Introduction To Boundary Layers
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Module - 01
Lecture - 33
BL separation with pressure- gradient-II

Hi. So let us continue with the Separation, we were are looking at separations.

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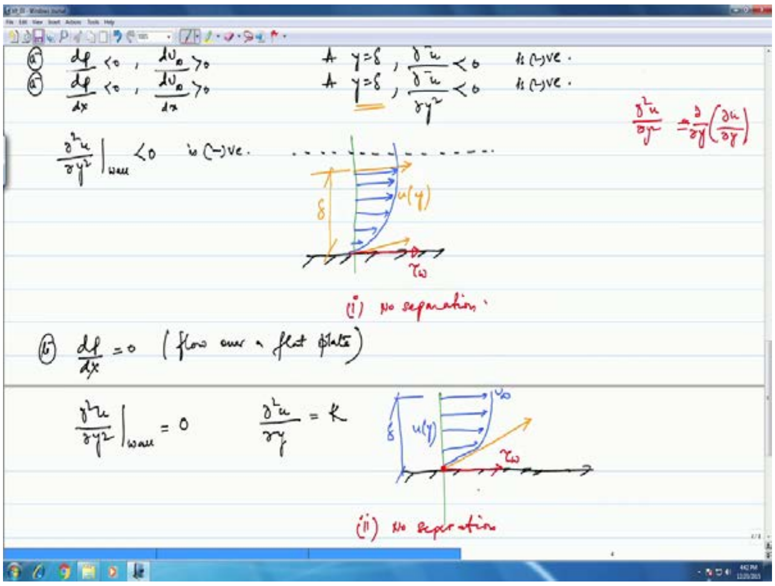

So, now again this we started off with this case, right. so what we are essentially looking at is the del 2 del y you know this value. The del 2 u del y 2 at the wall and at the edge of the boundary layer, and that is how we kind of defining that how the shape of the velocity profile should look like which is the profile of u, how the u should look like.

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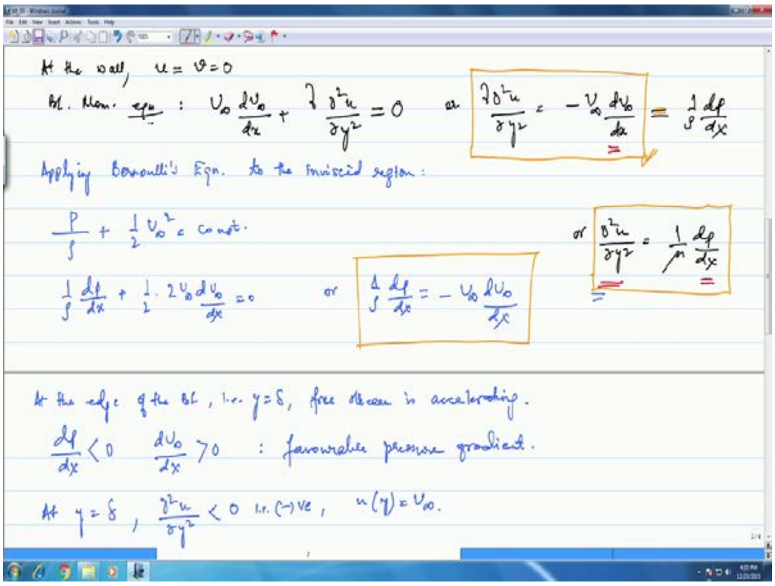

We are kind of looking at the value of del 2 u del y 2 which depends on as you can see the pressure gradient as well as the free stream velocity gradient and, what we are basically saying is that between the edge of the boundary layer; this one is the edge of the boundary layer and this one is the wall then how this slope should look like You know, so that we can comment on the shape of the boundary layer. And from here we are going to set of talk about how separation occurs.

Now the first case that we said is that both of these the del 2 u del y 2 is negative both at the wall as well as at the edge of the boundary layer. Now it is going to be negative always at the edge of the boundary layer you know to merge into the free stream smoothly. I think you could figure that out yourself, but I can come back and sort of you know if I have time I will give you little hints about how to do that. Except, basically when you say del 2 u del y 2 this is nothing but slope of a slop del y of del u del y, and so if you were to look at this, use this understanding and at the edge of the boundary layer where this u is very close to the free stream. Then this should give you an idea that why I am saying that del 2 u del y 2 near the boundary layer is actually negative is always negative.

Alright, so now going to the next one; we started with dp dx being negative, now we are saying dp dx is equal to 0. And what we are seen in the previous module that the dp dx being negative or positive is the basically effected by the shape of your body. So, dp dx is 0 you know because if the flow is accelerating you have dp dx is less than is negative and vice versa. If there is constant flow or you know non-uniform will be and uniform flow or you know stagnant then dp dx is 0 which happens for the case of a flow or flat plate, this is nothing but flow over a flat plate.

In this particular case then if dp dx is 0 then del 2 u del y 2 at the wall is 0 which means if I integrate this that del 2 u del y at the wall is a constant, which means that del 2 u del y as a constant. In this particular case which means that there is a linear growth. So, what I mean by that is that if I had to set off again draw this, so this is my wall somewhere. So before I do that let me draw this and say this is a linear growth, something like that. Your slope is something like that which means that in this way it is linear in this region that is what it means. And of course, here it is u infinity and so on and so forth. So, this is linear growth that is u infinity, this is u y and this is of course delta. And again of course, we have tau wall. So, tau wall is always going to act; this tau wall, right? So, what we are basically saying in this particular case again, so this is 0 del 2 u del y 2 is now 0.

In the sense now del 2 u del y 2 is now 0 at the wall, and this is negative of course, at the edge of the boundary layer. What we had earlier is that this is kind of negative at the wall, negative at the wall. You can think of the thing that the del 2 u del y 2 is basically it is a like I said del y of del u del y, so is the change in slope that you looking at. So, what you can see here is that between the wall, the first case between the wall and edge of the boundary layer both if there is no change both are negative so that means there is no change in slope as such here. But then this is negative here, so there should be an inflexion point which should be somewhere inside which is in this wall somewhere.

Now in this case however, you have a positive I mean it is 0 right del 2 u del y 2 is 0 here and negative here. In this case the point of inflexion is actually at the wall. Because then beyond 0 it will become negative. So, the inflexion is therefore, here. Here it becomes a linear and then at the edge of the boundary layer it is negative.

So, this is the second point here. Again there is no separation here. And now the interesting thing is that we are going to look at this, right. Now the next thing is we will look at basically the adverse pressure gradient.

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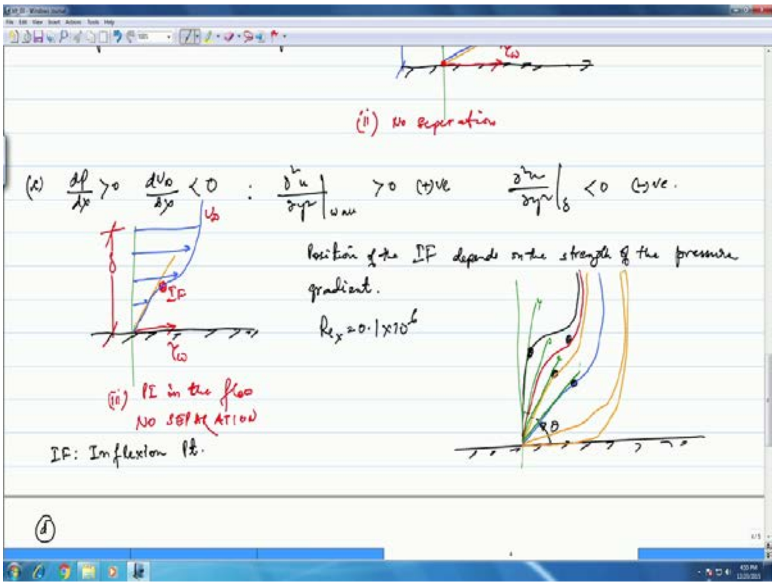

In this case we are looking at adverse pressure gradient where dp dx is this and d infinity. It is basically retarded flow; retarding flow. In this case therefore, as we can see that del 2 u del y 2 at the wall is actually positive, and we know that del 2 u and del y 2 at the edge of the boundary layer is negative. Therefore, there is a change in slope within the wall and the edge of the boundary layer somewhere in between. Where it goes from positive at the wall to negative at the wall. So there has to be an inflexion point somewhere. Let me see if I can sort of draw that picture.

What I mean is, so this is the wall and OK. So, now you have a positive something like that or maybe I make it a little more sort of steep say something like that and, in that case if I were to draw something like this; if were to draw something like that, you can see that there is a certain change in a slope around this point. So, this is a slope here. Let me erase that bit you can sort of and imagine that. So what you can see is around this point there is a change in the slope as you can see, because throughout this point it will be along this orange line that you see but after this here slope changes.

Let me sort of elaborate that on that little bit. Till before, till it reaches before this from the way I have drawn the curve what I see is the slope is somewhere like this, but after that this is positive, and after that around this movement it crosses that somewhere like that the slope is this. So you can see that the slope basically is now decreasing right, is positive and now it decreases and it becomes 0 around this point, and then it will actually you it will become negative as we go to the edge of the boundary layer. That is what this essentially means. So you have your velocity profile like this, so this is the inflexion point which we shall write here inflexion point of course, you have tau wall and this is again delta and this here is u infinity.

In this particular case, the point of inflexion is basically now in the flow, no separation; in this here. Now the question is where do think the position of the inflexion point should lie, would clearly it depends how much positive or how much negative as you can see here where how much positive is it and how much positive depends on the pressure gradient. So therefore, the position of this point basically depends on the strength of the pressure gradient. So, position IF is basically inflexion point; I do not think I wrote that anywhere, IF is basically inflexion point, right? This is basically the point where the curve changes a slope, that there is a change in slope which is happening at the inflexion point. The position of the inflexion point depends on the strength of the pressure gradient.

Now, for weak gradient flow does not separate, but is close to transition to turbulence. Well for a weak pressure gradient this will not separate if you have a weak pressure gradient, but it is actually close to it. So, for transition turbulence at for example, Rex around 0.1 million you will say velocity profile similar to this and it is close to transition to turbulence. Basically, the flow will transition to turbulence before it separates. So, here at around 0.1 million, so that will happen around 1 million.

Now the next thing is if I come here, so now moderate a gradient. So, a critical condition is reached. A critical condition is reached where in del u by del y is equal to 0. What is that imply? At a critical let us see here. At moderate gradient dp dx is kind of moderate. Now we are in this kind of place, so now the inflexion point is somewhere is in the flow. But that does not necessarily mean that right away it pulls separate.

The important thing however, now itself we can say that flow will separate only when the inflexion point is within the flow, that is one thing we know. But we need to know and where in the flow will it be located it depends on the strength of the pressure gradient. So, something that you see in three, I mean around 0.1 million or so it is close to transition into turbulence. At a moderate gradient nothing complicated. So, critical condition is might not, also I would like you to notice that del u del y; that is basically the slope say if you see look at the slope here del u del y, now if you look at the slope of this curve right now, now had this separation point be higher for a very strong pressure gradient del p del x.

Now the way is that, I could draw this curve in several ways instead of doing it like this. Let say I could just say I am going to draw on top of this to sort off I am sorry, give you the idea. So if I do that or let say if I had something like this, I am going to draw another location here so that you get an idea, so that I can give you better this is the (Refer Time: 19:11). Now I could have several plots here, so one is of course this I have one plot like this I could have this and so on and so forth.

These are all possibilities and if were to draw the slopes so this is something like that, this is something like this, this is something like this, this is something like this. These are all the lines of the slopes. So basically, 1, 2, 3, and 4 these curve. As you can see that these slopes are different and it changes, so it changes slopes may be somewhere here, may be somewhere here, may be somewhere here and may be somewhere here. So where it lies is something that depends on the strength, so what you can see is that this depending on how your height is. So, it could you know this is like the angle, right; this is the slope. Now this theta it could go here or it could extend up to here or here or here so on and so forth. This is one possibility.

Another possibility is also something like this; it could be something like this, this part actually. So therefore, what you can see is that this line is basically theta. Now it is a possibility that theta is 0 which means that I have flow which is like this you know. So therefore, slope, so that is the critical condition. What we trying to say here is that there is a critical condition.

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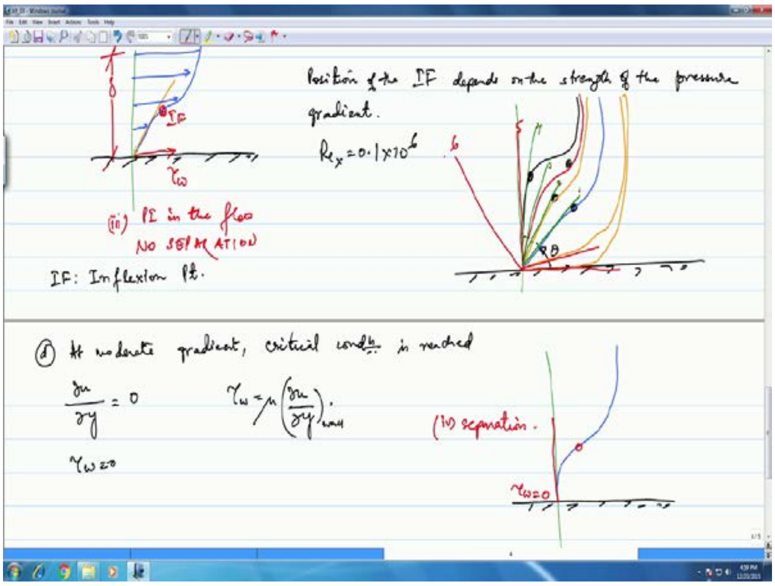

At moderate gradient, a critical condition is reached such that del u del y is 0. We have not seen that anywhere so far. So, del u del y is 0. So in that case what will be tau w? Because, tau w is nothing but mu del u del y at the wall; this is what we have been (Refer Time: 22:41) up with. Therefore, in here tau wall becomes 0. So what is happening is that the wall is no more exerting any frictional force at all or (Refer Time: 22:52) on the flow at all at this particular a point. This is when you know this is the point when tau wall goes to 0; this is the point when separation will occur.

So, now I think you can sort of gauge from what I am trying to say. If del u del y is 0 then tau wall is 0, is not it? Then you could probably have a curve something like this. So, you could actually have something like that, that is slope you got an inflexion point. In this case, del u del y is 0 and here tau wall is basically 0. Now if this is a point, this is essentially the velocity profile at which separation will occur. So, this here separation will occur. Now what is gone a happen next? I think by now you should be, kind of understanding so you could just set of take this.

So now, what I am saying is that we had this slope which was here, then it moves there, then it moved here in 1 then 2, 3, 4 then now it is right here like this which is let us say 5. Only thing it can do now is go like this, so say this is 6. If this is happening how you would draw a curve. So basically, we kind of a little back take a little bit; we are kind of trying to get an idea about the flow from the velocity profile.

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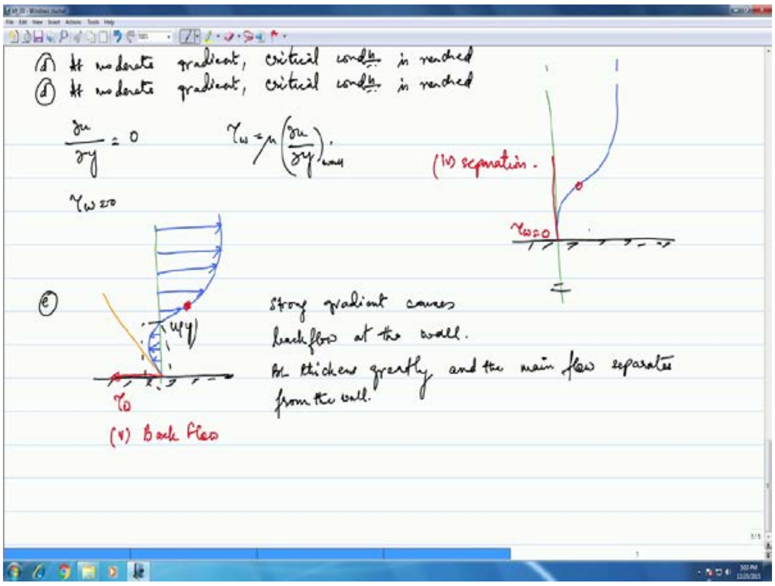

So what we will do here is if that is happening then what do we get? So then; and you have a slope like that. The only way this is going to be possible if I want to draw something like this then this should be my plot, then I will have a plot like this which means that I have flow and this something that you guys talk about back flow and so on so forth. Basically, the flow is in the reverse direction as you can see, and then again this is some kind of an inflexion point. So, tau wall is 0, in this case because of velocity is this way, so then my tau wall is somewhere here. So, this is here there is back flow.

We have back flow region. Once the flow separate, so basically we are able get to all this information. So basically, the strong gradient it causes back flow at the wall, and the boundary layer thickens and the main flow separates from the wall velocity profile is this. So, let me sort of write there that strong gradient causes back flow at the wall. The boundary layer thickens greatly and the main flow separates from the wall.

Again the velocity profile is what you here in 5. So, that is what it is and this is the region of back flow. Therefore, and this is your velocity profile again. I think, now the only thing which is important to mention here that the boundary layer equations which we have dealt with so far cannot be used in the back flow region which is this region; which is this region you cannot see that. And this has to be the boundary equations are only you know only up to separation, so only till say to the figure 4 that we see till tau wall is 0 so you can use boundary layer equations just before the critical condition which is del u del y going to 0 at the wall.

So, what I will do is kind of show you some pictures from my research. In fact, you can do that very easily using some available software also and that will kind of give you an idea as to how these equations are solved and it actually gives you nice pictures you know from something like fluent may be. So I will show you some of those results and discuss that a little bit in terms of separation. I thing we will do that.

So right now, I think we will stop having talked about separation in the presence of a pressure gradient and will want to do a different things elsewhere.

Thank you very much.