Introduction to Boundary Layers
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Module - 03
Lecture - 35
Effect of Prandtl number in thermal BL-II

Hey, I hope you are with me. So, let us continue this whole discussion on the effect of the Prandtl number. So we said, we are going to look at two the limiting cases.

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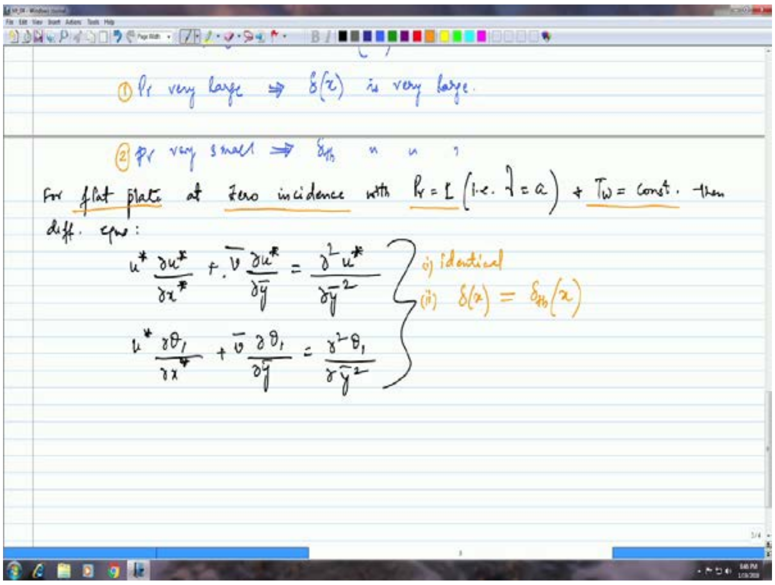

One were Prandtl number is very large one and the other case is when the Prandtl number is very small. Because, you know it helps us in getting lots of it simplifications in the calculations. So let us see, what do I you know a mean by that and here of course, the standard case where the wall temperature is constant is going to be considered that is what I am going to look at.

So, let us see. So now, when I say the Prandtl number is very small. So, what did I say here the Prandtl number is very small which means that delta thermal is very large and which also means that delta x is very small. So, let us set of draw that, you know whatever what does that if you mean? Let us set of draw that a little bit. Now, let us see if this makes sense to you.

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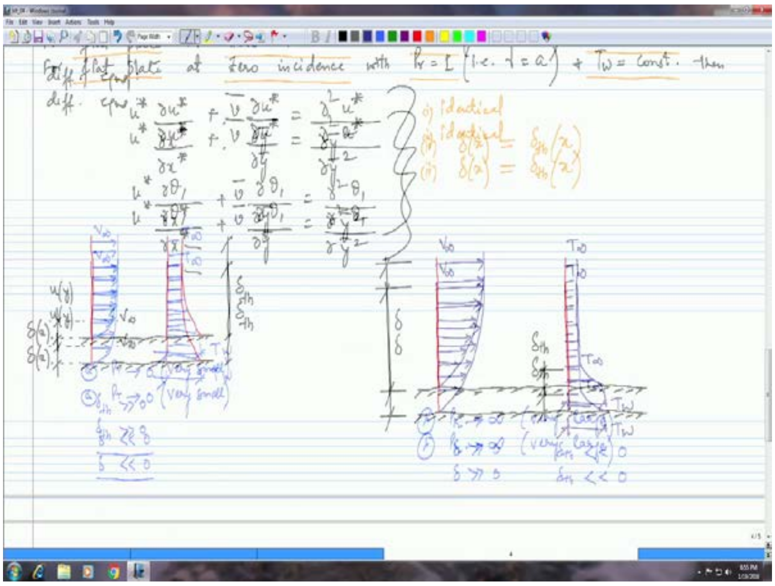

Now, that is my plate and this is case one, where the Prandtl number is very small. It is very small which means and what we have seen earlier, what that means is a delta h is very large, delta thermal. The thermal boundary layer thickness is very large which means, delta thermal is very large and delta is very small, the velocity boundary layer. So, what does that mean? Which means essentially, let us draw that; now, the term the boundary layer, the velocity boundary layer is very small. That is what we said which means that this is my boundary layer.

So, this is the free stream velocity, and essentially it is this height to which we have a velocity differential velocity or we can say it is in this region basically we have this and right after this the velocity here is again v infinity. So, it is as you can see delta x is very small. Delta x is very small now, the thermal boundary layer; however, it is very large.

So, what is that means? So that means, is that say I have this as the wall temperature and this is my free stream. Or even further if I were to set of you know exhausterate this a little bit. So, let us say it is something like this. Let me exhausterate that a little bit. So, what that means, I am not too while I am not getting straight lines here. I am trying to draw them as far as possible. So, this is the free stream and this is essentially the wall temperature. This is the wall temperature and this wall temperature therefore and it there is a certain heat transfer and at this height basically you get t after. This is the first term you get t in the free stream. Therefore, in this case this is the height required from for the temperature to go from the wall temperature to the free stream and that is the thickness of the thermal boundary layer.

So, therefore, this is a kind of a picture that one could visualize if the Prandtl number is very small and the Prandtl number is very small. So, the velocity will quickly reach the free stream velocity away from the wall. So, this is how it will look like.

However, the wall temperature you know it extends like we have said further earlier, that it extends quite large in to the full field and it goes it takes about this height delta t 8 to reach the free stream temperature. So, similarly we are going to talk about if the Prandtl number is very high. So, let us look at that. So, again we are going to draw the same thing. First we have flat plate not able to draw a straight at all. So, this one is again this is case b here the Prandtl number is very large. What is that mean? It is very large and so, which basically means that, delta is very large and delta t h is very small, is it alright?

So, in that case let us therefore, delta is very large. What can we say? Therefore, here something like that so therefore, and this is basically the free stream velocity. It reaches there the first times. So, here actually this is the height of the fluid. Fluid which is required for the velocity to go from 0 to the infinity, so therefore, this is the height of the velocity boundary layer.

Now, coming to the thermal boundary layer, here what basically happens is that is reaches the. So, this is the free stream. Which reaches it right here, this is also free stream. So therefore, it reaches right about so, this is your thermal boundary layer and this is your wall temperature. Therefore, this is your thermal boundary layer.

So, this is what we kind of looking at you know, in terms of writing these out. I mean you know graphically writing it a picture of what we are talking about. Now, if you see.

So, therefore, if you have a picture something like this. So, if you have a picture like this, one like a here right. So, the first thing if you could actually say: Oh! That is the case. You know delta x is very small, delta t it is very large that means, the Prandtl number is very small.

On the other hand, if you have something like this. Then you will say: Oh! So the delta is very high, that the delta t h is very small. So though therefore, the Prandtl number is very large. So, that is what physically, this means Prandtl number being very small and very large. So, now, let us see as we said that you know it also ranges some simplifications in calculations. Let us see if how that shapes up. So, this is the case where Prandtl number is very small. So, this is the case where, we will talk about this Prandtl number.

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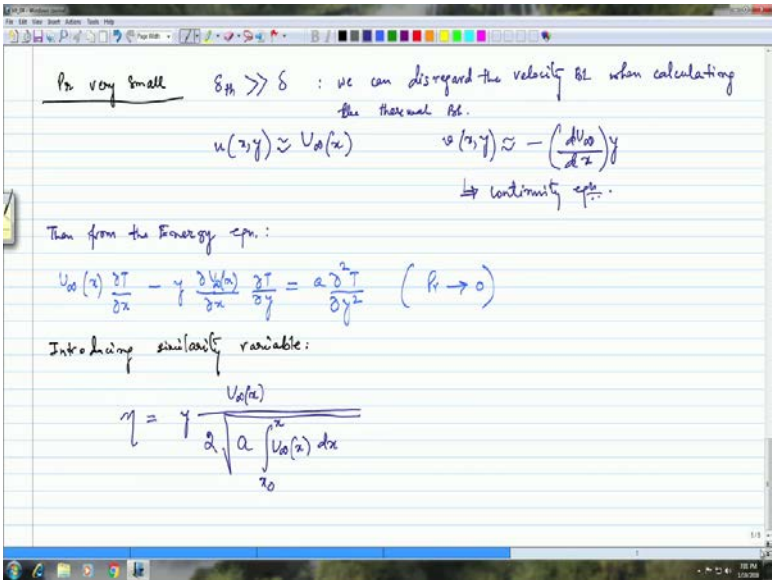

Now, in this case when Prandtl number is very small, which means now, you see the delta in this case, the thermal boundary layer is much larger boundary layer. Thickness is much larger than the velocity boundary layer. Therefore, what we can do here is that, we can disregard the velocity boundary layer when calculating the thermal boundary layer. So, what we going to say is we can disregard the velocity boundary layer when calculating the some more boundary layer. So, we can disregard the velocity boundary layer, when calculating the thermal boundary layer. Then in that case if we do that and what that means, is that the velocity can be written as, the u velocity is nothing but free stream and the v velocity.

Now, this is an expression; this is the expression for v actually we get this from the continuity equation. Then we can from the energy equation what we can write is minus y, the energy equation kind of looks like this you know this. Whereas, basically like I said we have introduced. So, what we said is that since, you know the boundary layer that was the thickness of the velocity boundary layer is very small so we disregarded. So, what therefore, what we will write is, so basically what we essentially saying is that, this you know this gradient in the velocity in this little delta x which is disregard that.

So, we basically say that is just the free stream the whole way through. So, therefore, we say that this u thing this small u is in this part right which we said. So, that u is nothing but the free stream and the v is nothing but this is we get from continuity. So, that and therefore, we use this into the energy equation and this is what we get. So now, what we will do is we will introduce a similarity variable as we talked about earlier.

Introducing similarity variable, when I do that; so, eta is y so I am just writing a lot of things. So, you must be wondering now, what is this now? Let us see what is this actually is? So, there quite of you terms will probably wondering what this is all about? So, I have taken the thermal diffusivity term here a and I am doing this interior going from x naught to x of the velocity u infinity. The u infinity seems to change as we from x naught to x, dx. And this is something that I am integrating and I multiplying it by the thermal conductivity. Now a just for a second, let us just go back and look at the formula with thermal conductivity. So, the thermal conductivity was something like this.

Thermal conductivity is something like that, fine. So now, let us see what this actually even you know means, let us define this a little bit. Now, what is this x naught x and so on and so forth mean? Now what we will do is, let me draw this flat.

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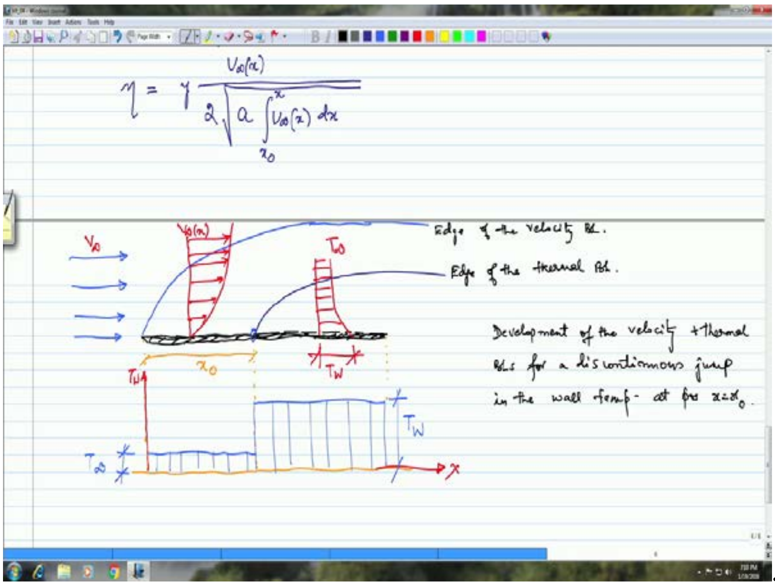

So, say I got a plate like this, I really do not know how to draw a straight lines any more. So, this is the flat plate now and you basically have a free stream coming in, you got a free stream coming in so on and so forth. So, what happens is that, you have a free stream coming in and there is something else here. Now this axis, I am going to draw this. This axis is basically of the wall temperature and this of course, is x. Then now here this bit, it is nothing but T infinity and around this point there is a jump and this is the wall temperature.

Now, here basically this is, this location is x naught. So, and you know this could be x for that margin. It could be anywhere between x. So, therefore, what you are saying is that this. So now, this is something this temperature profile thing is something that you have not seen or this is something we using right now is not it. So, you had been seeing this earlier. So, you have a flat plate and you have a velocity profile. Now it comes in and then it forms a boundary layer. So, let us do that, we going to do say that there it forms a boundary layer.

So, it just forms a boundary layer and if at any point you know at location. So, I was to draw the velocity profile, it would look something like that so on and so forth. So, this is v infinity x, this is v infinity. This is my velocity profile. Now this and it along and from this 0, this point is 0. So, or the origin of the flat plate till about x naught is what we get. Now from x naught you have a jump in the wall temperature is not it. So, you have a T infinity and T w. So, T w minus T infinity something which is positive therefore, you have a theta; so theta from x naught.

Therefore, what happens is that because there is so, it was at free stream temperature but now it is not. It is at a different temperature which is at the wall temperature. So therefore, it will try to get to T infinity and hence therefore, there will be you know transfer of heat across the boundary layer. And therefore, we will have a thermal boundary layer. So, that is basically the point, is not it. So, then what you are basically saying is that we will have something like this. Is these are not parallel or anything? So, then basically what I am saying, write that. So, this is basically free stream and this bit, this is the wall temperature.

So, then basically what I am saying is that this is the edge of the velocity boundary layer and this is edge of the thermal boundary layer. So, basically I can sort of label this. So, when I say x here, when I say x naught here that x naught is basically this. Now that x naught could be 0 for all you want. You it would have you know, they would be now discontinuous temperature jump. So, it will do basically start from there and yes. So however, what you can basically see is that every time there is a T w minus T infinity that is when we start having a thermal boundary layer and which is in this particular case starting from x naught.

So, I could you know a sort of this picture is. So, I could a set of label this and say this is development of the velocity and thermal boundary layers for this discontinuous jump in the wall temperature, at precision this. That is basically this picture what it is telling you. So therefore, if you see here what we are basically saying is that from x naught to x. So, basically depending on how far you want to go on the plate. So, starting form x naught that is the start of discontinuous temperature you integrate the free stream velocity and then, you multiply that by the thermal diffusivity.

And this shall give us; this will basically give us that is how we define this similarity variable, right? So, then what is this following ODEs are obtained for theta 1.

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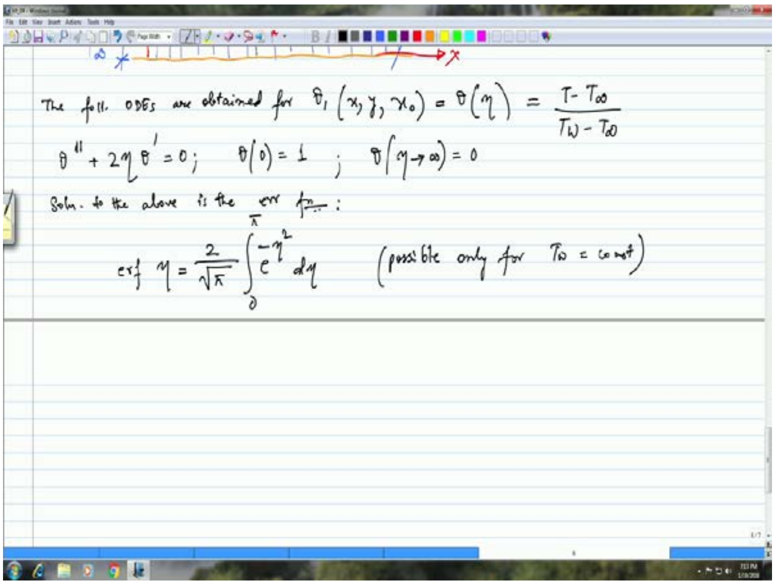

Now, the following ODEs are obtained for theta 1 which varies as x and y and also where of course, where it starts, is not it. Of course, I mean it varies because of that is well and who served as eta which is equal to eta and that is equal to T minus T infinity, T w minus T infinity. If I do that what I get is the following, theta double dash plus 2 eta theta dash is equal to 0 the boundary conditions being theta at 0 is equal to 1, theta eta tends to infinity is equal to 0. Now, that is basically my equation now solution. So, this is now coming down to math and calculus now solution to the above is the following in a function.

Now, solution to the above is the error function. So, what is that? Error function eta is 2 by under root pi e to the power and this is possible for this case. So therefore, when we solve this we do all math etcetera and we solve when we basically we use the math and we solve this then we get an expression for the Nusselt number. What is that?

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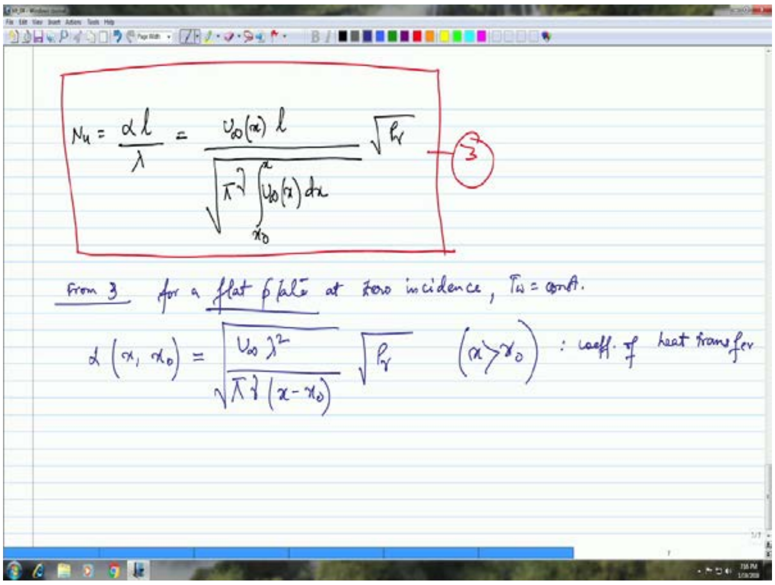

And that is by lambda which is equal to u infinity x into l by under root pi nu x naught x u infinity x dx under root Prandtl number. Now this is very interesting. So, we get an expression for the Nusselt number. I am going to call that as 3. So, what is that means? So, if I do get that then what is that even mean?

So, what that basically means for example that from 3; from 3 for a flat plate at 0 incidence alpha is a function of x and x naught is equal to free stream lambda square pi nu x minus x naught under root Prandtl number. And this alpha is basically co-efficient of heat transfer. So, basically what we saying here is that, when we look at these. So, you know when we look at these cases here you basically are able to write out simple equations you know very very simple. You are able to get these simple straight forward in a formulae with which then you can get more information about the some more boundary layer. So, in this particular case that is all you need to do. So, this is your co-efficient of heat transfer.

So, it should be your simple process to solve this. Now this equation that we wrote down for the flat plate here, for T w is equal to your constant. Now one could also write this in terms of you know if the wall temperature is a function, I mean it could be or like it could be a function an exponential function or it could be basically given. You could, one could also use that kind of a definition and do that.

So, that is pretty much what I wanted to talk about in terms of for the first case. Where, we talked about the Prandtl number being very small. So, if the Prandtl number is in this case is very small. This is the case. So, like I said if that happens we should be able to still ball it down to you know simple formulae with which we can get more information about the thermal boundary layer.

So, I will stop here and of course, will take up the next one which is Prandtl number very large like I said in the next module or so.

Thanks.