Introduction to Boundary Layers
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Module - 02
Lecture - 34
Effect of Prandtl Number in thermal BL-I

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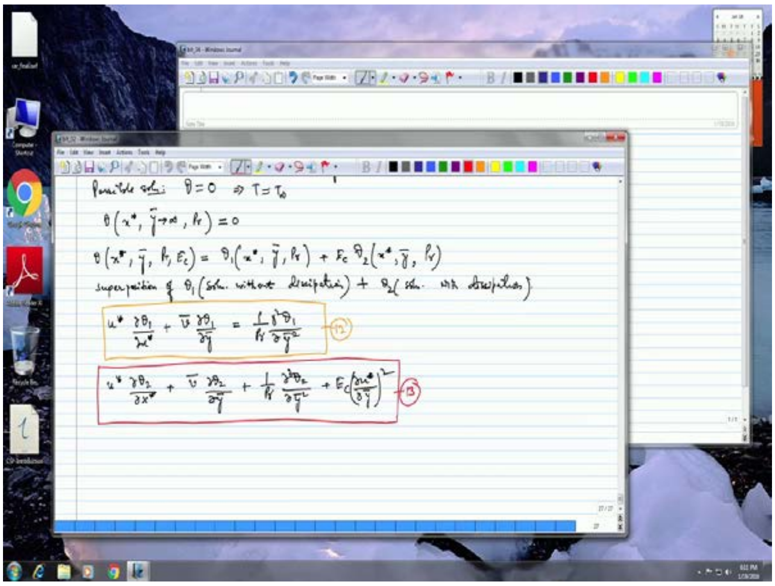

Hi, welcome back. So, we kind of talked about the energy equation and k map with the solution where we decoupled the velocity and temperature. And we came up, in the last time we stopped it was at this equations to (Refer Time: 00:37) 13. So, we also came up with this the Prandtl number and Eckert number.

So, what are we going to today is actually today you know couple of modules is to really understand what the Prandtl number here means and what does it set of mean in terms of physically what is it fall down to. So, because we relate a lot of things for example, the Reynolds number that something you more familiar with. So, we will also do this kind of similar studies to what exactly is this Prandtl number and how is it, you know what exactly is it mean.

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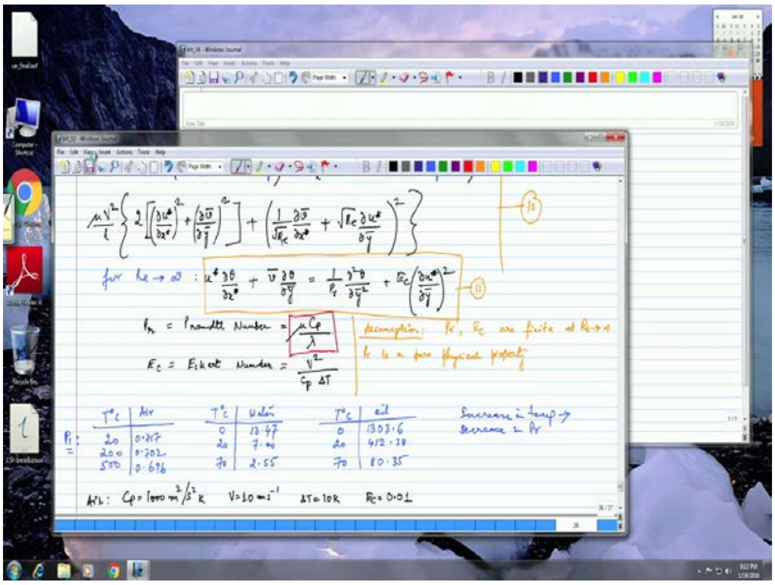

So, this is your Prandtl number that we had and this is the Eckert number. So, we will talk about that the Prandtl number and the Eckert number. So, let us begin to do that and see how things shape up and whether we can get some more inferences from this. Now, before I get to that just a little bit what do we mean you will hear this term a lot in the sense that forced convection, you know forced convection. So, I am going to define that a little bit.

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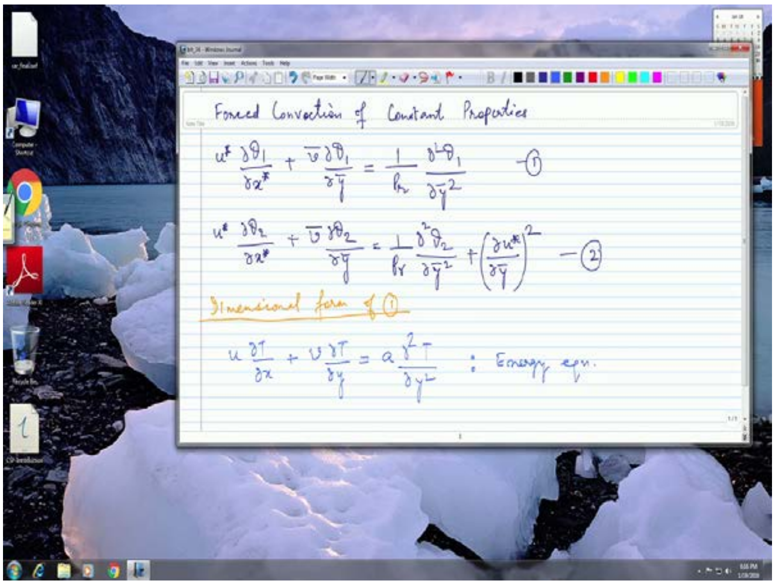

So, let me write down. So, let me first write this topic as Forced Convection of Constant Properties. Now this is the two equations which I just showed you, but I am anyway still going to sort of write it down again just to this for continuity I guess. So this is a non-dimensionalised form Prandtl number and let us call that equation 1. It is equal to one by Prandtl number del u star del y bar square, and let us call that as 2.

Now what we are going to do is to write equation 1 again in the dimensional form. So, basically what we are saying is that Dimensional form of 1 that is what I am talking about. So, we have done this map earlier you can do this to cross check, I am not going through the whole drill of it. So, it will look like this. So, if I were to do that it will look like this. So, this is essentially the energy equation.

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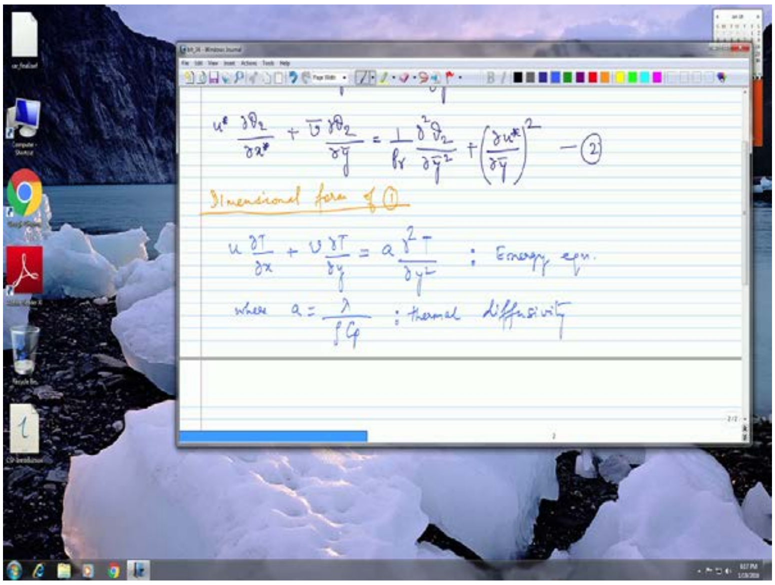

This is the energy equation where a is equal to, right. So what is a? a is the thermal diffusivity, and lambda is something that we said earlier, so lambda is thermal conductivity. So, we have done this lambda before so I am not writing that again. So, a here is essentially is thermal diffusivity. Now what happens here is as we emphasize earlier also, you can see here in order to if I were to solve this equation this energy equation in this form and if I would get a you know an expression for T across the boundary layer, I would need to know the velocity. I would need to know the velocity first. So, I would actually, so if I were to write this term let me sort of write this down.

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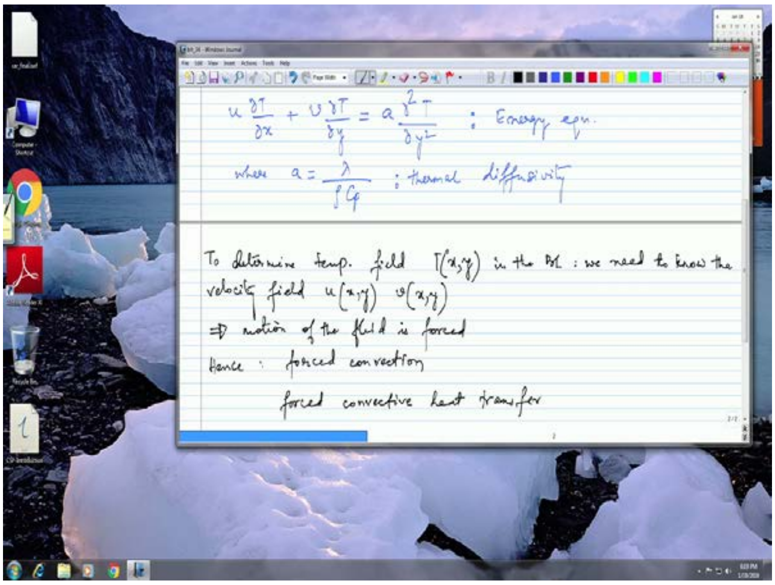

So, basically what I am saying is; To determine the temperature field that is T x y in the boundary layer, we need to know the velocity field, which is this. So, therefore, basically we are saying that there has to be a flow, so which means we are forcing the flow to have a velocity. So, we need the fluid to have a motion for us to determine the temperature field, so that is why we come up with this terminology forced convection or force convective heat transfer. So, what you basically saying is, so what this essentially means that the motion of fluid is forced you have to force the fluid to have a motion, so that it has a velocity field, I can calculate that and hence I can calculate the temperature which essentially means that motion of the fluid is forced. So, therefore, hence we use the terminologies forced convection and forced convective heat transfer.

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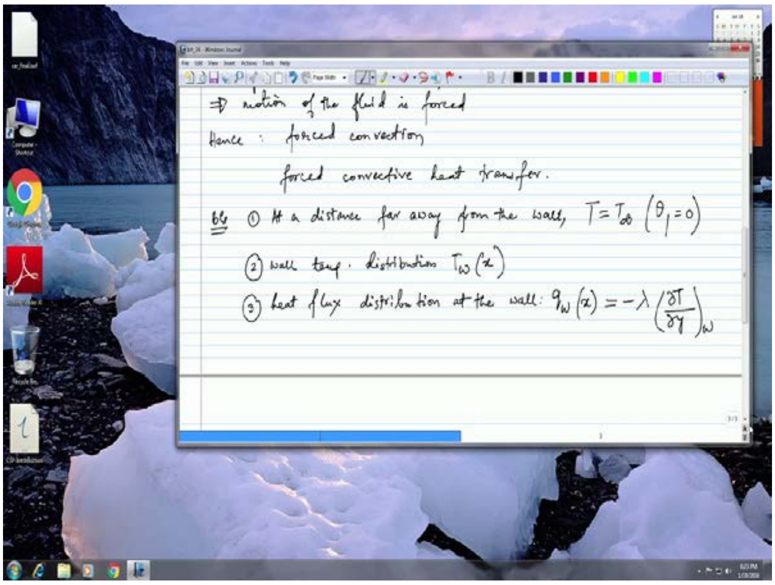

Now, we will need to have some boundary conditions of course, so what are those boundary conditions, what are those boundary conditions? Of course, at far away from the far distance away from the wall or the plate of the surface whatever you are looking at the temperature field. The temperature is equal to the free stream temperature. So, at a far distance from the wall, at distance far away from the wall that that would be the right English, at a distance far away from the wall the temperature is equal to the free stream temperature, so that the non-dimensionalized temperature difference theta 1 basically is 0.

We could have a definition or we could specify the wall temperature distribution. So, we could have a wall temperature distribution, easiest thing would be to make it a constant and we will do that actually. So, you could have a wall temperature distribution, you could also have the heat flux distribution at the wall, you could have a heat flux distribution at the wall, which is that is equal to; let me write that neatly is equal to.

So, what actually this is I am not sure if you are realizing this, but this is kind of you know, if you have a wall temperature distribution then it is basically you know the temperature the wall directly. But if you do not it, but if this is not specified; if this is the boundary condition not being used, then you could also have boundary condition like this third one here, where you are basically you know the heat flux distribution. So, which is you know at the normal component of the temperature at the wall.

This is similar to the Neumann and Dirichlet boundary conditions you know velocity if you are aware of it. So, that is you know either you say the property that you talking about; in this case the temperature is directly you know specified which is case two here or the normal components which is the Neumann and Dirichlet is the second one. So, the Neumann boundary condition is similar to the third one actually, so that is what we are saying.

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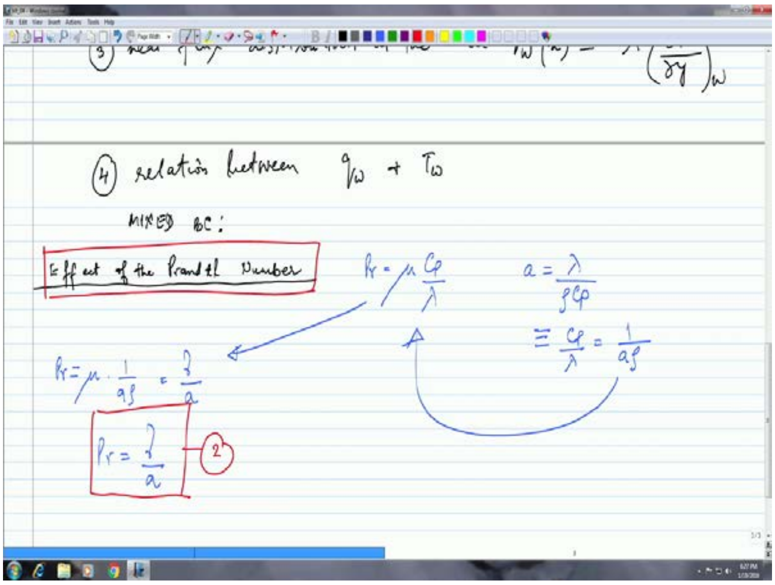

So, now another thing is that you could have a combination of the two, you know you could have a combination of the two as well. So, which is basically the there is a relationship between q w and tau w. So, there is a relation between tau w and this is actually called mixed boundary condition. So, this is a mixed boundary condition. So, basically we will have to calculate both simultaneously. So, temperature field and basically temperature field at the wall and the body have to be calculated simultaneously, so that is what we mean by the mix boundary condition. So, that is how we sort of go about doing that.

So, now, let us come back to what we started out saying that we going to look at the effect of the Prandtl number. So, this is what I am talking about the Prandtl number here. So this is what we are talking about the Prandtl number here. So, let us sort of call is that.

So, what we going to say here is effect of the Prandtl number. So, let us see where we get from here. Previously I mean when we do this earlier, we know that Prandtl number was defined as mu cp by lambda, but just now if you say thermal diffusivity we got a to be defined as lambda by rho cp. So, then from this expression what I can write is, so from here I can write cp by lambda is equal to 1 by a into rho. So, if I use this and put it back here, so what I get is that Prandtl number is equal to mu into 1 by a by rho, but we know that mu by rho mu by rho is nothing but kinematic viscosity a. So, this is important.

So, basically what we are saying is that Prandtl number is a direct ratio I am going to call this is as 2. So, Prandtl number is a direct ratio of the kinematic viscosity to the thermal diffusivity that is interesting. Now give this a little bit of a physical feel for what this actually is you know it is going to be like, so let us see. So, it is actually a physical property of the fluid and here of course, as we have seen similar to how the Reynolds number is something that we use for subsonic flows. So, Prandtl number here is characteristic number 4. Some more boundary layer and heat transfer in forced convection so that is basically was the Prandtl number is.

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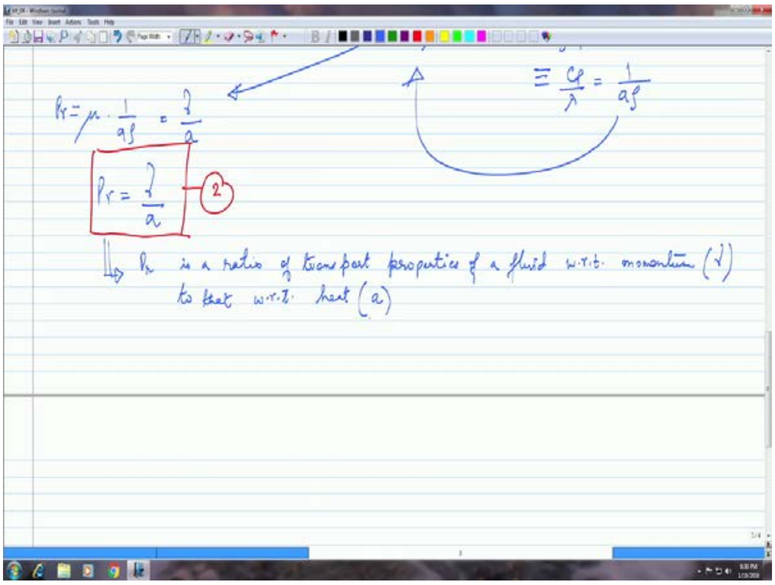

So, basically what we can say now infer from 2, from 2 what we can infer is that from 2. So, from 2 what we can see here is that it is a ratio of transport properties of a fluid with respect to momentum, because there is a momentum change across the boundary layer due to the existence of the kinematic viscosity. To that due to the existence of heat transfer which is a here thermal diffusivity. So, if I want to write that down the Prandtl number it is meaning Prandtl number. So, I can write that Prandtl number that Prandtl number is a ratio of transport properties of a fluid. So, there is a transport of properties of a fluid due to momentum so with respect to momentum and that happens due to the existence of kinematic viscosity to that with respect to heat and that is due to the thermal diffusivity.

So, basically we are saying that if there is a momentum, if there is momentum transport, we talked about this earlier, so we have a velocity profile across the boundary layer we have a momentum transport across the boundary layer and that is because of the existing viscosity. So, with a trans positive of our fluid with respect to momentum and we have a heat transfer across the boundary layer, because of the existence of thermal diffusivity. So, Prandtl number is basically is a ratio the transport properties of fluid with respect to momentum to that with respect to heat.

So, now the point is that as you can see you know very easily that if Prandtl number is very large, you can see from here that is the Prandtl number is very large which means that the kinematic viscosity is very large with respect to a. So, which means that if the kinematic viscosity is very large which means that we will have a very large boundary layer thickness, which means that this change in momentum or the momentum decrease that happens because of the surface pools on the flow, so it slows down. So, it slows down.

Velocity is 0 at the wall it increases to free stream away from it. All of this happens because of the nu and hence of course, there is a change in momentum there is decrease in the momentum. So, this effect is quite dominating and it extends far into the fluid. So, there happens if Prandtl number is very large. So, you one can sort of understand that which means that delta x which is the boundary layer thickness will be very large, if the Prandtl number is very large.

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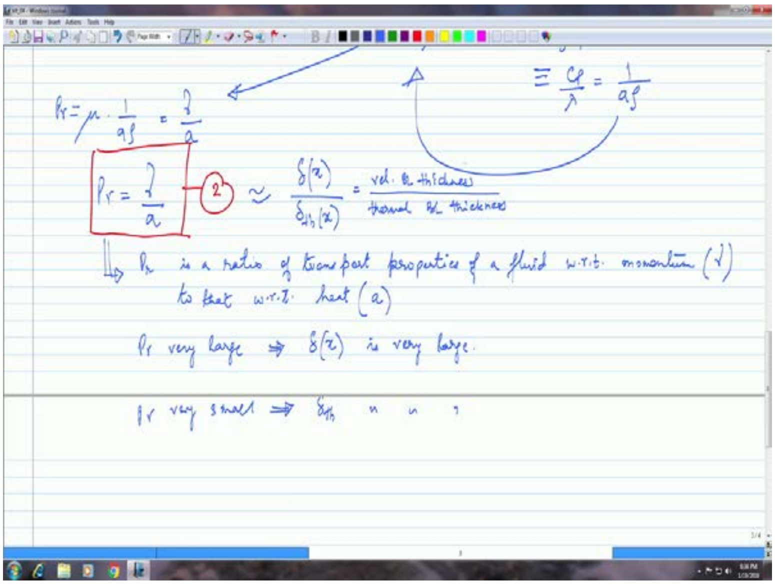

So, up right here again we can say that Prandtl number is very large implies that boundary layer thickness is very large. Thus similarly of course, if the heat dominates if the heat dominates, so when Prandtl number will be very small. So Prandtl number is very small what happens? Then the heat dominates, when Prandtl number is very large that means, delta x will not dominate, then the thermal boundary layer will be dominating. So, we can say that thermal boundary layer is very large or that dominates, I will expand on a little bit right now.

So, therefore, Prandtl number is also a direct measure of these two heights or these two distances like the velocity boundary layer and the thermal boundary layer. So, therefore, you know Prandtl number as we said is the property of the fluid and it gives you a ratio, idea of the ratio of transport properties with respect to momentum to that of with heat. And it is also a direct measure and it is also a direct measure of the boundary layer velocity boundary layer thickness to the thermal boundary layer thickness.

So, we can say that this is the velocity boundary layer thickness to the thermal boundary layer thickness. Now, so that is to talk about what is exactly now we are trying to basically understand what physically or what exactly we came up with this number, so Prandtl number. Now we need to figure out what exactly that means, and whether you know we can plot things you know against the Prandtl layer and make inferences out of it like we do for Reynolds number. So that is what we trying to do here.

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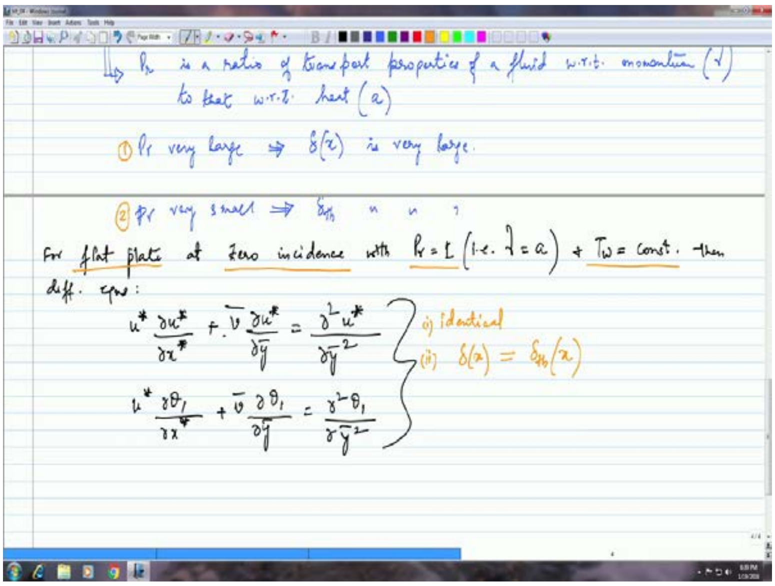

Now the next thing, now for a flat plate at zero incidence, for a flat plate with Prandtl number equal to 1, which means that is what nu is equal to a. And we had boundary conditions, so the wall temperature is to use this. Then the differential equations, then if you see I am reproducing this you can look this up yourself, we have sort of. So this is for the velocity. So, this is for a flat plate at zero incidence, at flat plate with zero incidence basically that it is inline the flat plate is kept in line with a free stream it is line for the free stream the Prandtl number is one and the wall temperature is constant. Then the differential equations turn out to be like this. And here we will basically is you can see that there actually identical, they are actually identical. And in this case, they number one that they are identical the differential equations are identical.

Secondly, in this case, delta there is a velocity and thermal boundary layers thickness are same that is so that is for a flat plate, so that is what it comes down to if I were to look at Prandtl numbers. If I had a Prandtl number which is equal to 1, and I use a wall temperature which is constant then what it balls down to is that the equations look exactly, they are exactly identical and the two thermal and velocity boundary layer thickness are same. Now please remember that, this is essentially for the conditions that I have said it is a flat plate, it is a zero incidence, Prandtl number is one and tau wall is constant.

So, now, for any other flow; however, given the conditions, so these are of the same order of magnitude. So, they not really fold a part or anything. So, now what we going to do is sort of continue on this a little bit and talk about two very important tool set of limiting cases. It is kind of just talked about that one is the Prandtl number is very large and secondly, the Prandtl number is very small.

So, we kind of have an idea what that means, now because what happens here is that it brings about a lot of simplification, you know simplification in the type of equations that in the type of a solutions that we can come up with that a formula actually. And therefore, it is of lot of interest. So, I think that is something that we will start looking at and so what I will do is and I will stop here at this moment and take it I mean take that up in the next module.

So, I will kind of see you there. So, let us see, I have forgot to make this full screen at the beginning I hope you kind of got that. So, basically we talked about forced convection, and then we came up here we talked about the thermal diffusivity we wrote down the dimensional form then the equation we got that. And we talked about why we are saying, what exactly it is term forced convection mean if in, right? So, forced convective heat transfer, we looked at what kind of boundary conditions are possible. Now, we started talking about the effect of the boundary the Prandtl number.

So, we will stop here and take it up in the next one.

Thanks.