u rho infinity u infinity. So, I think let just think about this in a little; so that is an expression which you get.

Now, this is another way of doing that. So, another way of doing that and if you look at this picture again, then what you do is you say, that if for the inviscid case if there is an incoming flow which is entering up till the height h naught then height require to exit is the same h naught, we do not need anything else. However, if we are considering the viscid case then you cannot exit in the total incoming flow at height h naught by the same height you need little more height which is; so therefore, h naught plus delta star.

Therefore, the mass which enters here and whose velocity profile looks like this will be leaving section 2 of the exit using a velocity profile something like that, using a velocity profile something like this; at the exit. And this velocity profile, this total velocity; so therefore, the total mass which leaves at section 2 at this height which is h naught plus delta star with this kind of a velocity profile that is equal to the velocity profile at the inlet which come in till about height h naught. In other words, you could equate the mass coming in at height h naught to the mass leaving at the exit from the height h naught plus delta star, you would get the same results. So, I think I would advise you to go and sort of do that again.

And, before we close this, let us just look at this formula that we kind of developed here. Now say, if this h naught, we have written here if this was just outside the boundary layer. If this h naught that I am talking about; here h naught is away from the boundary layer, if this h naught was just at the boundary layer somewhere here, so this h naught is somewhere here. The h naught is something which you can derive, so what is that mean? So, if h naught is just outside the boundary layer then h naught is basically equal to delta, if that is so; so therefore, h naught is equal to delta and what would delta be? So, delta would basically be h plus delta star. So, h naught is delta and the delta would be h plus delta star.

Now in that case, if we use the velocity profile which we introduced in while calculating the boundary layer thickness, if we do that, then we would come across delta star to have value something like this. So here basically, you can solve this integral provided you have an expression for u. So, we will use what we had done all here, so if you look at your notes, you should able to see that. So, then delta star comes out to be around one-third of delta. So, that gives you some idea. So, if you know the boundary layer thickness at a certain distance then the displacement thickness is about one-third of that. That is what this relationship is telling you and the other expression is that delta star with x also (Refer Time: 30:08) 1.83 R e x. So, the displacement thickness, boundary layer thickness is about 5.5 by R e x and the displacement thickness is about 1.83 by R e x. So, we will close this module here and pick it up again.

Thank you.