Introduction to Boundary Layers
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Module – 04
Lecture – 40
Similarity Sols to thermal BL: Overview

Hi. I am going to give you a brief overview of the similarity solutions to this thermal Boundary Layer.

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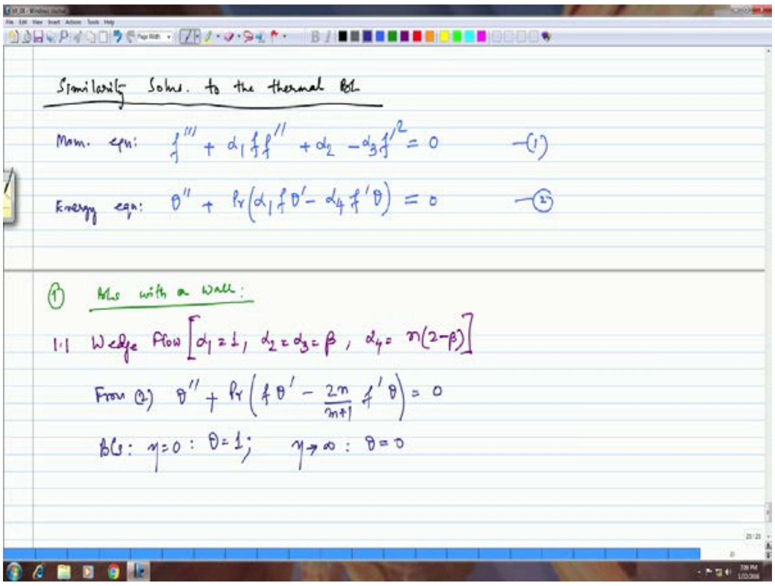

Without going into too many details, but I think this is important. So, that you have this information at handwrite, so how basically do we use the similarity solutions which is similarity variables, how do we use the constant similarity to get some solutions to the thermal boundary layer.

Let me call this as similarity solutions to the thermal boundary layer. Now, we have talked about the Reynolds analogy earlier. So, I am going to sort of pick up from there. Now, from the momentum equation which we have written earlier, so we can write this hopefully we can recall. For the momentum equation, let us write that; momentum equation. So, we can write this, and the energy equation. I am just reproducing these 2 equations here. Now what will do is, we will look at several cases and the what should be the specific values of these constant there and possible solutions, that is what we need to do right now.

So, number 1 is boundary layers of the wall. So, in this case the first that we go to is wedge flow. So here alpha 1 is 1, alpha 2 is equal to alpha 3 is equal to beta, alpha 4 is n into 2 minus beta.

So, then from 2 what we get? From the equation 2 what we get is, alpha 1 is 1 f theta dash minus alpha 4 is n into 2 minus beta, but we have seen that. So, we can write basically the connection between m and n; m and n beta, so we can write that. So, 2 n alpha 4 f dash theta, so basically we just write out beta. This and that is equal to 0 and the Boundary Conditions then become, now eta is 0: theta is 1; eta is very large: theta is equal to 0.

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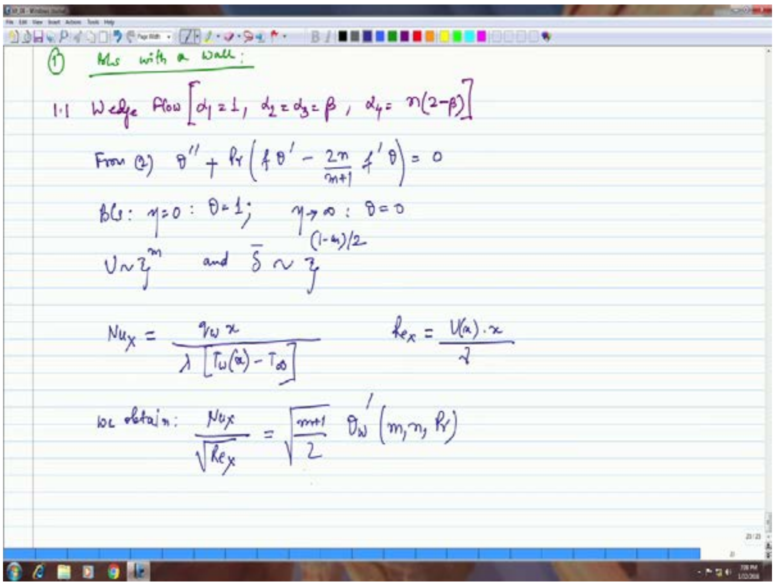

So in this case now, what we will do is in this case we will write the velocity in terms of the local x co-ordinate and the thickness boundary layer thickness zeta to the power 1 minus m by 2. Now these are some of these are we have used earlier. So, please just remind yourself and if you go back into the (Refer Time: 06:05) you should be able to see everything, I am not repeating what we have done before. Now then, of course, Reynolds number. Therefore, from this consideration we obtain, per under root Re x is equal to m plus 1 by 2 this.

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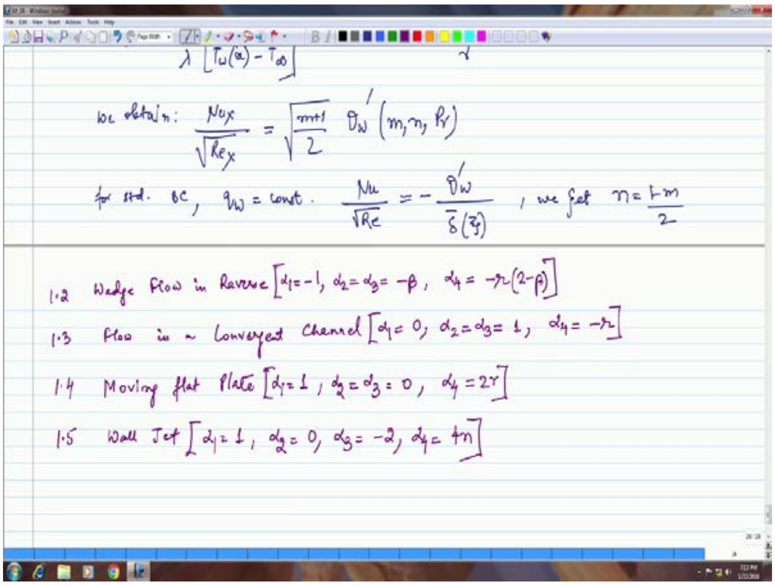

Now for standard boundary condition, flux of the wall is a constant. So, that is equal to this. So, we get n is equal to 1 m minus m by 2. So, that is basically for the wedge flow. So, this is for the wedge flow.

Now, then the next thing is we will Wedge flow at the wedge flow in reverse. So, reverse wedge flow. So let us see, how does that look like? This is wedge flow in reverse. In this case alpha 1 is minus, 1 alpha 2 is alpha 3 is equal to minus beta, alpha 4 is minus r 2 minus beta, so that is that.

The next thing is Flow in a convergent channel, so alpha 1 is equal to 0, alpha 2 is equal to alpha 3 is equal to 1, and alpha 4 is equal to minus r. Moving flat plate: If you had a moving flat plate, so alpha 1 is 1, alpha 2 is alpha 3 is 0, and alpha 4 is 2 r. Then again Wall jet, so alpha 1 is 1, alpha 2 is 0, alpha 3 is minus 2, alpha 4 is 4 times n. So, that is basically, give you an idea as to this is basically.

So, if you can see that finally, balls down to solving this equation and if you have a readymade code you can actually basically be able to handle a lot of these cases, is not it. At least so, all you need to do is change the alpha 1. This constant of course, the nature of the numerical solution also changes and that is resulting something to, the point here is that you may or may not need, all of these things you might just do a wedge flow or may be just a flow in a convergent channel or say for example, a moving flat plate. You might just do one of these, any one of these or may be little more of these or may be all of these who know.

But this is so that, just to give you an idea as to like we were changing the other day when in the module, when we took the equation. Then we put it different values of beta and we were seeing how the solution was changing. So, that basically also means that, the value, when you change the value of beta. So, you are also looking at different physical flows.

Because the code itself, the code does not know, code just treats the beta as just any number. So, you just changing a number it is giving your velocity profile. But you can make the inferences that; so, if I make this beta that actually physically corresponds, for example, if I have beta is equal to say minus 1 here or beta is equal to 1. So, that alpha 2 and alpha 3 is equal to minus 1. This is wedge flow in reverse. So, in whatever velocity profile that I get, that corresponds to physically a wedge flow in reverse.

Now, the next thing is this is without a wall. So, that is interesting. What do you mean by there is a boundary layer which is forming and there is no wall?

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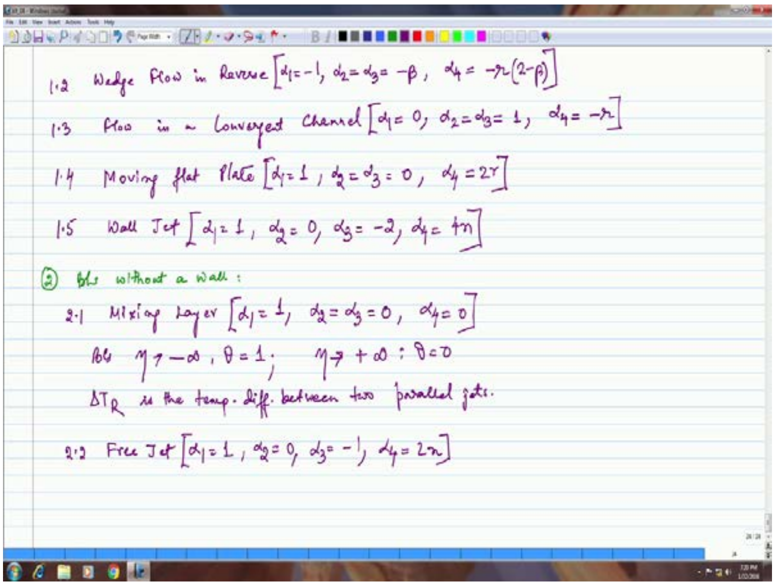

So, let us see Boundary layers without a wall, a mixing layer. So, basically we have 2 things here: one is a mixing layer and the other one is the free jet. So, that is all, we basically have here. Now for the mixing layer here we have got alpha 1 to be 1, alpha 2 is equal to alpha 3 is equal to 0, and alpha 4 is equal to 0. With the following Boundary Conditions, 1 and this is a temperature difference between 2 parallel jets, is not that right?

So, basically what you see here is that you got a mixing layer. So, what is your computer code do? The computer code, you just tell the computer code that alpha 1 is 1, alpha 2 is equal to alpha 3 which is equal to 0, alpha 4 is 0. Now, what you could basically do is look at the equation here, look at the equation here put in those values and see what the equation would (Refer Time: 16:35) do. That is what we are talking about here. So, you need the velocity field and you also need the temperature field. So, you need the velocity field it is like a post convention thing.

So, the computer code basically understands that you giving it some values for the constants alpha 1, alpha 2, alpha 3, alpha 4. Then you give it the boundary condition saying that. So, if eta goes to the negative side far of on the negative side, theta is 1 and theta is 1 and if it goes on the positive side. It goes positive further side theta is 0, so theta here that we kind of write down.

Well, theta is your non dimensional temperature difference. So, what we basically see is that. So, basically there is a temperature difference between 2 parallel jets. So, you go from one end to the other end. So, that is your eta. You go from one negative one far end to the other far end. And your non-dimensionalize temperature difference goes from 1 to 0, and this delta t r is basically the temperature difference between 2 parallel jets. The next thing is 2 point 2; we will talk about free jet. So, this is a free jet. So, in this case, alpha 1 is 1, this is the free jet and these are the values of the constants. Now if the fluid on each side of the free jet has a different temperature, then the Boundary Conditions become so, let us see.

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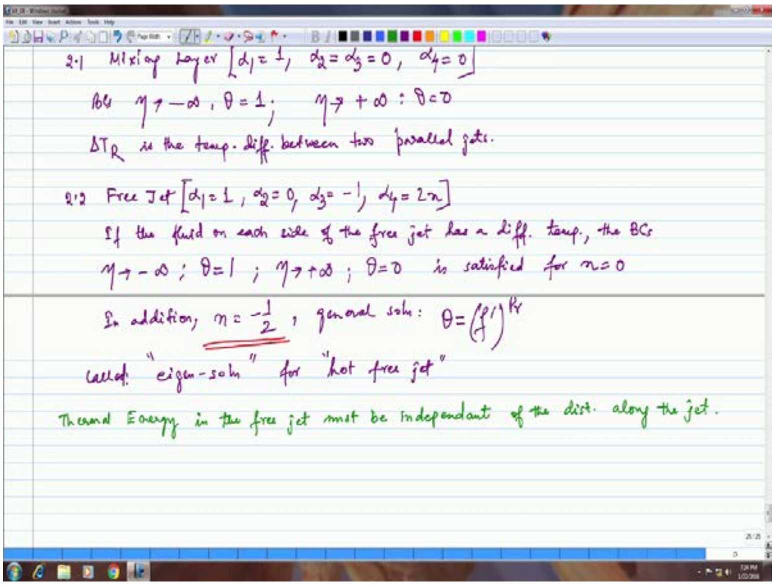

Now, in this particular case if the fluid on each side of the free jet has a different temperature the Boundary Conditions are the following. The Boundary Conditions go to the far negative side, so non dimensional temperature difference is 1 and if you go to the positive far of side. So, if the fluid in each side of the data in different temperature obviously, this is whole for that and this is satisfied. This is satisfied for n is equal to 0. Now, in addition to this let us well basically satisfy n is equal to 0. Now, in addition to this for n is equal to minus half, if you have an n is equal to minus half the general solution rise to the Prandtl number. This is called Eigen solution for the hot free jet. So, this is the general solution. So, in case you have n is equal to n is minus half and this is called the Eigen solution for hot free jet.

And the final thing that I would like to say on this is that, thermal energy in the free jet must be independent of the distance along it. Let me set of you emphasize that a little bit. Now, thermal energy in the free jet must be independent of the distance along the jet. So, that is really everything that I really wanted to talk about in terms of the similarity solutions to the thermal boundary layer.

So, this is basically in overview. So, for students who are going to be, who are going to research on this there are several of you (Refer Time: 23:20). All saying even masters program PhD programs and who want to future in this. So, this is to give you an idea that is when we do see the basic equations and we develop the similarity solutions. Because I do not want, because there is a lot of math involved and we also some of the math is also quite rigorous and it is slightly complicated, I will acknowledge that, but I think hopefully, slowly you begin to see the pattern.

Hopefully begin to see the pattern. That once you get the basic equations and place, then the effectiveness of the similarity solution. So, you break this equation down, similarity solution down. And you have 1 for the momentum equation and 1 you have for the energy equation. So, you need both of those for the thermal boundary layer of course. So, in order to predict the temperature field you need the velocity field. So, you are using both, is not it.

So, now once you do that, you take this and then you basically change the values of the constants. You change the values of constants and what you get out of it is that you get different equations. You get different equations for different cases. One, we have this wedge flow, we have 5 of these. Wedge flow in reverse, we have flow in a convergent channel, we have a moving flat plate, we have a wall jet.

Now the difference between all of these things as far as the computer code is concerned is just the difference between alpha 1, alpha 2, alpha 3, alpha 4 beta m n whatever, so it is just those. You kind of take that now fill it into your computer code. There of course, when the movement you give different values of these constants your equation also changes. It will also give you a different numerical nature of the equation, but you once you got the Runge Kutta method in places and you are able to solve these things over certain domain. And to define that domain what you need is a boundary condition.

So, for example, for the wedge flow we had a boundary condition where the eta went from 0 to infinity. So, 0 is basically at the wall, where the velocity the non dimensional temperature difference is basically 1, and in this case there is none because this is because the temperature is equal to the free streams. So, there is no difference. So, that basically tells the computer code. Computer code is basically solving 2 equations between the Boundary Conditions.

Now, what you get out of that for you to make the inference. So, you should know that when I use these values and I use these Boundary Conditions. What I come up these the solution that I get physically corresponds to a wedge flow. I mean my request to you is not to get frustrated with the entire math and a lot of derivations. It is important I mean I could not do these. So, if I bored a little bit I apologize, but if I do not sort of do this I do not know how else to do this.

So, some just basically was trying to impress upon you that, if all the computer if you have a basic generic computer code, where you keep the computer code in terms of alpha 1, alpha 2, alpha 3. Keep it in terms that and then the movement you change this and then you change your Runge Kutta method. You should be able to use that. Remember we were changing beta and however, using that, so that was very interesting, is not it. So, that and again here as we are able to express, we have done this earlier in this particular case also, so we are able to express.

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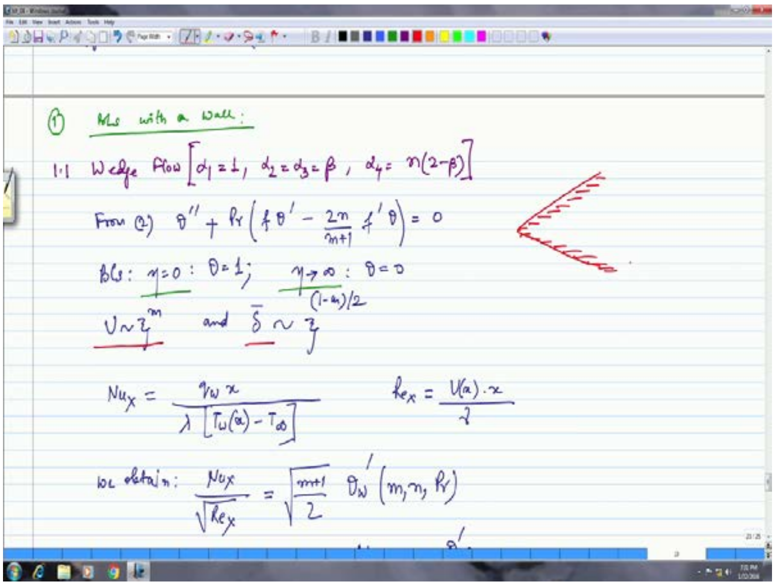

For example the velocity and the boundary layer thickness right in terms of loco zeta, and therefore, from there we are able to obtain values for the enough of number; Reynolds number and the temperature profile.

Similarly, when you have these cases, so all you basically need to do is make sure that you change the, and one of the fun things to do is, I showed you a in one of the models is to just keep changing this values and see what you get. So, is that is I think that is the first step 1 would do you just build up a code and keep changing things and see how things are changing and all that. And slowly once you set of say, fine. Now I played enough let me go and do this, let us take a particular case well say for example, I have something like this for a convergent channel. So, alpha 1 is 0, alpha 2 is 1, alpha 4 is minus r, and then try to interpret the velocities and the output that you get, the temperature output that you get in terms of the actual physical flow in a convergent channel. So, that really is the objective.

So, one more thing that I wanted to impress upon here, so this is here. So, boundary layers with the wall, see actually have a physical wall on which the boundary layer is developing. So, that is what we mean. So, whether is a wedge, it could it is a wedge like that and you have a boundary layer. sorry in other way all right, so not that way. So, you actually have solid surface like that, and then you have boundary layers forming.

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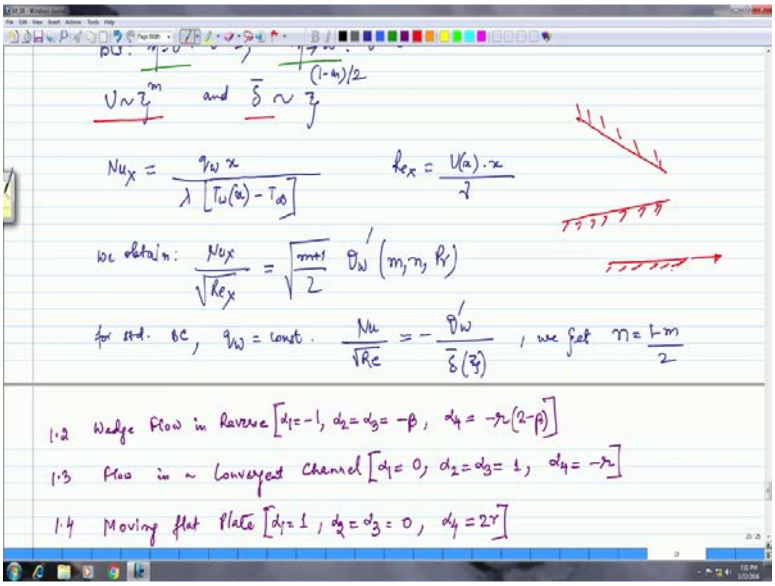

So, that is a wedge and of course, you have a convergent channel. Then you have boundary layers forming. Obviously, this is a channel. So, boundary layers are forming on the inside here, then you have a moving flat plate. So, you have a flat plate which is moving. Then you there is a boundary layer forming. There is and then of course, is a wall jet. There is actually a solid wall on which the boundary layers forming this is more in twitter this is what we been doing in flat plate and things like that.

Now the boundary layers without a wall, now that is kind of almost like a miss over right do you like how can I, boundary layer form without a wall. Well, I thing you also heard of immiscible fluids, immiscible liquids. So, you have like water and oil. So, they will not mix and they from a layer which is which is basically the boundary. So, therefore, in this case if you see the Boundary Conditions. So, again for these constants, again these constants will change.

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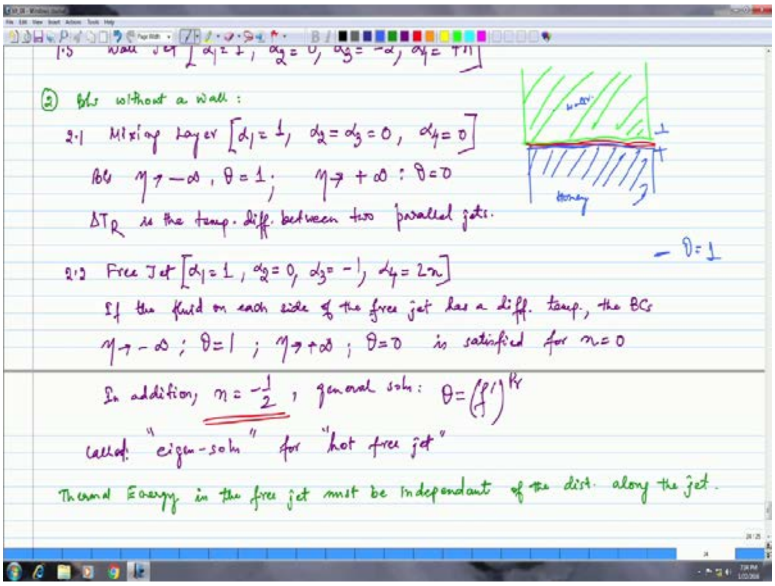

The boundary conditions, for example if you see then that change, so if you see the Boundary Conditions it goes from the. So, if you have for example, this is one fluid, one kind of exaggerating this a little bit and you have another fluid here. So, this is a fluid and we got this fluid in here. If you add something like that, so the Boundary Conditions, it goes from essentially from here. So, negative far off from this edge where the non dimensional temperature difference is this, and then it goes to the other end where the theta is 0.

So, there since there is there is no like solid wall over ever. So, basically then we are saying and then this here you know. So, this is the parallel jet, this temperature there is certain temperature difference there. So therefore, similarly for the free jet and you could actually have these 2 fluids it could be at a different temperature. So, you could if they could have they could be at different temperatures. Yes. So, that is basically boundary layers it is without a wall, because if you want to think about it say, for example, this is honey and this is say water and it would probably wonder flows.

So obviously, just think about what should happen is something like this is going on, if the water comes in here I mean, I think it is a also you could think about how that would look like if you have a thick layer of honey and you allow throw a little water on that, how will the velocity look like will flow as nicely or as freely or as you would expected to flow over any other say a smooth flow or for that matter would be the same or different compare to if we were to just say through what and all flat table like that.

So, anyway we have not done this exactly in detail. So, boundary layers without a wall I just wanted to sort of give you an overview of the whole thing. So, all of this so, but this something to think about especially, for people who want to do research and things like that. Something to think about that how this thing will develop, but the whole idea of all of this is that the computer code does not know all that it does not know whether you thing there is water or honey, whatever it does not know all of that.

So, all you have to tell the computer code is take this alpha 1, alpha 2, alpha 3, alpha 4 give with these Boundary Conditions and look at the results and look at the output and see some inferences from there, and that should give you a lot of inferences about; what I just ask, what I just said that how will the velocity profile look like. Well you can actually if you get an output from this computer code you should be able to see that how at velocity boundary layer is behaving, or how it now the thermal boundary layer is behaving.

So, similarly here for the free jet you have several conditions and yes, Eigen solution hot free jet etcetera. So, well we kind of what I mean by this, the thermal energy in the free jet must be independent of the distance along the jet. Well I guess, we see, we do not see the energy packet kind of changing as it moves along, as you move along the jet we do not want that happen. We kind of assume that, it is independent of the distance that it moves along with jet. So, like I said I have not really this in detail, but this is just to give you an idea that if you have a basic code, and place you can still run the code and get some results and see some outer from that. That does not stop you from doing this, and it is a do able thing I have given you an idea of how the code works. We can actually make that work. So, it should be interesting to see how you should be able to do that.

So, I think I will go head and stop here. Hopefully, we will talk about something interest taken in other time.

Thanks.