Introduction to Boundary Layers

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Module – 04
Lecture – 36
Effect of Prandtl Number in thermal BL-III

Hi, hope you are all set to discuss little more about Prandtl numbers.

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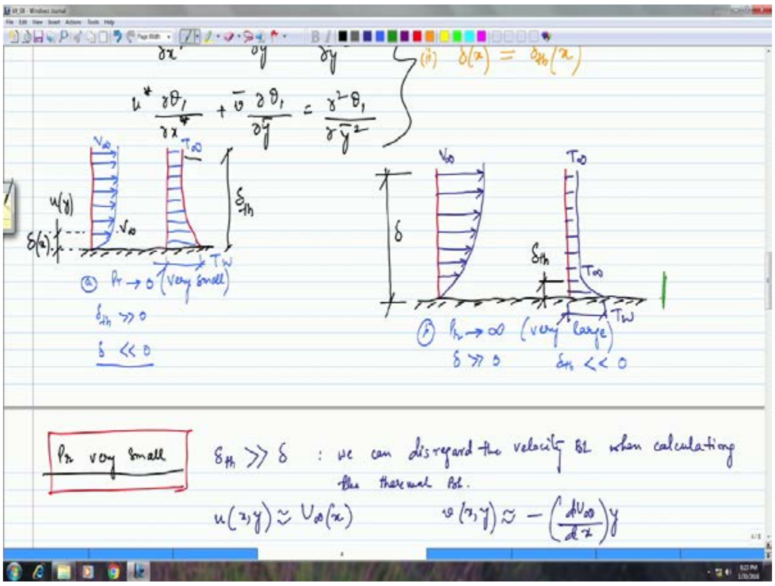

So, where we stopped was that we were discussing was the effect of a very small Prandtl numbers. And what is a very small Prandtl number mean, which is a picture like something like this. This is actually for a very large Prandtl number, where the thermal boundary layer thickness is like very very small compared to the velocity boundary layer thickness.

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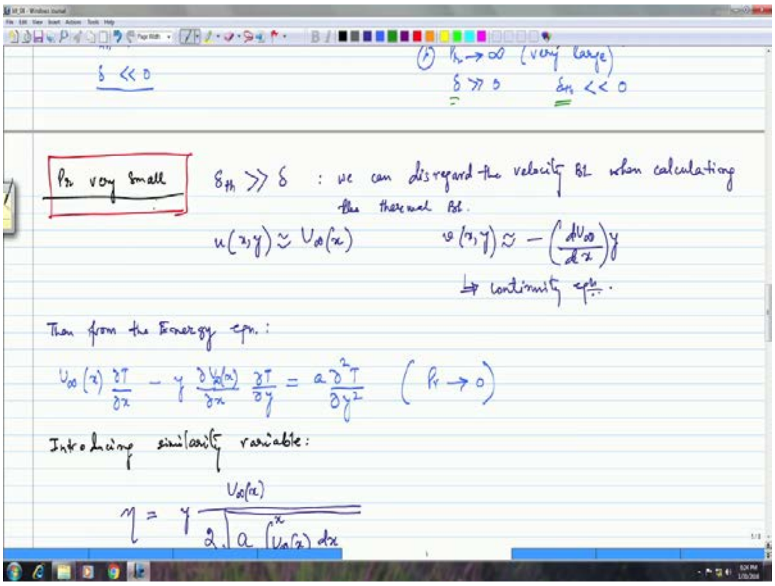

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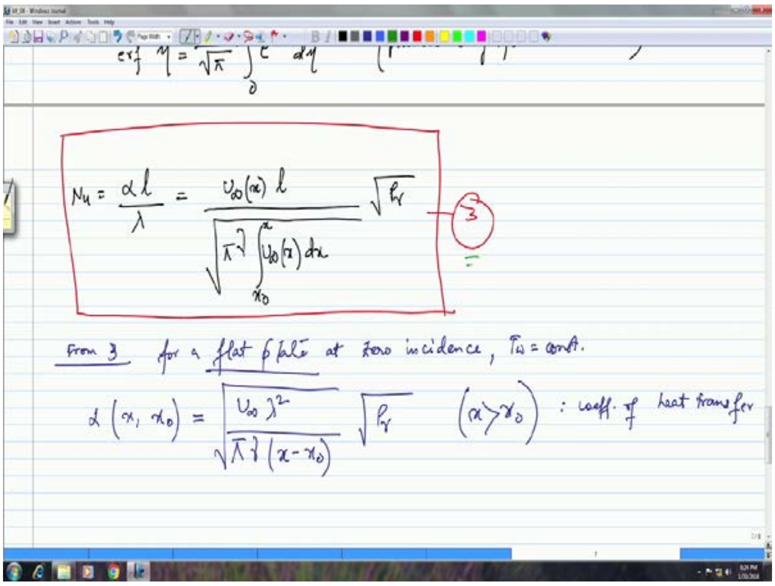

And we then looked at little bit of the math what we can do, so we looked at Prandtl number very small and we looked at the math and what kind of simplifications it brings about, and we saw that we could actually write the Prandtl number, I mean the nusselt number for a large Prandtl number in this fashion. And hence, you come up with equations, simple formulae which one can actually use from this consideration, so that is that.

Now the next thing to do is basically we are looking at the two limiting cases. So, this was we discussed about when the Prandtl number is very small and so where the thermal boundary layer; we discussed this, thermal boundary layer is much higher, is much large compared to the velocity boundary layer thickness. So, what we are going to discuss now is about this, where the Prandtl number is very large which means that the thermal boundary layer thickness is very small. Let us sort of do that now. So, we will go and discuss this, so let us see.

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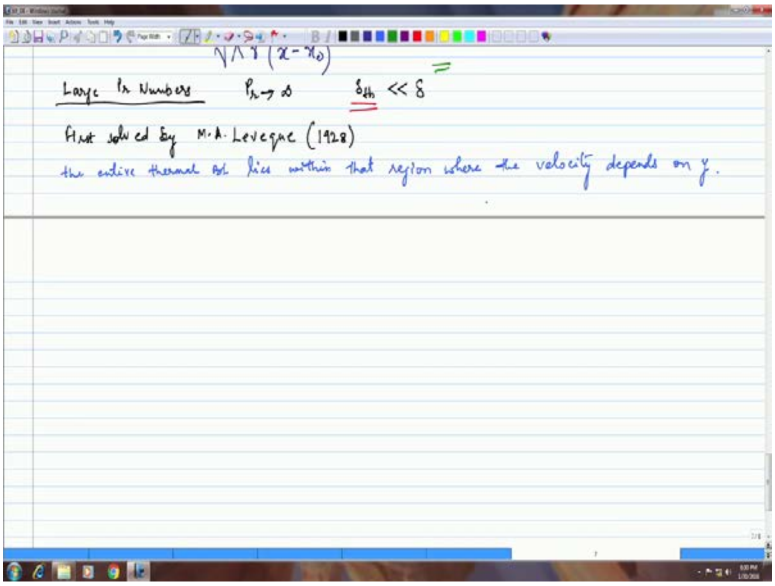

So, we are going to talk about that, we are going to talk about large Prandtl numbers, now so large Prandtl numbers that, Now, this was actually this sort of problem was first actually not even going to try and pronounce that, so this scientist basically and so the thermal boundary layer thickness is much smaller and the velocity. So, basically the velocity boundary layer thickness is the one which terminates.

In other words we can really say that the entire thermal (Refer Time: 03:46) because this thermal boundary layer thickness, because this lies, this is very small, if it is really very small compared to the velocity boundary layer. So, we can basically say that the entire thermal boundary layer lies within that region where the velocity depends; velocity is the function of y. So, I mean let us just go back and look at this diagram right here, now these are the two cases.

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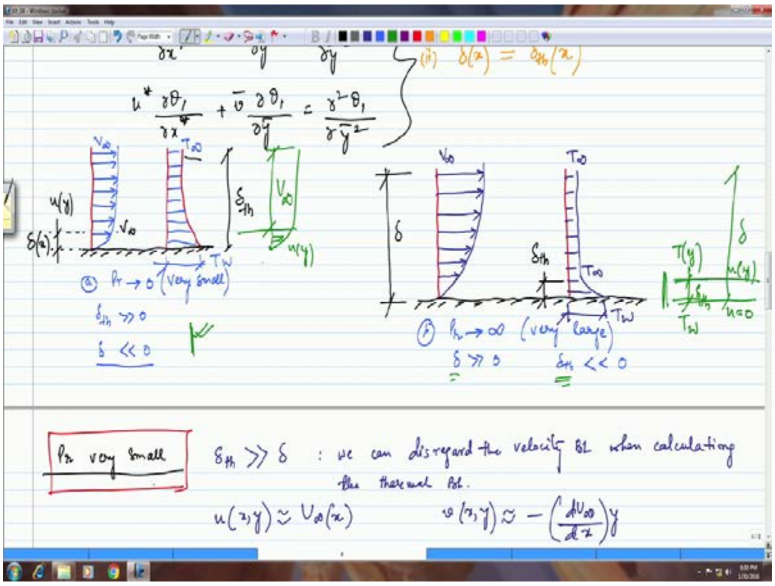

Now, if you say this was the Prandtl number very small case, this is the one that I am talking about here; this is the one that we looked at earlier. So, this is where the Prandtl number is very small and you can see that here the thermal boundary layer is large. So, it spreads from the surface to about this height and for the majority of the distance which is like from here to here actually, the velocity is constant; the velocity actually is v infinity is not it because this is delta x, because delta x is the boundary layer thickness, the velocity boundary layer. So, that really goes up there after that the velocity remains constant, therefore this is the u.

So, that is what we are talking about right now, that here delta t h for the case that we are considering now which is for very large Prandtl number. So, delta T h is the thermal boundary layer thickness, thermal boundary layer thickness is just this small, just this much. But since the velocity boundary layer is very large, now that extends from here to this much, this is the entire delta, but delta T h is just this much. So, what basically you can see that this, thermal boundary layer actually overlaps the delta in the sense that here there is a difference in the temperature. So, you go from the wall temperature to this and the velocity also goes from or you know some temperature, so I can say that or say u, let me call that as u and here u is 0, let me write that properly.

So, basically what I am saying is here u is 0 and here it is u y. So, v u have both, unlike you know in this a case where; so that is kind of different from what we did earlier. So, that is what I mean actually when I say this; depends on y.

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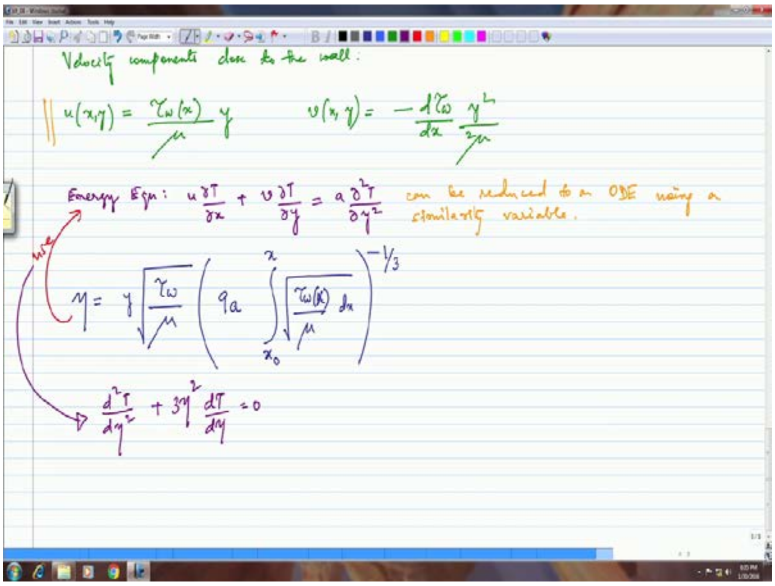

So, if we then can write the velocity components, you know close to the wall. So, we can write this we will write the velocity components close to the wall, so we can write that. If we write the velocity profile, velocity description here like this then the energy equation which I will just write this again here, so the energy equation; this is basically, it can be reduced to an ODE which is an ordinary differential equation, so, this is a PDE, partial differential equation. So, we can reduce this to an ordinary differential equation, using a similarity transformation. If I use these velocity, if I use this description for the velocity components close to the wall and the energy equation can be reduced to an ODE using a similarity variable or similarity transformation or variable or whatever it is.

So, let me write that, what is that similarity variable and that similarity variable can be written as y, 9 a; x; tau w x, where I hopefully you remember what the x naught is, x naught was the location of the temperature jump. Let me just go back and show you just remind you that, so if we took a flat plate; so this x naught was the location at which where we had this temperature jump, so we basically then again in the integral is basically from x naught to x.

Now so, if we use, so this is x actually. Now so, if we use this similarity variable then my energy equation reduces to, so therefore using basically what I can say is that, if I use this, then what I get is this, then the energy equation becomes d 2 t, d eta 2 . Now, basically this is getting a little mathematical, so I will try to keep it simple because it would be nice if we had enough of mathematical back ground, but that is OK if the progress in fluid mechanics is always been related to the development in math and the mathematical progress is so important because you just come up with all sort of weird equations and then we have to sit and solve them. So I mean we have to be grateful to the mathematicians to help us do these things, so the entire numerical methods etcetera you have never seen much day light without the contributions from mathematicians.

So, now that I understand all of it; but I will try to keep it brief and give you just a little over view as to what we do here. Now this is a stand, so this has a solution, which is for in terms of an incomplete gamma function. I will explain briefly about that let me first write this solution for you.

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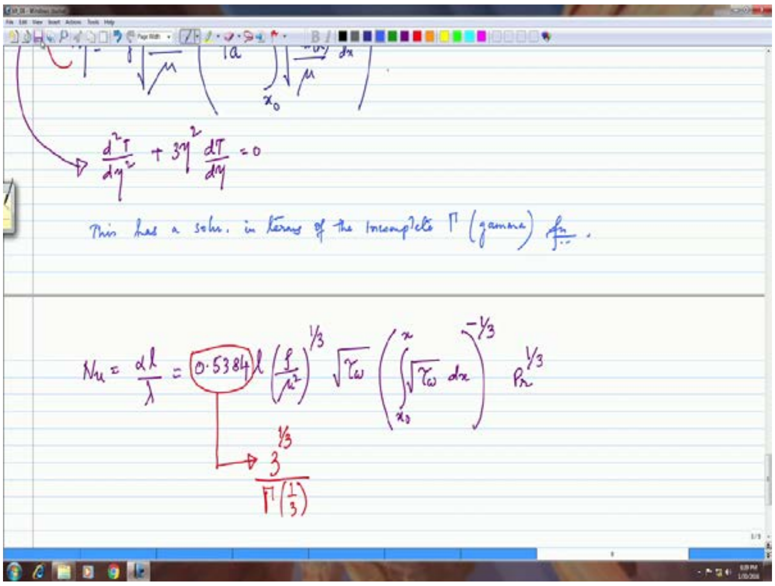

So, if I have this, this equation can be solved; this equation is solved and what you get from there is this is a solution l rho by tau w and so this is basically the solution. And here, so this 0.5384 this term, so this thing is basically by this. So, therefore, this gamma that capture gamma that you see this is nothing but the gamma function. So, what I will do is; so this is basically the solution that you get. So, the whole (Refer Time: 15:30) here is, that for large Prandtl numbers, if you use this velocity components description close to the wall, then the energy equation is reduced to an ODE using this similarity variable. So, the equation becomes this and this has a standard solution in terms of the incomplete gamma function, let me just sort of write there. So, this has a solution in terms of the incomplete gamma function, let me just you know tell you what exactly is this gamma function, I will do this very briefly.

So, now the gamma function is, so what is exactly a gamma function?

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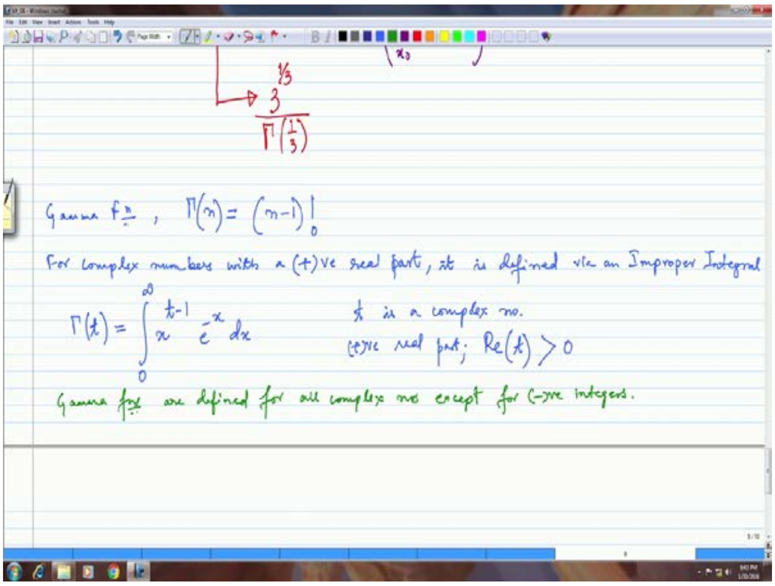

Now, gamma function; gamma n is nothing but n minus 1 factorial, so basically you know similar to the factorial except it is like n minus 1, factorial n would be factorial n. So, gamma n means one less you know than n factorial. Now let me sort of explain, what do you mean by the incomplete gamma function or something. Now, therefore, complex numbers with a positive real part, it is defined the improper integral. Now what exactly is an improper integral, so let me sort of write this down.

Now, for complex numbers with; this is important, a positive real part; it is defined via an improper integral which looks like this. So, that is gamma t is equal to right; where this t is basically a complex number, so t is a complex number with positive real part, which is that real part of t is greater than 0 and where gamma function is basically defined for all complex numbers except negative integers. So, I can write that as well, so let us just say gamma functions are defined well for all let us start do that, for all complex numbers except for negative integers. What is nice is that this is basically this is an integral, so in terms of the gamma, we said the gamma 1 by 3, as you can see, it is a positive number. So, now this basically this converges, absolutely to what is called all are integral of the second kind.

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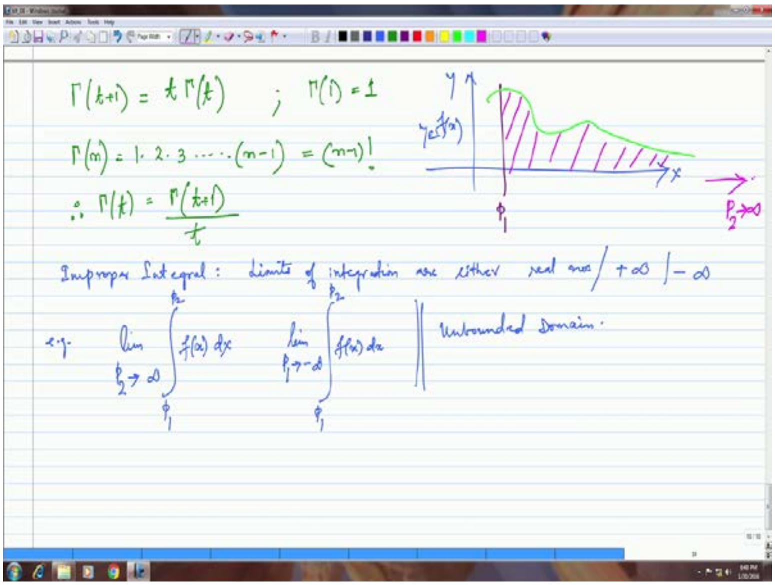

Now, then if we use sort of, I mean integration by paths. When we use integration by paths what we can, using integration by paths what we can say is this, now along this if I combine gamma of 1 is 1. Then what we can write is gamma of n is basically 1 into 2 into n minus 1. So, which is nothing but n minus one factorial, therefore we can also write gamma t is, gamma t plus 1 by t. So, it is a little bit about gamma function, now what we said something about improper integral. It is defined via an improper integral, what exactly is now improper integral? This is just defined a little bit.

Now, improper integral basically the limits are it is actually unbounded. So, the limits are infinity so, give you an examples for that. So, improper integral so, the limits; if I give you examples you will find better. Limits of integration are either a real numbers or very large or very small, what I mean by that for example, if I give you the examples for example, say we have limit, so we have functions like this say p 1 to p 2; f x d x. Then we have function we will write this again, again p 1 to p 2; f x d x. So, I can say limit for example, p 2 tends to infinity or I can say limit p 1 tends to negative; negative infinity. So, basically what we are saying is, it is unbounded domain, this is basically an unbounded domain.

Well, I can explain that little bit, you can kind of understand that, you could have say; some I do not know some function. So, say this is your x and y or something, this is y in the sense this is actually your f x, y is equal to f x. So, p 1 here is some boundary this is say you know this one, but p 2 is infinity, so say I have this function; so this function is something like this, something like that. So, then if I had to draw, if I had to integrate this, so what I do is basically take the area under the curve. So, if I had to take the area under the curve, so this is going to be my area under the curve then I would know where to stop because this is unbounded p 2; p 2 is really somewhere you know it is all I can say is that the next limit is p 2 which is infinity. So, this is what kind of physically means it is an improper integral, so that is what we basically use.

Now during; the point is, it can get a little scary you are like fine you showed us some real complex equations and then you came up with an even more weird sort of thing and you calling this as solution. Now how am I going to even do this? How am I even going to use this and all? Well, the interesting thing is that all of these things they reduce, they actually reduce to very nice formulae from here, so we do not have to worry about all these complex math. However, at the time when we are learning the subject and it is important to understand that what goes in, you know to make something as comprehensive with this as this. But finally, when it comes to applying it you would be using formulae to do that. So, I will give you briefly these formulae although you can find this in; I got this from the (Refer Time: 26:16), so the reference is there you can get that.

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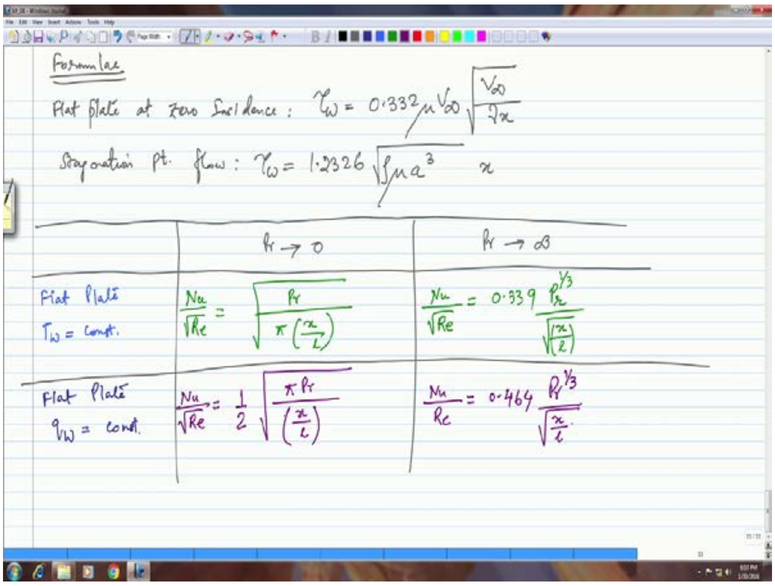

Let me just give you a couple of examples for this; I am not going to write all of this down. So, this is a kind of formulae that you can land up with, this is the formulae you can generate. Now, for example, flat plate at 0 incidence; so wall shear stress is given by this formulae or for say stagnation point flow, if I have a stagnation point flow, so there my wall shear stress is given by this. Now let me give you some these formulae, so from here is basically; actually I would not write out all, but some actually. So, this is for the two cases, for the limiting cases here Prandtl number is very small and here Prandtl number is very large.

These formulae is basically for specific cases, we have this for examples for flat plate, so numerical solution, you need boundary condition. So, one boundary condition is where T w is equal to constant, so this is more like the (Refer Time: 28:23) condition that I was talking to you about.

Then the solution here for this, Prandtl number this case is basically given like this; the nusselts number by this you get a pretty little formulae, nothing complicated you pick this up and just use it, that is it. So, that and then if it is very large then what you get is this, that this is the cubic root of the Prandtl number or if this is more difficult to understand best to write this is Prandtl number raise to the power 1 by 3 by square root of x by l. So, that is a like one of these. Then the next one is for the normal condition where the heat flux is given that is constant, so basically the gradient of the temperature. So then here again the same thing, that is equal to half by L and this is 0.464, Prandtl number raise to 1 by 3 by L. I have got a stagnation point and I got heated wall jet for both these conditions, so I could you know give you those as well.

So, basically the point is; so I am going to not go further and not write any more formulae, you can actually pick this up I am writing this only for the flat plate; just to give you an idea that what it kind of reduce is to. So, you get similar formulae you have similar formulae for this and the heated wall jet and things like that. So, basically that is what it kind of reduces to and you should be able to use these formulae to solve the problem. So, what I am trying to say is that this kind of usable formulae something that we can trust is something that you can come up after lot of research and thought behind it.

So, that is what the objective was to kind of give you a little insight into what went behind all of this research, and understanding so that we can come out with formulae like this and this and then we should be able to solve our real life problems with this.

So, I think we will stop here and I think we will be talking about the dissipation, effects of dissipation in the next couple of modules. So, I think I will stop here for now.

Thanks.