Introduction to Boundary Layers

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Module - 03
Lecture - 39
Effect of Dissipation in thermal BL-III

Hi. So, we are here to continue with some more concentrations of the Prandtl Number and Dissipation. So, continuing our discussion of the viscous dissipation for a flat plate and we stop to this equation, our expression that we cut for this, we said this r which is a ratio of the thermal Tad minus t infinity by u infinity square by 2Cp.

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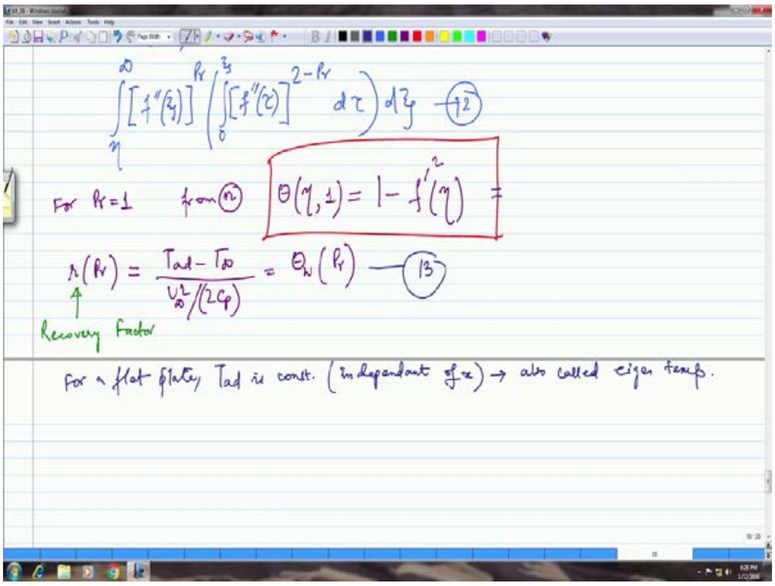

So, this ratio is basically something that I can calculate as a function of this Prandtl number by solving this equation. And this is called the recovery factor and for a flat plate, the adiabatic wall temperature is constant meaning at it is independent of x and hence it is also called Eigan temperature; this where we stopped, OK.

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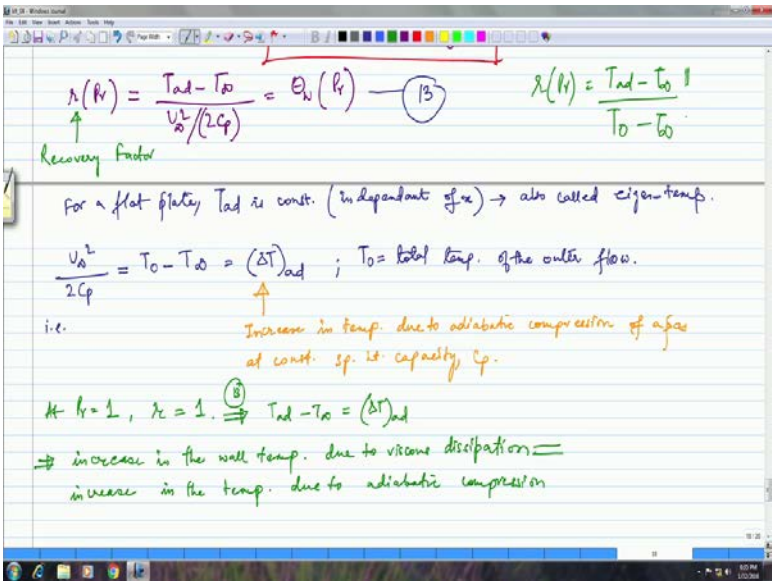

Let us continue from there. And now in this expression here in 13 this u infinity square, right by 2Cp. Now that is actually equal to t naught minus t infinity which is essentially this, where t naught is basically total temperature of the outer flow, and what is this basically means is that the dimensionless yes, arise in temperature of the adiabatic wall due to dissipation is called the recovery factor. Yes well, basically we are looking at the rise in temperature, rise in temperature of the adiabatic wall temperature and we have non-dimensionalizing it by this term in the denominator. And we are calling that is recovery factor.

So, now let us see what this thing is, the thing that we wrote in the bottom, the denominator rather. Now, this delta Tad I am just denoting that is delta Tad is essentially the increase in temperature, increase in temperature here due to the adiabatic compression of an ideal gas it comes in specific heat, this is the Cp constant specific heat capacity. So, now, this thing actually is the increase in temperature due to adiabatic compression of a gas, at constants specific heat capacity, that is Cp. So, increase in temperature due to adiabatic compression of a gas at constant as specific heat capacity Cp.

Of course, now at Prandtl number 1 the recovery factor is also equal to 1. So, if from 13 basically I can see that the, which means that this, which means this from it was from 13 essentially; minus t infinity is equal to this, which means that the increase in the wall temperature. Increase in the wall temperature due to viscous dissipation is equal to increase in the temperature due to adiabatic compression. It is exactly the same as so. Now, the thing is basically what we are doing here if you see. Now let me write that down first, what this essentially means that increase in the wall temperature due to viscous dissipation is exactly equal to increase in the temperature due to adiabatic compression.

Now, So, we will give you some numbers in stuff. Now the thing is that, basically what you see is that the recovery factor. So, I am taking the temperature here, the temperature difference of the adiabatic wall temperature with the free stream temperature. This is at, we are kind of trying to measure this in terms of the local velocity and this also happens to be equal to the bottom you can see here, that this is basically effect the velocity and that is also equal to this change in the temperature.

So, this is the total temperature of the outer flow, minus the free stream and this is the adiabatic wall temperature minus. So, essentially what I am saying is that the recovery is adiabatic wall temperature minus the free stream by outer flow minus free stream. This is a recovery thing. So, this is the increase in this is the increase in temperature, the numerator is increase in temperature due to viscous dissipation and this is the increase in temperature due to adiabatic compression of a gas at constant specific heat capacity.

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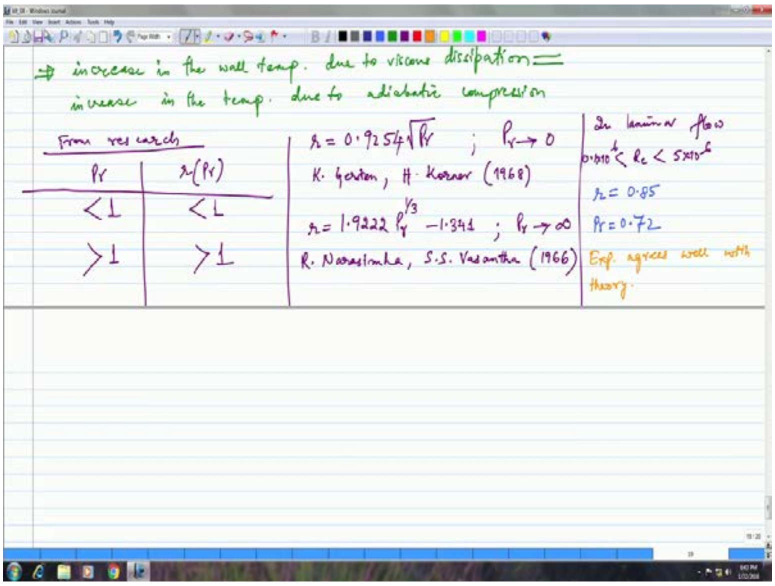

Now, some output from research. So, I can say basically I have got Prandtl number and I have got the recovery factor. Now this Prandtl number is less than 1, the recovery factor is also less than 1. Prandtl number is larger than 1, recovery factor is also larger than 1. So, if you sort of look at this for example, Prandtl number less than 1 recovery factor is less than 1, which means in the first case that the increase in temperature due to viscous dissipation is less than the increase in temperature due to adiabatic compression. So, due to random and that is because if the Prandtl number is less than 1. If Prandtl number is larger than 1, the increase in temperature due to viscous dissipation is larger compare to the increase in temperature due to adiabatic compression of the gas.

So, I will give you some values. This is from researches for your reference. For example, now values like this, r is equal to Prandtl number for very small Prandtl numbers and this was given by Gersten H Korner, this. Again, this is for very large Prandtl numbers and this was given by. Now I am going to do is, now look at a plot which you can pull up the actual reference in sort of look at that, or this is I got this from book actually.

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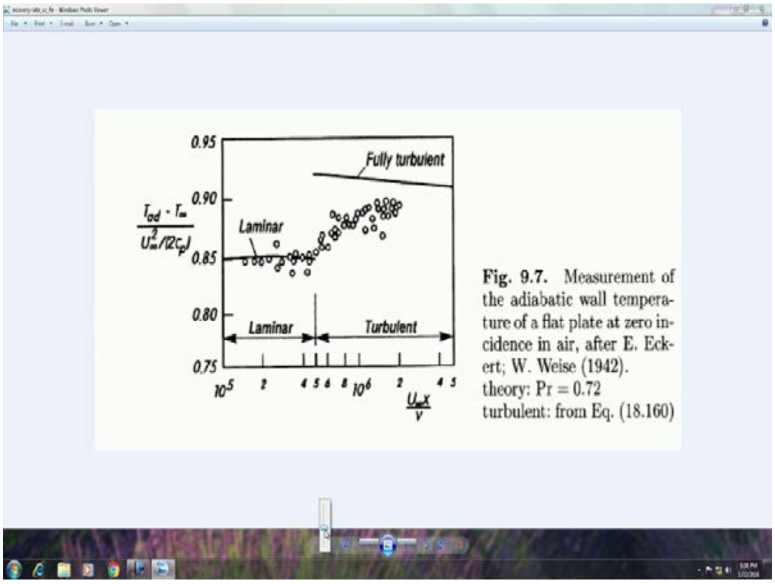

So, what I am going to do is; look at this. Fine if you sort of look at this, now if you look at this plot here what you basically see here on the y axis, this is recovery factor. So, Tad minus t infinity by infinity square by 2Cp and on the horizontal axis is essentially the Reynolds number, this is for the Reynolds number and you can see that between this is the Laminar sound because, this is between 0.1 million to 5 million. So, this region from this plot that you can see and these are the data, which is been plotted.

Now this here, what you can see in the Laminar region. The recovery factor is 0.85 and then this corresponds to a Prandtl number of 0.72 and this experimental result actually agrees well with theory. So, it is from this plot that you sort of can see that, and this is like I said the measurement of the adiabatic wall temperature of a flat plate at zero incidence in air, after Eckert; Weise 1942. So, then in theory Prandtl number is 0.72. So, it explains it well for that. So, if I go back here let me sort of write that down. This is just a note, so that speaks; I will just write that here. That in laminar region or in laminar flow say; there Re is less than this.

Now, the recovery is 0.85 and the corresponding Prandtl number is 0.72. So, just of the matter is that the experiment agrees well with theory. So, having talked about that, now we will just sort of slowly look at some wedge flows and how the temperature profile or the temperature and exactly how we are going to arrive at an expression for the temperature field, in case of wedge flows. You got wedge flows, you got a wall jet and we are going to talk about a little bit about the nusselt number.

Now, when we were going to talk about the next thing that we will talk about is Wedge flows. So, we basically use the equations that we have been, use similarity solutions if we have used earlier. Then solve that and get an expression for the wall temperature field and that should give us an expression for the wall temperature which is our main purpose here. Now, if you remember. So, let us sort of do that.

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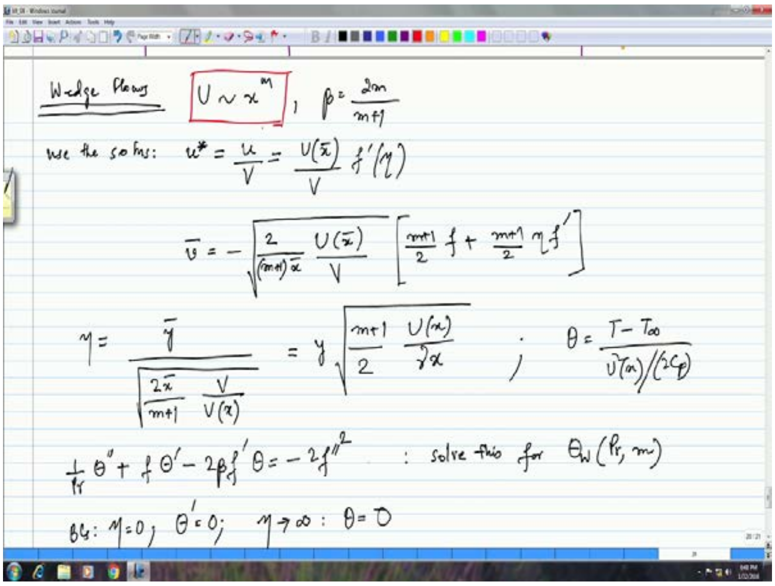

Now, if I were to that, so let us just do that. Wedge flows, so here we are going to write the velocity in an exponential form of the x and beta is this and then we shall use the solutions. So, following solutions which is that, then that. And of course, we have an expression for eta, which we write as that, this, which is also that and yes, then of course, we have. So, if I do this then the equation that I get is so, so I get this equation. So, if I use these solutions and so and so forth, so the equation that I get is this. That, and the corresponding boundary conditions and eta very large, is this.

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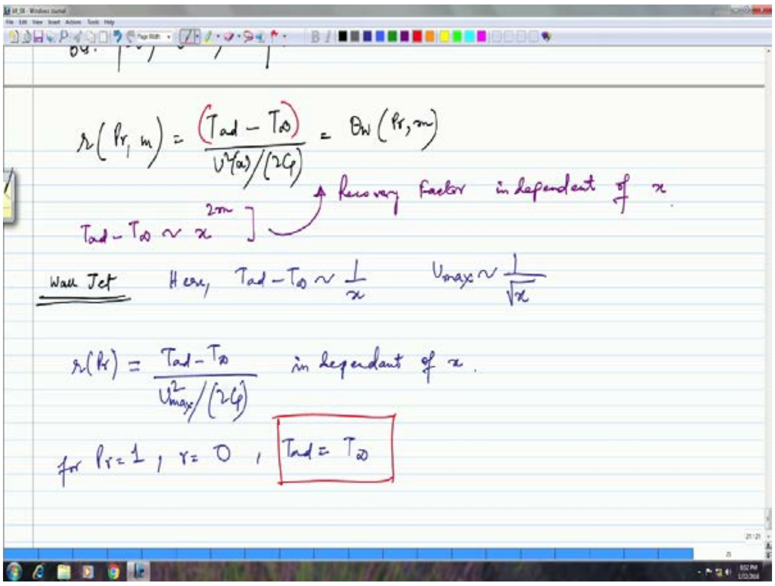

Then of course, we solve this equation for; we solve this Prandtl number and m. So, then we get the recovery. So, the recovery factor is a function of both the Prandtl number and m. So, we get. We have to solve for the wall temperature that definition and then we get the wall temperature field actually, and then we get this.

Now, it is interesting here that. So, we get this expression, now the only thing is that we have been able to, we have written the velocity like this. We write the velocity in terms of the x. Now, if the increase in temperature due to dissipation is also related to the local velocity which can be written in terms of x. Resource in power of m. So, if this also is something that I could write in that fashion then we can get an expression for the recovery factor which is independent of x. What I mean is that if were to write, if I were to write this then we will get. If I were to do that then you get a recovery factor independent of x. Now that becomes interesting, fine.

So, that is that about the wedge flows. If I were to write it like this, the next thing is let us talk about wall jet. We will talk about a wall jet, so here in wall jet, well it is actually proportional to the inverse of x and u max is proportional to the inverse of root x. Therefore, the recovery factor which is the function of the Prandtl number. So the recovery factor based on the maximum u, so this term as you can see here the numerator is a function of 1 by x and the bottom actually is a function of under 1 by under root x. So, this is actually independent of x. The recovery factor actually is independent of x. That is very interesting.

Of course, now for Prandtl number 1 recovery factor is 0. Now if the for Prandtl number 1, in this case of Prandtl number 1 recovery factor is 0 which means that the adiabatic wall temperature is equal to the free stream temperature. So, where we will just talk about the Nusselt number and then we will stop.

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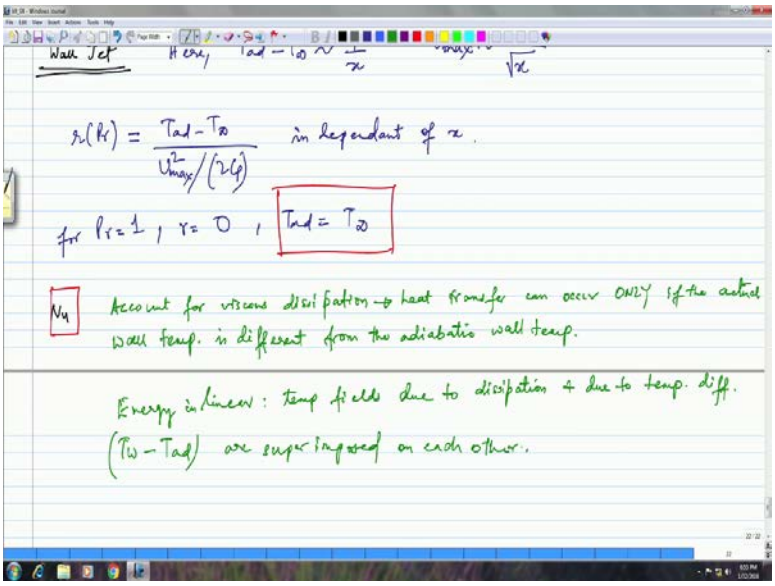

So, let us do that. Let us talk about the Nusselt number; we have not done that at all. So, that should be the last thing which we talked about and then we will stop for this module. Now, the accounting for dissipation if we account for dissipation now the heat transfer can occur only if the actual temperature of the wall is different from the adiabatic wall temperature. So, let me write that down.

So, if we account for viscous dissipation. So, heat transfer can occur only if the actual wall temperature is different from the adiabatic wall temperature. So, if you account for viscous dissipation. So, the heat transfer can actually occur only if the actual wall temperature is different from the adiabatic wall temperature. Now, what we do here is that now the energy equation is linear. Now, since the energy equation is linear the two temperature fields; one is due to the dissipation and other is due to the difference in the wall temperature and the adiabatic wall temperature are super imposed on one another, since it is linear. So, temperature fields due to dissipation and due to temperature difference are super imposed on each other.

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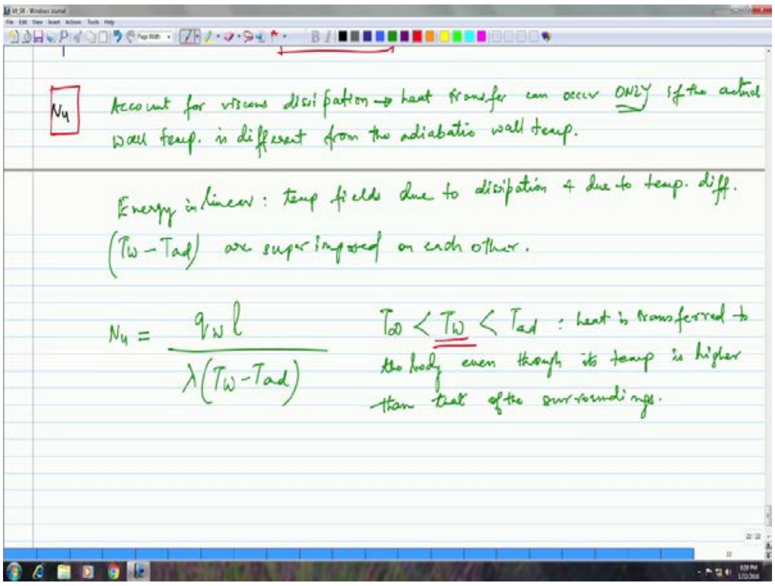

So, if I do that, then we get Nusselts number something like this. Now, if the wall temperature is greater than the free stream temperature and less than the adiabatic wall temperature, heat is transferred to the body. Heat is transferred to the body although its temperature is higher than that of the surroundings. Yes, well this like a interesting thing here. Now if you see here if the wall.

So, basically like we said that if you account for the viscous dissipation heat transfer can occur only if the actual wall temperature is different from the adiabatic wall temperature. Now, if this is larger than the surroundings and less than the adiabatic wall temperature, heat is transferred to the body. So, heat is transferred. This is the wall temperature which is less than the adiabatic wall temperature, so in this case heat is transferred to the body even though its temperature is higher than that of the surroundings. So, just notice what I am trying to say here. So, this is the wall temperature, which is larger. So, this is the temperature of the wall which is greater than that of the surroundings.

However, the adiabatic wall temperature is larger than the wall temperature. The actual wall temperature and we said on if you account for viscous dissipation, heat transfer can occur only if the actual wall temperature is different from the adiabatic wall temperature which means that in this particular case, it is so, right? Due to the viscous dissipation there is actually a difference in the actual wall temperature and the adiabatic wall temperature. Hence, heat is transfer to the body. The only catch is that heat is transferred to the body although the body temperature is actually higher than that of the surroundings.

So I think, that is all I need to discuss about dissipation effects and hopefully you get a picture of the interesting things happening here, and also along with the theory hopefully you get an idea of how the math shapes up so that we get information for the math is also. And this gives you an idea if we do take into account the viscous dissipation then what transpires and compare to if we do not. This is what we did for most of the part before we started this one.

So, I am going to stop here and hopefully that you can make some (Refer Time: 32:24) out of this. So, I will stop here and I think this is probably the close of this course hope you if picked up something's, and I will try to answers my questions as you may have. And hopefully I can also some of them at least.

Thanks.