Introduction to Boundary Layers
Dr. Rinku Mukherjee
Department of Applied Mechanics
Indian Institute of Technology, Madras

Module - 01
Lecture - 37
Effect of Dissipation in thermal BL-I

Hi, welcome. So, we have been talking about the Effect of Prandtl Number and we went on length to understand few things about that. If you were to kind of remind ourselves a little bit.

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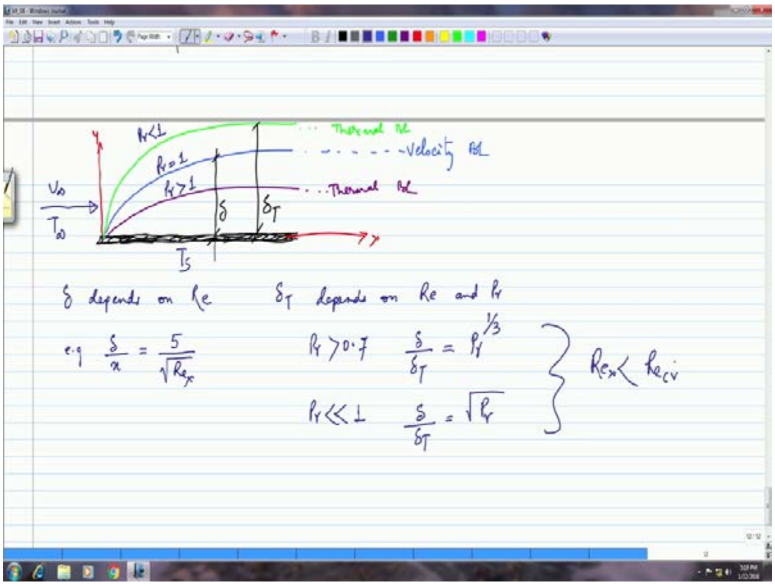

This is the flat plate. Now, I am going to draw an axis system and then I am going to draw three things here, that. Now, what we have coming in is a free stream, so you have free stream coming in. Let us say the temperature of this surface is given by T S. Actually this blue line, now that is essentially the velocity boundary layer and see at some location, this is the height of the velocity boundary layer thickness. Both these are also some more boundary layers; the green and this are both several boundary layers. Again, this is the several boundary layer thicknesses. What I am essentially saying is that this is also a thermal boundary layer, so is this. This is also a thermal boundary layer. Then what is the difference? Now, why do we have two of these and what does these things it will mean.

Well, the difference is that for the velocity boundary layer, the prandtl number is equal to 1. The boundary layer thicknesses which are less than the velocity boundary layer, we kind of did this yesterday, in the previous module. Here, it is greater than 1 and this is prandtl number which is less than 1. So essentially, what you see is that if prandtl number is equal to 1 then the velocity boundary layer thermal boundary layer thickness is are same. If prandtl number is greater than 1, then the bound velocity boundary layer thickness is larger than the thermal boundary layer and if prandtl number is less than 1 then the thermal boundary layer thickness is larger than the velocity boundary layer.

Again to remind ourselves, the velocity boundary layer that depends on the Reynolds number and the thermal boundary layer depends on both Reynolds number and the prandtl number. For example, that, this is something that we sort of done this quite a large earlier on. Now, for prandtl number is larger than 0.7, then we get a relationship something like this and for very small prandtl numbers, this. This is a case where the Reynolds number is less than the a critical Reynolds number. It is less than the critical Reynolds number.

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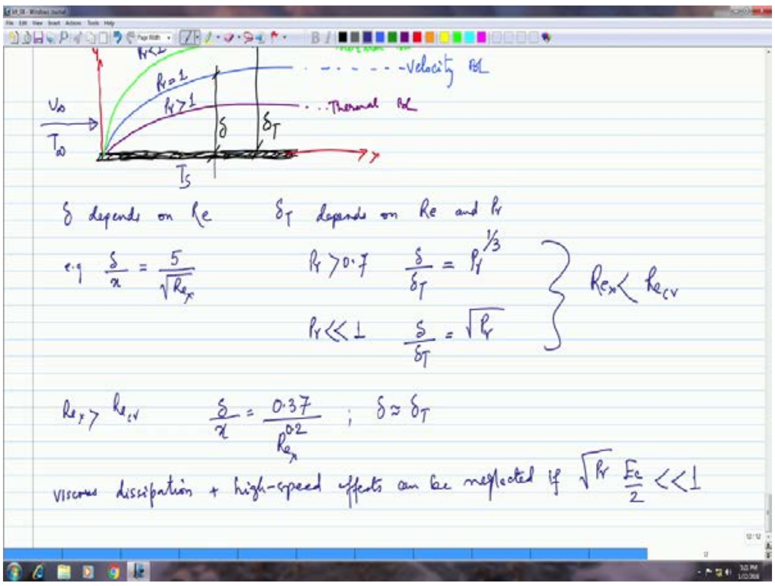

And if the Reynolds number is greater than the critical Reynolds number, then and that. The velocity boundary layer and thickness and the thermal boundary layer thickness were same, the viscous dissipation, now we going to talk about dissipation. Now, the point is that the viscous dissipation and high speed effects can be neglected if, so let us just write that down here. Viscous dissipation and high speed effects can be neglected if prandtl number. If the root of prandtl number into the Eckert number and half of that is very, very small. We should be able to disregard the facts of dissipation and high speed.

However, what we going to do now is that go ahead and look at the effect of dissipation. So far what we have done is we have neglected it. We are not going to do that anymore and we going to see that when we cannot neglect dissipation effects what really intakes. So that is our job today. Let us go and see you know what that means.

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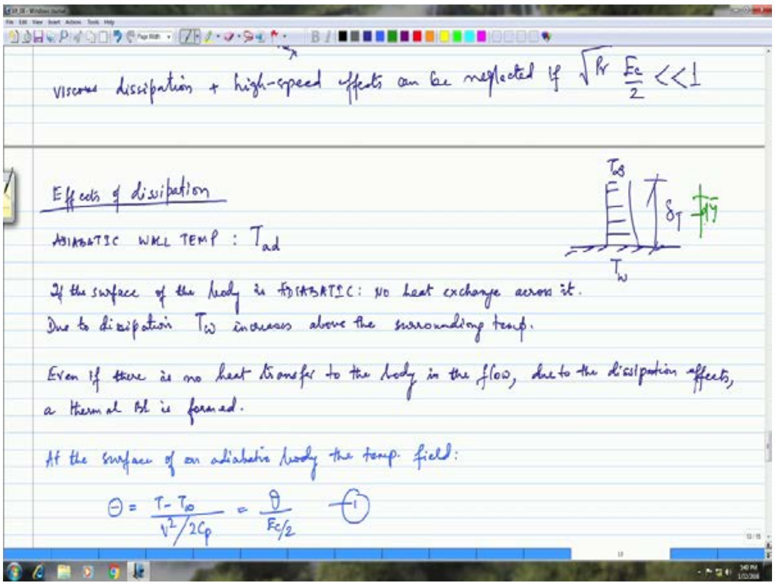

Let me say this is effects of dissipation. Of course, now I mean so far effects of dissipation we have been neglecting that. Now, in regard to this the distribution of the adiabatic wall temperature becomes very important within the context of the effect of dissipation. The distribution of the adiabatic wall temperature becomes very important, and we going to call this as T subscript ad for adiabatic. Of course, a first question is what exactly dissipation is, what do we mean by dissipation? And why are we studying this in context of the thermal boundary layer?

Well, the point is that even if we have a say a flat plate or anybody, if there is no heat transferred to the body, but due to the dissipation effects a boundary layers still forms where there is a difference in the temperature. Essentially meaning that there is a thermal boundary layer where the wall temperature becomes different than that of the free stream temperature. Therefore, a temperature feel develops between the flow attach to the surface that we talking about. So, dissipation essentially means that there is no heat transferred to the body in the flow. But due to the effects of viscous dissipation, there is a viscous flow happening so as a result of that the thermal boundary layer develops where the wall temperature is different, and that kind of develops into your thermal boundary layers so that it reaches the free stream temperature, some distance away from the body, which is what we call as delta T or the thermal boundary layer thickness.

Now the point is, if the surface of the body is adiabatic, I think you understand what we mean by adiabatic. Before I do that maybe I should just write what I meant by dissipation, fine we will do that. If the surface of the body is adiabatic, what is that even mean? There is no heat is exchanged across it, then due to the dissipation effects. So, that is what you mean by adiabatic. However, in this particular case due to the dissipation effects, the wall temperature increases about the sort of in temperature due to these increases well above the surrounding temperature.

Now, the point is that there is no heat going in or coming out, there is no heat exchange happening. In the sense how do I kind of define this a little bit? Let me first write down this what exactly is dissipation. In continuation of this, let me just write. Even if there is no heat transfer to the body in the flow, due to the dissipation effects thermal boundary layer is formed. The point is that you could just have, just think about this, that you take just a cup of tea and you take a glass of water. Now, the glass of water is at room temperature, so you can put your hand in that and that is fine you are with. But if it is like really hot cup of tea if you put a hand in it will burn, is it not? Now, if you hold the saucer or the cup itself it is also warm or at least hot.

Now the thing is that, if you leave this cup of tea after sometime the temperature will drop. Why? Because, now you have the ambient temperature or the normal temperature is less than the temperature of the hot tea, so the temperature is moving. There is a certain heat transfer flows from higher temperature to lower temperature simple. Therefore, now that happens. That is another way that there is a change in the temperature. There is a change in the temperature, in the sense that the tea becomes less warm and that probably around that if you put your hand on it this is going to get warm, because there is a certain heat exchange to it. So that is the way a temperature can change.

In this case, there is actually no heat exchange across the surface, nothing of this sort is happening, but this dissipation is really the viscous dissipation. What do you mean by that? This is also similar to the fact that if you put your pumps together and rub against them you probably see it will sort of warm up a little bit. So, is probably a little crude of explaining that because you have layers of fluid, you got layers of fluid on top of each other and they are kind of moving because of this viscous effect due to layer and it is this effect which causes a viscous dissipation as a result of which, while there is no heat exchange across the body in the fluid.

This effect which is the viscous dissipation effect where even if that is not slow, that wall temperature increases above the surrounding temperature due to the this viscous fluid wanting to move over the body. Therefore, because of that, and since the wall temperature in increases there is a thermal boundary layer because then at the temperature at the wall is this and then so that decreases and this becomes T free stream. This height at which this sort of thing is happening is the thermal boundary layers sort of talked about that. That is kind of what we mean by the effect of dissipation. This is essentially viscous dissipation that we are talking about.

Now, let us do a little bit of math, because now all this understands. Now, how do I get an account of this? We will do a little bit of math and see at the surface of the body we will define a temperature field.

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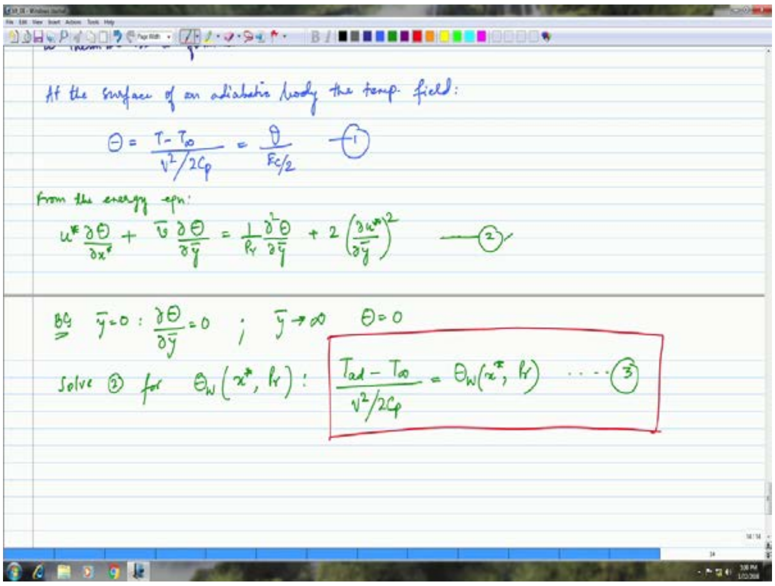

At the surface of an adiabatic body the temperature field, I am going to define it like that. This is essentially a capital of theta. This is something how we you know define this. Then from the energy equation we get this and the boundary conditions are the following. The boundary condition at the wall basically, is not that obvious. So there is essentially no temperature field. It is basically this is equal to the free stream so then we do not have one. Then what we do is, we essentially then solve this equation 2 for the wall temperature field. So, what we saw for then we solve 2 for this which is the function of the prandtl number and location, so that we get an expression for the adiabatic wall temperature. Which we do this so that we get that right so we do that.

Obviously, now what you see here is that essentially that this again depends on the prandtl number. We can see that now this depends on the adiabatic wall temperature depends on the thermal boundary layer. Again, what we will do is that we will go ahead and look at the two limiting cases; small prandtl number, large prandtl number and see what that means. Let us do that and see what information we get.

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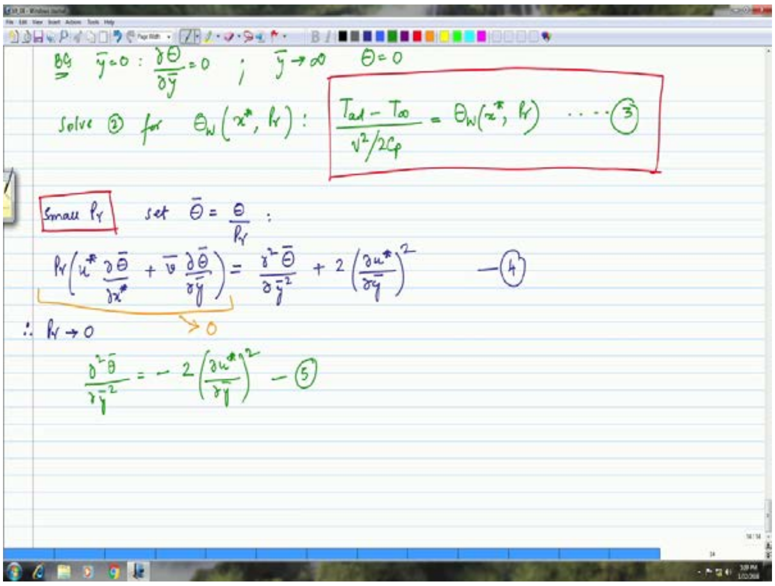

We are going to start off with small prandtl number. If I start with that, so that is my small prandtl number. Then here, what we are going to do is, let us keep this equation. Let us keep looking at equation 2, what we are going to set? We are going to set theta bar by prandtl number. If we do that what we get is this. It is actually very interesting if you see this. Just simple playing around with the equation gives you interesting information. What we basically do is, we just set the theta bar, which is theta bar as theta by prandtl number and we just change that and put that into 2 here. What we get is this. So we get equation 4. Now, for a small prandtl number, therefore, if small prandtl number, what you think happens if you look at the equation 4, then if you see here the entire left hand side disappears, if prandtl number goes to 0. This basically then tends to 0. Then if that happens then this from the equation 4, what we get from this equation is the following. This we will do a very simple math to solve this.

Now, what we will do is, we will just do a very simple math to walk around this. So, what we shall do is integrate this, or say a very small elemental or to the boundary layer so y bar. What I am trying to say is, we going to integrate this over say a small height of y bar. Then we are going to call that is d y bar take a small little element like that and integrate this thing.

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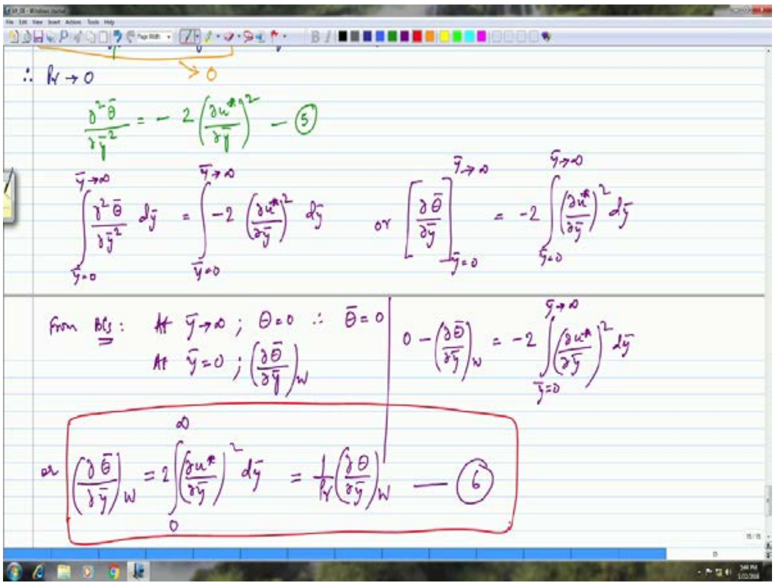

Then if I do that, what I mean by that is this, that I say del 2 theta bar d y bar is equal to minus 2 d y bar. And we going to integrate this that y bar is 0 and y bar is very large. Again, y bar is 0 and y bar is this. Now, we know if I do this from here therefore, what this if I integrate this, what I will basically get then is del theta del y the limits y to 0 that which is equal to minus 2 del u star del y square d y so we will have that. Now from the boundary conditions, now at y bar this right therefore, this is also equal to 0 and at 0 we just know we are going to just say del, it is denoted like that.

So, basically that, so we going to find that out, if I do that, again from here what do we get. So if I do that, the left hand side of this basically then becomes let it may continued here, that so at infinity if I go there so what I will get here is, this becomes 0 so I get 0 minus wall is equal to minus 2 0, that. So finally, I can write this. Or from five basically, what I can do is. Therefore, the negative signs cancel out so I can write from 5 del theta bar del y at the wall is equal to 0 to infinity 2 del u star del y bar and this can also basically, if I were to write this and this will become 1 by prandtl number at the wall, is not that so.

This is an expression that I am going to call as 6. We will discuss this a little bit when we come back into the next module, that what is this even mean. Now that if you do have for very small prandtl numbers and we have taken into account the dissipation effects as we just said that this in here, the wall temperature or the thermal boundary layer thickness will depend on both Reynolds number and prandtl number. We have use the energy equations here and we have come up with the expression; we have come up with the relationship for the velocity field near the wall. Let us come back and look at this little more in detail in the next module. So, we will stop here for the movement.

Thanks.