Introduction to Boundary Layers
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Module - 08
Lecture - 31
Similarity solution to thermal BL-III

Hi, we were looking at the energy equation, and what we basically started out doing is by saying that the 2D steady state energy equation is given by 1 here and 2 here. And is basically, if I were to look at this equation on the left hand side. So, this is the change in internal energy, left hand side of this equation here.

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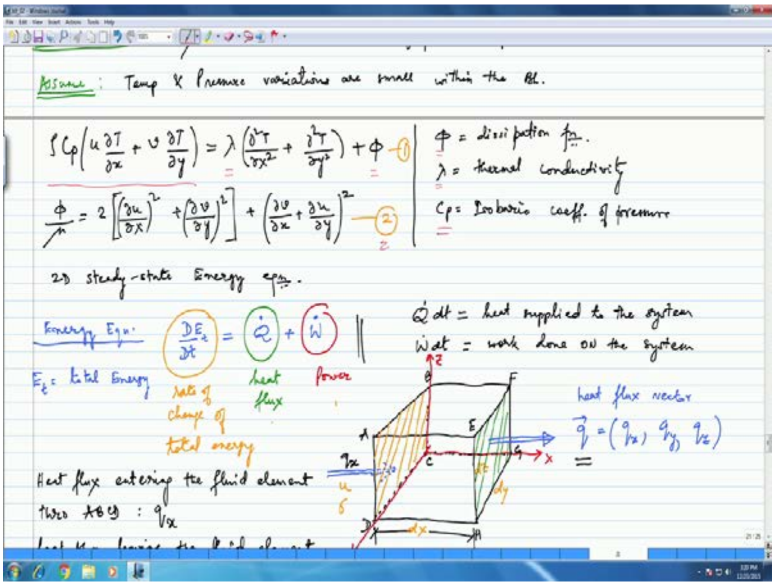

Change in internal energy and you can see that this is, like the convective change in the temperature. So, del t, del x, del t, del y and u and v are the velocity components. Now, and this term and c p is the isobaric co-efficient of pressure, as we defined earlier. Now, this change in internal energy, which is essentially the convective change in temperature can we written in terms of; the term which contains lambda as well as this phi and the phi looks something as given in equation 2 here.

So, essentially we have a conduction term and a dissipation term because lambda is essentially thermal conductivity and phi is the dissipation function, which is given in 2 and so this is the form of in a 2D steady state energy equation. We started out with the conventional energy equation saying that rate of change of total energy is equal to heat flux plus power and we went ahead and looked at a fluid element and saw how we could get the equation now.

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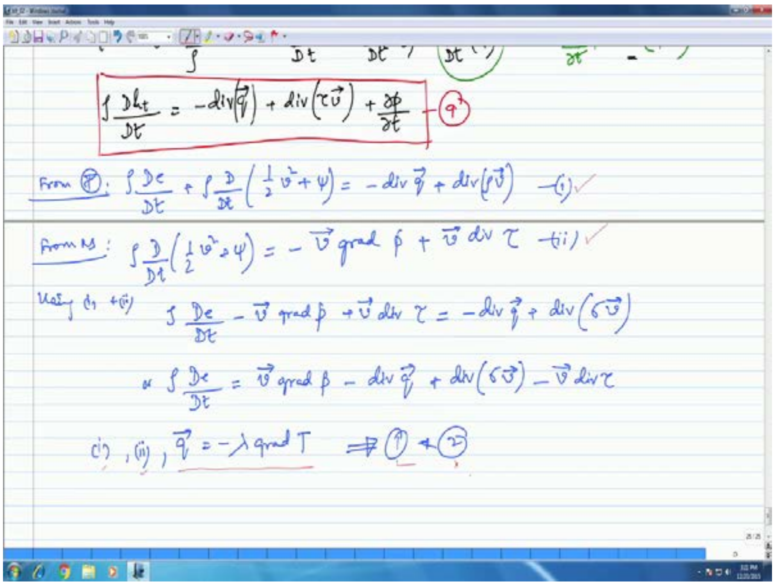

We finally, did all these calculations and we came upon this expression. Once we came up on this and also therefore, using 1, 2 and the expression, this we basically, if we solve this, we should be able to get 1 and 2, which is the 2D steady state energy equation. Now, detail of this derivation is something that I will post on the web page. So, you can take a look at that, and of course I would definitely suggest that you go through it. Do this yourself, it always have set of a better understanding. Now, that is where we kind of stopped earlier.

Now, of course, as we have seen here, that in order to solve for the temperature, we will basically need to know the velocity field. Now, if you look at this equation. This is our equation and we said that the total change in internal energy which is left hand side. Now, that can be written as the conduction term and a dissipation term. How I have for the dissipation term therefore, you can see that has a definition of the velocity. Therefore, in order to get the thermal field in this particular case, we need to know the velocity field.

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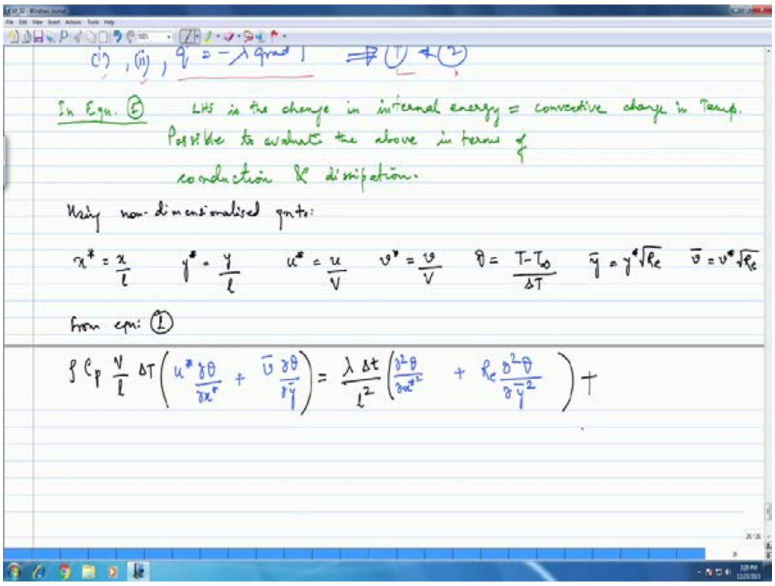

Let me jolt down few points for you; let me jolt down a few inferences that we made so far. In equation 1, let us say, the left hand side is the change in internal energy, which is essentially convective change in temperature and this is basically possible to evaluate the above, in terms of conduction and dissipation. So, essentially now the point is that we need to know the velocity field. Therefore, let us see how if we can, we could probably right this down as we have done earlier using non-dimensionalised quantities. So, let us see how we will kind of do that.

Let us consider non dimensional properties. If I do that, let us use these and well, we will assume a high Reynolds number for this case, so that the flow will have boundary layer character and we will use the following non dimensional quantities. Using this is similar to what we have been doing so far. Non-dimensionalised quantities which is x star then theta, now this is delta t is basically some suitable temperature difference. Now, what we seen earlier there it could be the wall temperature, but the difference between the wall temperatures in the free stream temperature that is a possibility. But it is essentially whatever is suitable to the problem at hand, this and again y bar and v bar this. Now then, basically equation 1 then we can write the size state 2D energy equation as follows.

Then we shall write it as rho c p v by l delta t into let us write this in a, just to give me a second. This is taking little long to derive, so u star del theta, del x star plus v bar del theta, del y bar is equal to lambda delta t by l square. Just try to pin point, where we are using this theta because that is what we interested in. We are really interested in the temperature field. So, del 2 theta, del x star 2 plus r e, del 2 theta.

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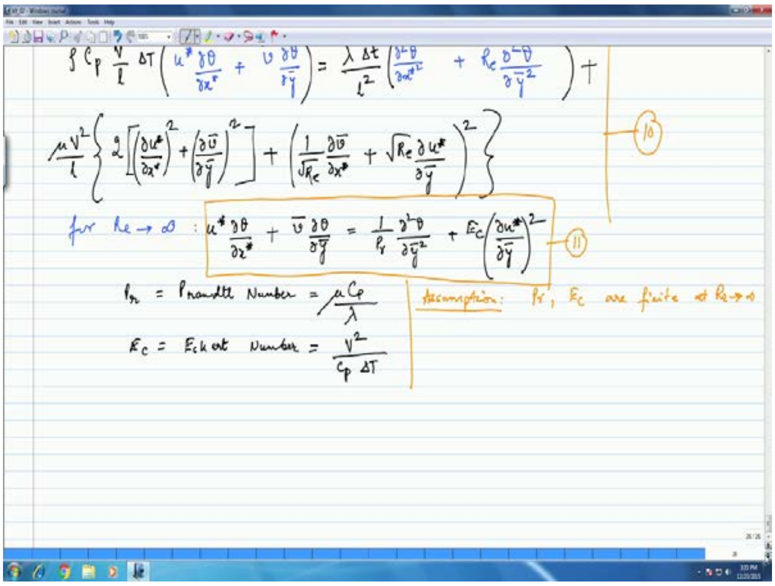

This, when plus mu v square by l, 2 plus and this term I am going to write this out 1 by r e del v bar, del x star by and well, here two. Then this is del u star del x star square plus del v bar, del y bar square. Now, the detailed derivation of this is something that you can look at I will basically posted. Then this is essentially, this is equation 10 and this is equation 10 and so, basically we writing the 2D steady state energy equation instead of found.

Here, you can see that instead of the temperature, we said we need the velocity field and what we do is, we write this down. We did have these in terms of the temperature. As we convective change in temperature and what we use now the usual, non dimensional variables and we are able to get something like this where, what we have essentially is the non-dimensionalised temperature theta. Now, if I do that, now what we will do is, will since we said this is going to be at high Reynolds number. So, that we have the boundary layer you behavior. We are able to see boundary layer behavior. So, we are going to take this further limit. So, for this, then this 10 equation you can see it becomes so and so.

Then, it becomes u star del theta, del x star plus v bar del theta del y bar is equal to 1 by p r, is a Prandtl number actually write it down for you, del u star del y bar this. So, essentially if I say that Reynolds number is very high. Then we looking at each term and this is something we have done earlier and I would like you to sort of go through, each term and make sure that, you agree with what I have written. I think you should be able to do that. So, high Reynolds number, what happens?

So, we write it in this way, where p r is the Prandtl number, which is mu c p by thermal conductivity and the Eckert number, which is v square by c p into delta t. Of course here, if you have this equation, this is now an important equation for us and this is equation 11 and of course, here this equation will have to assume that at a high Reynolds number, the Prandtl number and the Eckert number are finite values. That they do not blow up for I think. So, that is essentially kind of while we assume. That the Prandtl number and Eckert number are finite at large rho numbers. Now, a Prandtl number actually is a pure physical property and what I will do now is give you just a feel for. I will give you some of the typical values of the Prandtl number.

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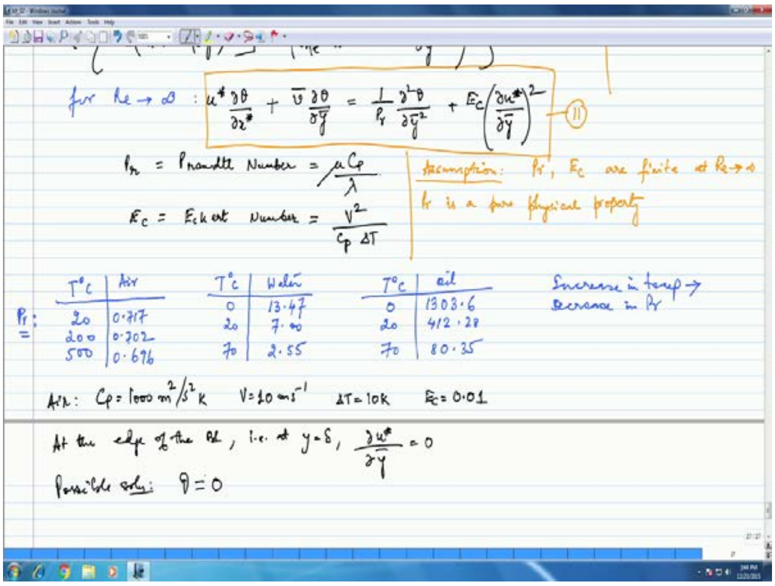

The Prandtl number is, it is a pure physical property meaning, it is the property of the fluid itself. It is not something that comes on and off due to the fluid flow, but it is a property the fluid itself. It is a pure physical property and for example, let us write here, so this is the Prandtl number I am write it for various cases. Now, this is T degree centigrade. So, I got at 20, at 200 and at 500 degrees.

This is for air and the Prandtl number for 20. For example, is 0.717 a 200 is 0.702 and for 500 is 0.696 which is interesting. If you look at that with increase in temperature, the Prandtl number decreases what I can see form here. Again, you got temperature and this is prandtl, I mean this is for this is for water. This is for water and this is at 0, this is very interesting 13.47 I got 20, which is 7 and I got 70 I got 2.55. Again, this is for oil, this is 0, 1303.6 for 20 it is 412.28 and for 70 it is 80.35. Now, what is interesting is you can make your inferences from there. If you see, I would have compare let us just say, that just between water and oil.

Now, water and oil are both way more viscous than air and also the density is way more heavier, in the density of water and oil is much larger than that of air. So, what you see is for at least water and oil that in here two with increase in temperature this is Prandtl number, establishes instead of Prandtl number.

Now, for both, for all air, water as well as oil with increase in temperature, the Prandtl number decreases. Increase in temperature is decrease that I can see. Now, oil is more denser than water and you can see that this is almost 100 times more than, the Prandtl number is almost 100 times more than water. Now, what you can see here the Prandtl number is got the coefficient of viscosity here you know. So, that kind of gives you a hint right away. It is directly proportional to the co-efficient of viscosity so; obviously, this is way more viscous.

This is almost like 100 times more than that of water. Again at 0 degrees as you increase 20 degrees then, this is 412. So, again it is more than that of water at the same temperature and it is more than that. Now, for air you see that even for 20 degrees, you can compare that 20 degrees it is extremely less it is 0.7. If you go up to 500 then you kind you know, you decrease the temperature if you go up to 500 then, you get around 0.7, which is no more even like a close. So, if you are 20 degrees, then you get like 0.7. So, so definitely I mean that is kind of. So, that is what we can see. Of course, it is a direct Prandtl number is directly in proportional to the co-efficient of viscosity.

Hence, we see this now the Eckert number of course, is the measure of the dissipation effects and as you can see that it this grows, as a score of the velocity which means, that for a very small velocities, you can neglect this. For example again, air is like seems to be this spoil spot to its peak. Now, for example; for air, let us change that, now for air c p is 1000 meter square second square kelvin, v is equal to 10 meter per second, delta t is ten kelvin and Eckert number is 1 by 1 100 basically. Now, what essentially because of the dissipation therefore, because of the dissipation there is no heat transfer, but there is a there is a temperature field which emerges because of that.

And example for that could be, insulating walls which are antibiotic in nature. Of course, the walls of higher temperature, than the outer wall and than the outer surrounding fluid which is a free stream t t infinity and this is something that we call as antibiotic wall temperature. Now, we know that at the edge of the boundary layer that is at y is equal to delta is 0. Now, if that is so, del u star del y bar, which means that the term containing the Eckert number which is here which is basically the dissipation term in equation 11, if you see. So, that is going to go off. Therefore, we will be left with this equation, which is only in theta. Now, therefore a possible solution, a possible solution is for 11 is theta is equal to 0 and theta is 0. Which means what? Theta is defined as t minus t infinity by delta t that is what I have defined.

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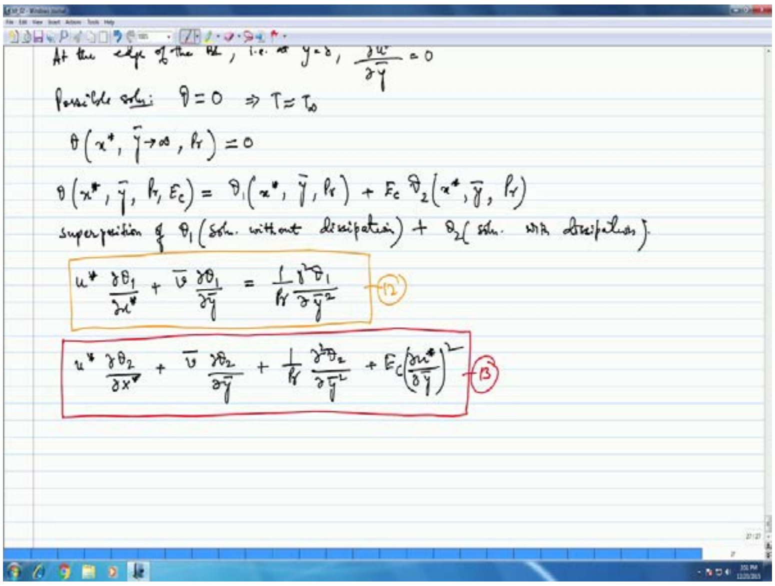

If theta is 0 this implies, the t is this, which means that at large distance away from the wall the temperature of the boundary layer. Boundary layer edge is basically is equal to the field stream temperature. So, in other words theta is a function of x star, y bar which is in this case is far away, and the Prandtl number and that is equal to 0. Now, equation 1 which we have seen earlier, so mathematical equation 1 is a linear differential equation, which has this standard solution theta, which is x star y bar Prandtl number Eckert number, this is what it depends on.

It has a standard solution theta 1 which is x star y bar Prandtl number plus Eckert number into theta 2 x star y bar and Prandtl number. This is a Prandtl number now, which essentially means that so, theta 1 as you can see, we have the theta 1. This is a solution which is not; contain the dissipation which does not any dissipation and the second term which contains the Eckert number essentially contains dissipation. So, this is your equation 10 and 11.

This is the general solution to the equation 1, so equation 11. Now, therefore, let us see that this essentially is if I may clarify here that this is a super position of theta 1, which is solution without the dissipation, plus theta 2 which is solution with dissipation. Therefore, we can write the solution, we can write the equation 1 therefore, we can break it down and what we can write is this. We will break that down in terms of theta 1 and theta 2, del x star plus v bar del theta 1 del y bar is equal to 1 by Prandtl number, del 2 theta 1.

These are nice, in an equations which we can write and plus this is the equation which has the dissipation effects. When I write that, this is the equation 12. Now, 12 and 13 can be investigated separately. So, essentially what we doing is, we have breaking down equation 1 into 2 equations, which is easier to solve or basically it is the solution of the linear differential equation, where we define these 2 functions theta 1 and theta 2 which is essentially solution without dissipation and with dissipation.

Now, the way we can look at equation 12 is the in here, this is a linear equation dissipation to this holds y because the dissipation terms are small. The dissipation term is small this dissipation effects are small, because the velocity is a small and since this is a. So, therefore, the Eckert number is basically negligible. It is similarly 0 and this is a linear equation is not a problems if I want to solve then all I need to do is just get back the dissipation term. Therefore, it basically breaks down to 12 and 13 and you can see this is an easy equation to solve from the equation that we started with which is 1 and 2 so. Therefore, I think will close here and the some of the detail derivation is something that I will post for you and you can take a look at this. So, we have come up with equations in terms, in order to solve for the some more boundary layer. So, will stop here and move on to new things in the next module.

Thank you.