Introduction to Boundary Layers

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Module – 02
Lecture – 38
Effect of Dissipation in thermal BL-II

Hi. So welcome back. We were discussing about the Effect of if the Prandtl Number is Small. How do these viscous dissipation effect looks like or how do they look like actually. Now, from the energy equation we got this equation 4.

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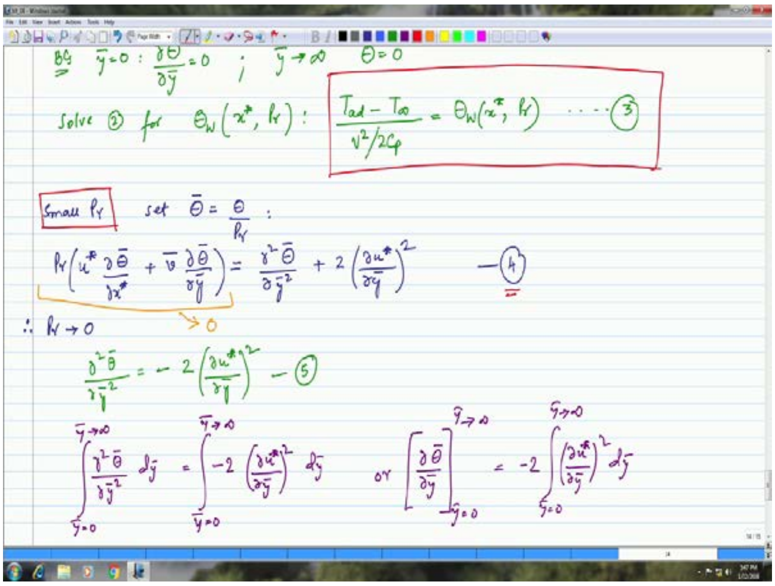

We got this equation 4 and you can see that the left hand side basically, this dissipate here is actually the convective term. But this convective term as you can see disappears for small Prandtl numbers. So there is essentially, if you see the theta bar here so that is not been convected. The temperature change is not been convected because this term according to at least this equation if a small Prandtl number the convective terms disappear. So, let just say that for the limiting case where the Prandtl number is very small the convective terms disappear.

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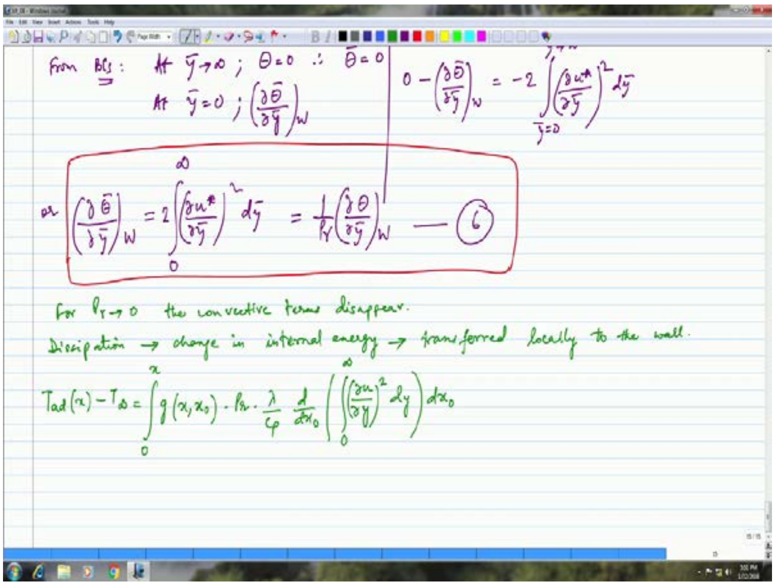

But however, because of the viscous dissipation there will be a change in the internal energy. There will be a change in the internal energy and which is basically stands from the very definition of dissipation, as a result of which this will be transferred locally to the wall. You can see that there is no convection of the temperature difference, but due to the dissipation there will be change in internal energy and this is going to be transferred locally to the wall. There is going to be a change in the internal energy is going to be transferred locally to the wall.

Now, locally to the wall, and if that happens, so in this dissipations there is no convection terms as we can see from 6 here for the temperature field. So therefore, there is no convection happening, however because of the viscous dissipation there is going to be change in internal energy which is going to be transferred locally to the wall. So, if that is going to happen there is going to be a change in internal energy transferred locally to the wall. So therefore, the adiabatic wall temperature should adequately comprehensive for it.

Now, when we do that so that kind of comes up with, so I write out the solution like this. When that happens I can actually write which is, now let me explain this a little bit. So, we get this expression, this is little bit of scary expression. Well, there is lot of math involved in this. Now, hopefully you remember this thing that we have talked about earlier, so let me just remind you of that diagram; this one.

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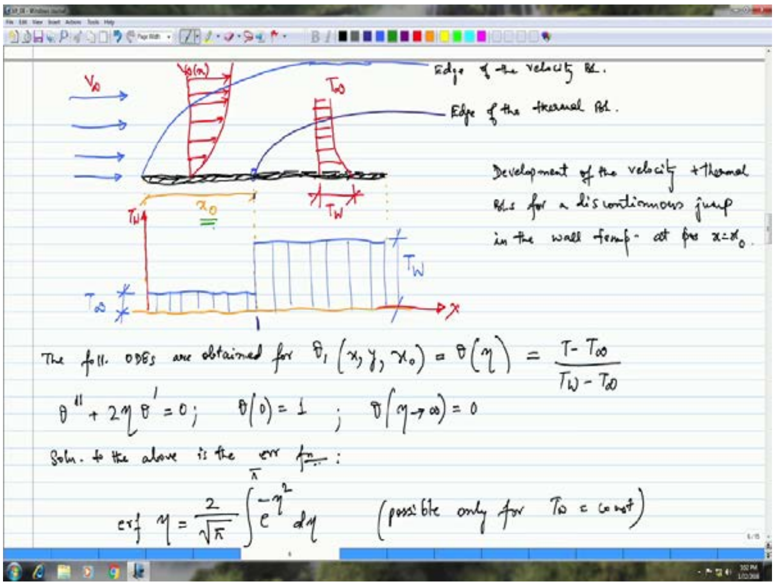

We had a plate, and this x naught is the distance for were the wall temperature is basically the free stream temperature and it is x 0, it is a jump where if the wall temperature is higher than the free stream. Therefore, this is happening why and the movement that happens and this is happening because of you know viscous dissipation. So there is no heat or anything been supplied to the body, but because of that there is a jump in the wall temperature which you can see here, T w.

And obviously, because of that a thermal boundary layer develops where the wall temperature is higher than the free stream temperature. Therefore, because of that I have the temperature difference across the flow which is nearest to the wall here, and it finally reaches the ambient temperature. And this is basically is my thermal boundary layer. So, this is the x naught that we are talking about and this is the wall temperature and the free stream temperature.

Now, having said that for constant heat flux, so this is a standard solution for T w is constant where; is 0 obviously.

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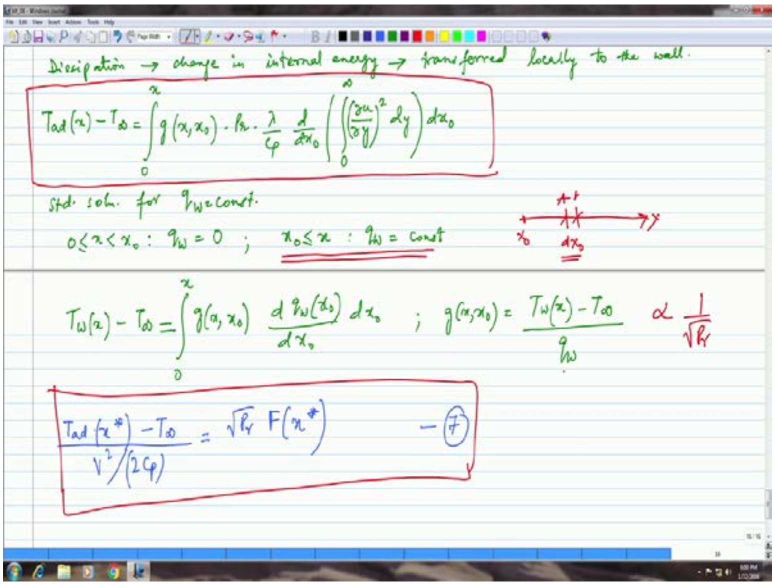

So, this is essentially 0 to x naught. The heat flux is basically 0 because the wall temperature is the free stream temperature. So, when you go past x naught however, so we have a finite value, this thing. Then the solution this is the standard solution T wall so 0 to x. Again explain that a little bit. Now if you see here, basically what we saying is the wall temperature minus the free stream temperature. Now if you see this term, what is this mean? This basically means that I am looking at the total change in this heat flux when I go over a small distance past x naught, because this is where only when you go past x naught that is when see there is a heat flux which is constant. So there is at the other wall.

So, now the point is by how much does that change. I take a small elemental dx as I go along, this is my plate is I go along x, I take a small element so this is the x naught and this is where I have the temperature jump. When I go pasted I just take a small element dx naught, then I say that this of course the heat for that changes, so this is a rate of change and then how much of the thermal change for the small element dx naught. So, that is what it means.

Therefore, this gives you the total sort of flux and you multiply that essentially by this value this is g function where this is nothing but q. So there is a certain distribution of the wall temperature what you can see here, otherwise we would not have to do this, we would just say that is q w; the q end minus q start. So, q basically I would have said q w at x minus q w at x naught I would have said that. But the thing is here this is a function of x naught, it is not constant. So that is all you do. This entire term, this part actually is just summation of the total so it will ball down to that. So, then these two will cancel out you know this q w and q w will cancel out, it will remain T wall and basically you are integrating it. So, that is what this integral actually means here if you want to look at it.

So that is how, which is what we are using in here for our calculations here. So, this is what we get. Now the point is that, this distribution of the standard solution this is actually proportional to the Prandtl number, is proportional to one by square root of the Prandtl number which means that if you look at this the adiabatic wall temperature minus T infinity; this is proportional to 1 by Prandtl number, if I do that here then basically what I can say is that under that this, so this thing is proportional to the 1 by square of Prandtl, so then I can say is basically some multiply that Prandtl and the remaining I would just write it is a function of x star.

Basically, I am trying to get an idea as to how this actually behaves. So what basically I can see that if, because of this dissipation effects if the Prandtl number is very small then, we are going to talk about this a little more in the left hand side of this. So, when the Prandtl number is very small then what we essentially see is that the left hand side you can see here is the adiabatic wall temperature, the difference in the adiabatic wall temperature and the free stream temperature. So, temperature field is essentially is a function or it is dependent on the square root of Prandtl number.

So, having talked about that one of the boundary limiting cases that we said, so it does depends your velocity field, does depend on the Prandtl number and we can see what is going on in terms of, how much in what way so that is what is given by equation 7 for the limiting case for the (Refer Time: 14:18) the Prandtl number is very small.

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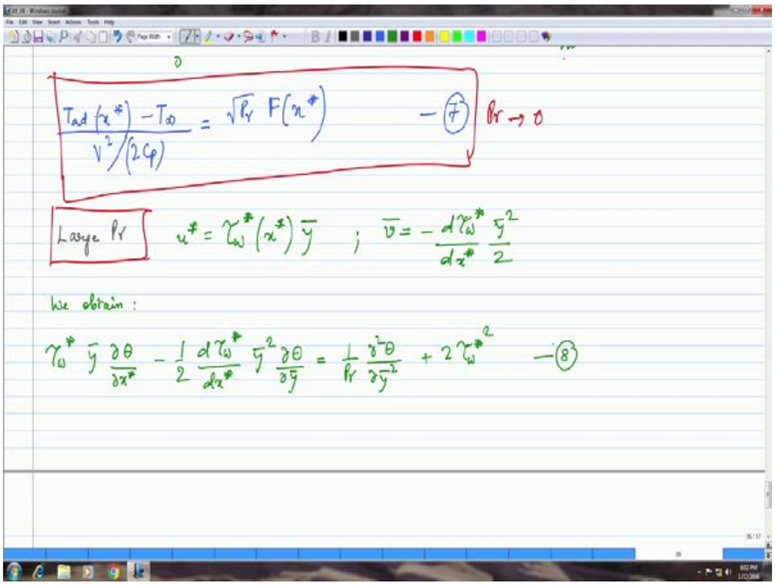

Now, the next obvious step to do is to figure out what is going on for large Prandtl numbers. Let us do that. So, what is the case with large Prandtl numbers? That is the logical things to do. So in here again, we want to go directly to the math and then sort of talk about this. If I go through this, these are things that we have been kind of doing for some time now. What do we obtain? What we obtain is y bar minus half this is equal to Prandtl number, so this is what we obtain and we are going to use this transformation here.

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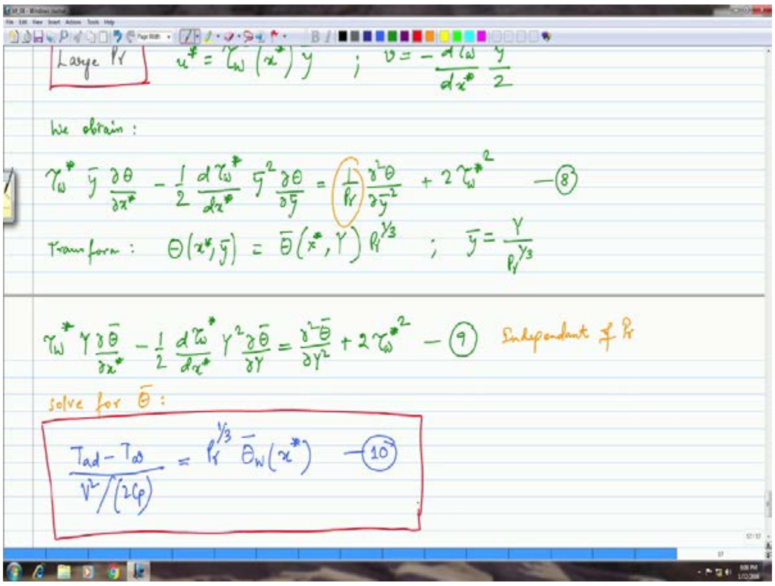

We are going to use this transformation here. So we are going to use, and y bar is essentially y by. Essentially, this y here is y bar into Prandtl number is to 1 by 3 so that is what is this balls down to. If I use this transformation what we get is the following equation and you will see something interesting here. That is equal to, so what do we get here? I mean we use this transformation and we get this equation 9. So what do you think what is so wonderful about this or you know what do we get here. What you see is that this equation is actually independent of the Prandtl number, there is no Prandtl number here involved at all. So, we had this Prandtl number in 8, so if you see we did have this term here, we have none of that you know directly involve. So, this is actually independent of Prandtl number.

We will solve for a theta bar. So, when we solve for theta bar. If we solve for theta bar we get this then is equal to, so this is what we get. So, if we solve for theta bar this is what we a get. Hence, from this equation 10 what you can see that if the Prandtl number is large the increase in the temperature so, increase in the temperature meaning you know this part of course, right. Due to viscous dissipation is quite large is not it. So, this is for Prandtl number very large. Now, what we can see in this case is for this limiting case that is when Prandtl number is very large then the increase in temperature due to the viscous dissipation is quite prominent and considerable actually for large Prandtl numbers.

So, now having done that so what I will sort of do next is basically, now then go and look at a couple of the cases that we looked at earlier. For example, for a flat plate, for a wall jet and also have the Nusselt number looks like we have not sort of talked about that at all. The next obvious things to do from here is to could of talk about how these things will look like for a flat plate and we shall sort of a discuss that a little bit, and in terms of equations. So, let us see how that looks like.

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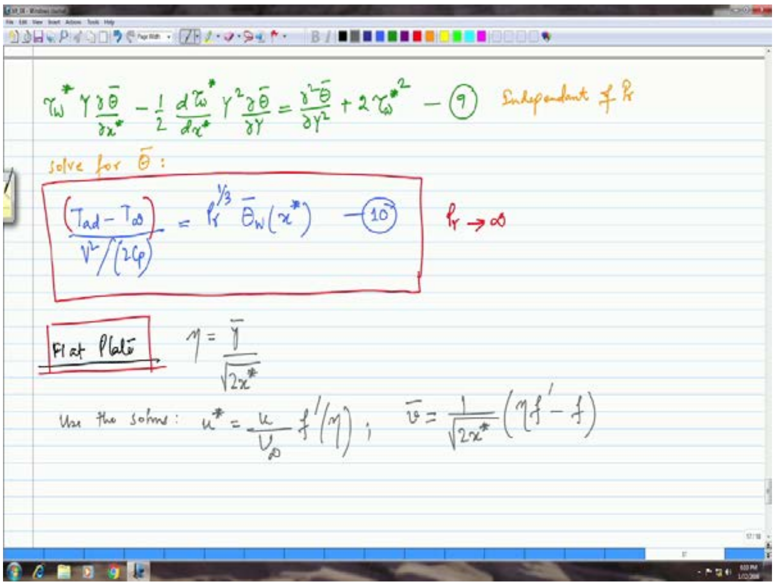

So, let us say if this is for now a flat plate. So now, we were using this, right? So then we shall use the following solutions. So u star, this is as we have been doing this equation for the similarity available is, so v bar is, so we will use this solution. I hope you can remind yourselves, this is what we and that how I got this is from essentially I use these just to remind ourselves.

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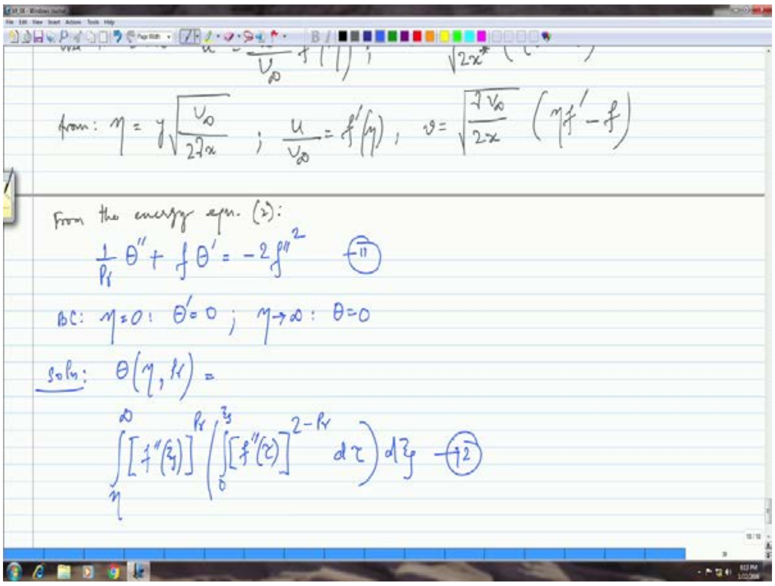

So eta is y, this; then u by U infinity is f dash eta, and v is nu V infinity 2 x eta f dash minus f, so this is where we get this from. Then from two which is the energy equation, let us go back and look from the energy equation. Let us go and look at equation 2, which is equation 2 let us remind ourselves a little bit, so this is the energy equation. So from this I am going to write this out.

From the energy equation two what we get is this and the boundary condition eta is 0, theta bar is 0. These are the boundary conditions we use, say for the solution that we get here. The solution that we get here which is a function of those eta and Prandtl number that looks like this. So, that is essentially equation 12, this is my solution. Now, what is interesting here is, so this is a solution that we get. We have to use this transformation into the energy equation this is for a flat plate.

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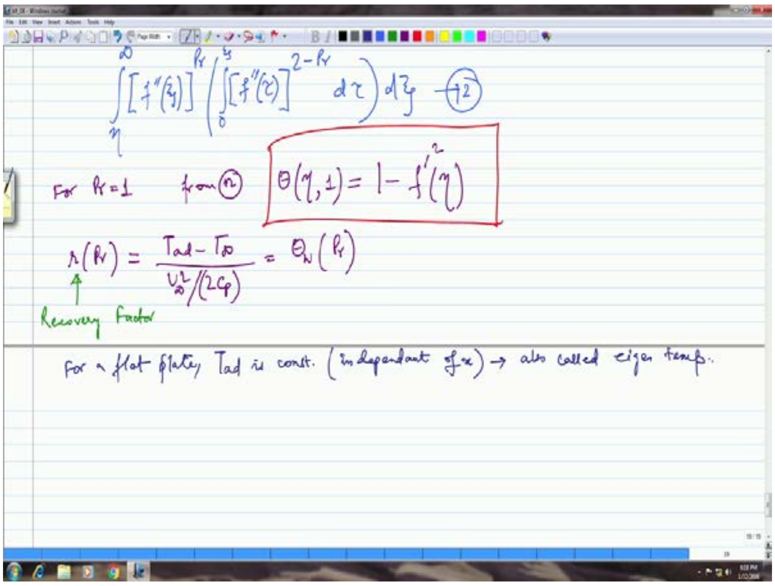

And we just come up for Prandtl number in here, for Prandtl number equal to 1. If that is the case from 12 what we get, right? Prandtl number is equal to 1 we get; this is the equation to solve for. It is really that simple, I mean I have been showing you how whether it is the focus scan equation or the boundary layer equation that we solve this is in similarity for able to note down the q code and then solved it and I showed you the parts this is even similar to solve equation.

Now, there is another thing here. So this is an important thing I want to talk about before we end this part. So, there is the something so therefore that is that. Now, I am going to write this term here and then will come back and discuss this a little more in the next module. This is basically as we have been, this is the non-denominational field. So this is essentially the temperature field at the wall which is a function of the Prandtl number; is not it. If I solve this equation of course, from this equation if I solve will get see once I solve this for eta at the wall I should be able to get theta w at the wall; is not it. That is what it will be for me if I would solve this equation. So when I do that this theta wall is basically equal to this and let us call this stuff as r.

I am going to just say here, just write down a few things and then kind of stop. Now, this r, this is actually called Recovery Factor. Now for a flat plate this Tad which is the adiabatic wall temperature is constant. It is constant it does not change along x which means it is independent of x. Hence it is also called an Eigen temperature. Basically, why Eigen temperature? Because that is the temperature, which makes the temperature field here Tad minus T infinity, it reaches a particular value; a single particular value. So, let me sort of write that down here. For a flat plate the adiabatic wall temperature is constant which is basically independent of x and it is also called Eigen temperature.

So, we are going to stop here and continue to discuss this a little more in a couple more modules. So, will stop here and I will see you next time.

Thanks.