**Introduction to Boundary Layers**

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**Module – 02**
**Lecture - 30**
**Similarity solutions to thermal BL- II**

Hi. So, let us go ahead and look at the energy equation.

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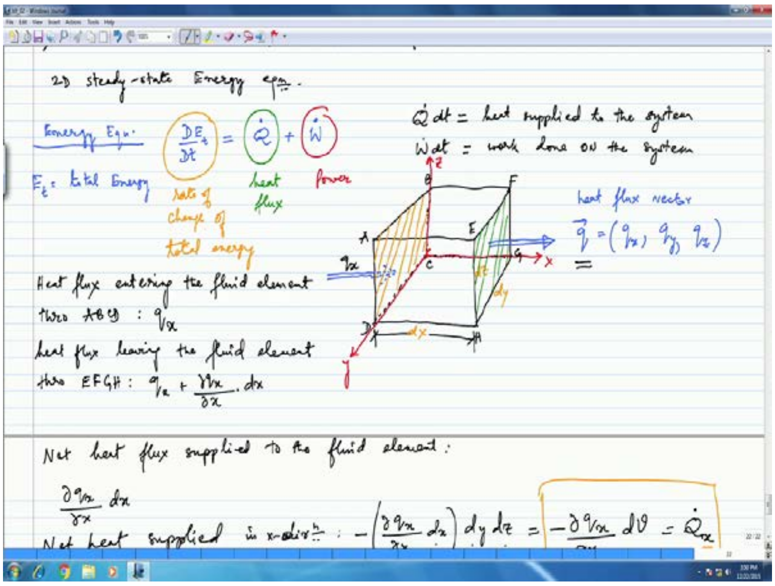


We were basically looking at the energy equation of course and we wrote down the rate of change of total energy is sum of the heat flux in power. So, we looked at a fluid element and we have got an expression for this Q dot. Now, let us look at the other two how we going to do anything about it. This is where we had stopped last time. Let us look at the total energy, so that we can find an expression for the left hand side.

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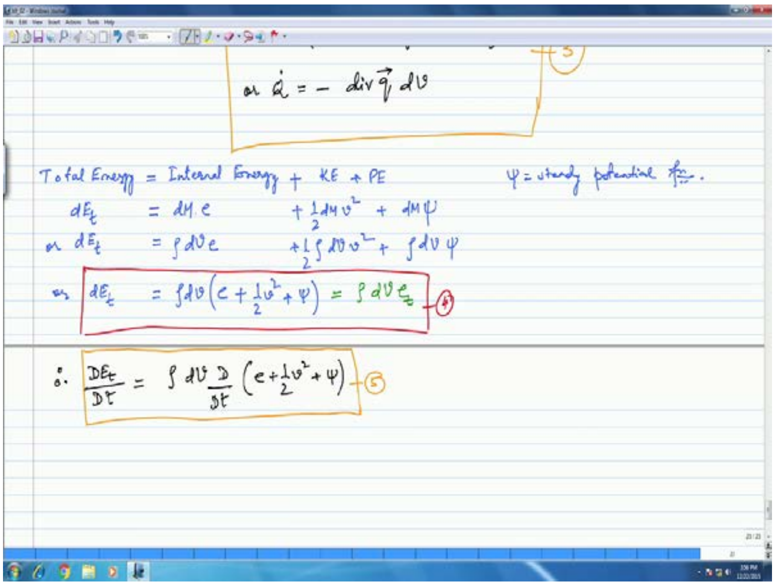


Now, total energy is equal to internal energy plus kinetic energy plus potential energy, is that right. Let us call this as; therefore, if I take this is an element of the total energy, so I need to set of integrate this. So, this is the energy in a small volume or small fluid element. Therefore, I write dE t, dE t is the total energy is equal to internal energy which is an elemental mass into e. This is usually internal energy per unit mass. So, I think this is also something I have talked about at the beginning. So, internal energy dM into this plus KE is half Mv square and this we are going to write as elemental mass into. This is basically, a potential function.

This is potential function and it is a steady state of potential function. If I were to write this is equal to; I am going to break this mass up and say this is equal to this density into volume into e plus half rho into again dv v square plus rho dv and psi. So, I can write this as rho dv. I can write that as e plus half v square plus psi. This is what we can set off, write this as and I can also sort of say, that this bit.

Let us see we can also say that if I were to write this and I can write this whole as a total energy. I can write this or say total yes, I can write this in this fashion. I can also write this as rho d v small e t. So, that is essentially this part. Then I can write I am going to set of work on this. I am going to call this as 4, we get this expression. Now, we have to find out d e t d t. We need to find out the rate of change of the total energy. So, rate of change of total energy is equal to rho dvd dt of e plus half v square plus psi. Therefore, you can see that I can actually write out the left hand side of the energy equation in something like 5.

Now, next thing is let us go ahead and again look at w. So, now worked on I mean worked on a fluid element. What is your idea is of worked on a fluid element? So far we have been talking about drag. We said one of the reasons to study boundary layers in a velocity boundary layers is because the viscosity is not dominant and it exerts certain pull on the body on which it has developed fluid and is trying to move over a body and it faces a certain resistance and this resistance is from the fluid and this is due to the viscosity of the fluid, and if you kind of make a sum of that on the total surface area of the fluid which is experiencing that it essentially develops into the expression for drag. So, that is what it drag is all about and if you are going to make a fluid flow happen, you will have to encounter the drag is going to be pulled back and flow is going to be retorted.

Therefore from that context, one can understand that and we also talked about the shear stress. We calculate co-efficient of skin friction and we calculated in terms of the wall shear stress. Therefore, what we are doing now is going near the wall and looking at taking a fluid element and see that how exactly what is going on with that. So, when I say work done, I basically mean if when it is like applying due to the viscosity, it does apply a resistive force which comes out to be shear stress now. The fluid is in layers, right and it shears off. So, I am talking about a fluid element. Then there is certain shear stress which it is experiencing. Of course, source the normal forces.

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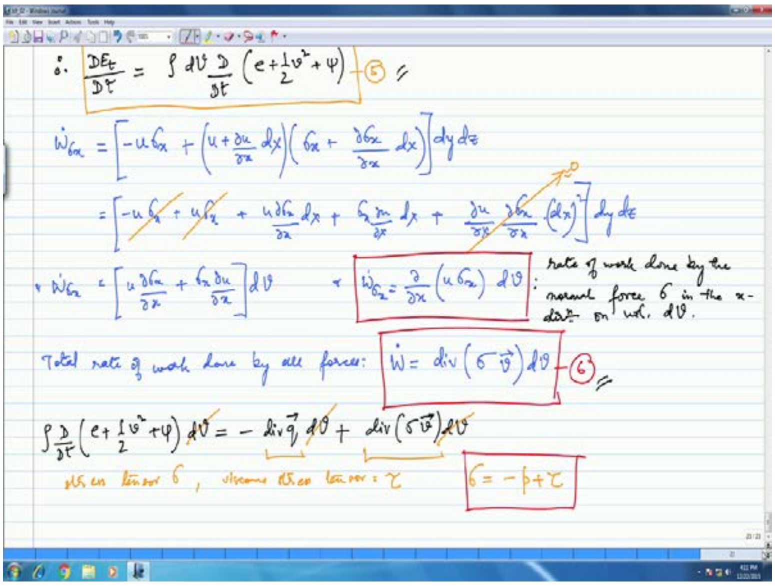


Let me set of write this here. I am going to say this is normal force in the x direction. So, see if this makes any sense, this is kind of under at knowledge. I am not going to repeat that, but if you find it is a little more not very easy to understand, do let me know and maybe I will kind of elaborate on this part. I think this is something you may have done around in the first year of your engineering program. This is this plus into sigma x plus this. It is not very difficult to understand from here I hope because what we are basically saying is that you have like I have shown q x here coming in. This also has a velocity which is here and the sigma is basically the normal force. So, sigma x basically is sigma in the x direction. Now, when that happens, this is on the left phase. I mean the way I am calculating the work done, this sigma x for sigma in the x direction, so for sigma in the x direction.

Like we did q x on the left boundary is just a q x and what is the flux which is leaving from the phase e f g edge across the x. So, that is again q x plus this. Now, similarly you have u here into sigma x. U into sigma x on the left hand side and at the right hand side u also changes. So, u also changes by u plus and similarly, del u del x into d x that is the u which has changed in sigma x which is again sigma x plus del sigma x del x into d x and then, we multiply this by the areas across which it is leaving. We get something like that. So, that is what we have. That is how we are calculating the power or rate of work done.

If I do that, you can probably see here.

If I do this minus u sigma x plus, I am going to multiply that u sigma x plus u del sigma x del x d x plus sigma x del u del x d x plus del u del x del sigma x del x d x square and all of that into d y d z. If you see this I should be able to two set of cancel each other out and I also want to take d x out of the brackets. Then what I get? This is equal to u plus sigma x and also if you look at this, we have a square of d x which is anyway small and we have a multiplication of two derivatives.

In all likeness ignore this. These set of that can be ignored. It is very small compared to these. Then that time goes, so what I have is value. I write this way, you can see that I can also set of write this the term inside the bracket as this is del del x of u sigma x into this. If I just add, this is nothing, but what is this term is even mean it is a rate of work done by normal force sigma in the x direction on the volume dv.

Let me set of write that down what this means. What this means is the rate of work done by the normal force sigma in the x direction on volume dv. Now, that we have done is, we shall now write it down for total for, sorry. Why total force? Total rate of work done by all the forces is nothing, but we will sum there up, so that plus in the y direction and x direction. It will be, well I am not trying any extra step. So, w dot sigma x plus w dot sigma y plus w dot sigma c and this is essentially I can say it is divergence of sigma into the velocity vector for this. If I do that, this is this.

Therefore, now we can set of down the energy equation in the following way. We can now write down the energy equation going back. Therefore the energy equation which we had here basically we called it something. So, this is the equation that I am talking about now. Let us see we have got an expression here in 3, we got an expression here in 5 and we have got an expression here in 6. Let us put all this together and see what we get.

Let us write that down. Let us look at 5. What we going to write here is rho d d t e plus. This is equal to we had q dot which is minus divergence q d v divergence sigma this. Let us take out the d v throughout. Let us take that out. If do that, I am not going to erase it just do set off. We can take that out, cancel set of throughout. The left hand side is the total change in energies that you can see in tau energy, kinetic energy and potential energy. This is the total change in energy. Right hand side is energy supplied due to this is this term. This is the energy supplied and I do not know if you noticed, but when we first started out we said that we going to assume that heat is going to be transferred to the walls. It is going to be transferred through the walls. I probably not mentioned that. I have not written it, but I did set of mention there. So, what I mean by that is that heat is going to be there. There is a way in which you can transfer the heat. This is by conduction. This is the left hand side is total change in energies.

The second term here is or the right hand side, this term is basically the energy supply due to conduction and this is the work done by surface forces. So, to do a little bit of set of math if I were to do this bit of math, we will say that the stress tensor sigma and the viscous stress and the tau, all are related. We kind of just write it like that. So, stress tensor this and viscous stress tensor is equal to tau and this and these are related as sigma. It is varied as sigma is equal to minus p. What is p? P is just the pressure, this is the equation. Let me set of write this out.

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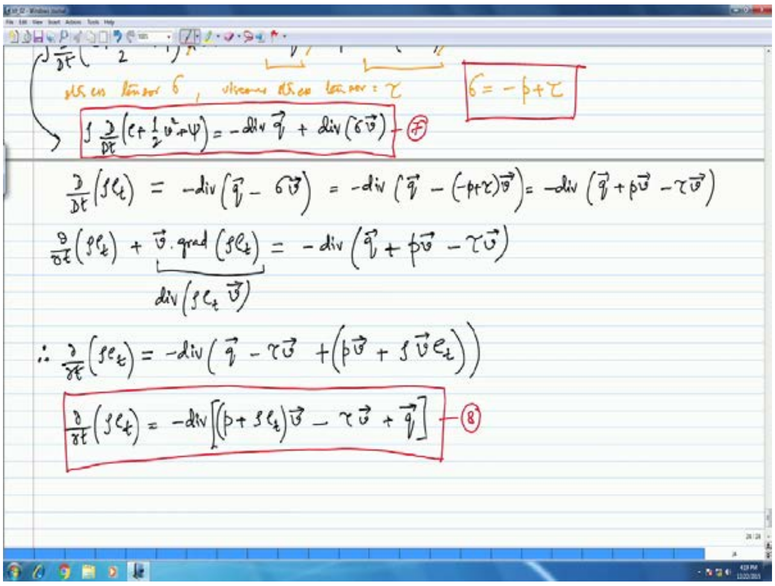

What we get? Therefore what we get from here is rho d d t. This we are going to call as

7, is it right. From this equation from 7, what we can now write is that ddt. We can write the left hand side like this, because we have written earlier that this is the total energy and I can write that as small e t, and I took the rho inside. Therefore, I am writing it in this fashion. Did I mention? I think I wrote that the total energy e t. If I write that, that is equal; let us keep that equation, there in the right hand side from here, I can write minus divergence q minus rho v and we have also said that sigma is equal to minus p plus tau. Therefore we can also write this as minus divergence q minus minus p plus tau. This is that.

Again, divergence this is q plus this minus this. Now, I am going to expand again the left hand side and write that down as del del t of that plus gradient. This is nothing, but the total derivatives some set of breaking that down. This is instantaneous and this is the convective change in the total energy rho 80 that is equal to divergence. I am going to just write that down this. Now, if you write, so I am not expanding this part. I hope you kind of get it right. If you do not realize how I got this, just drop your notes and I will explain that.

Now, what I am going to do is, I do this v dot this grad. If I write it like this, so essentially I can write this term. I can write this term as nothing, but divergence of rho e t v, right. I can write it like this. Therefore the above equation del del t is equal to minus divergence q, sorry minus tau v plus this. Let me put this inside of bracket. So, I am essentially taking this to the right hand side. Yes, I do intend that to be negative that is going to be set of a negative.

Therefore, now I am going to just set of rearrange this a little bit and write it that del del this thing is rho e t is equal to minus the divergence of here. I will take the common v common. So, plus rho e t v minus that and this is I am going to call that the equation. If I did that, now we slowly set of get into a place where we are you know introducing new stuff here. We have got the total energy term in here, the pressure term here, shear stress here and heat flux factor here.

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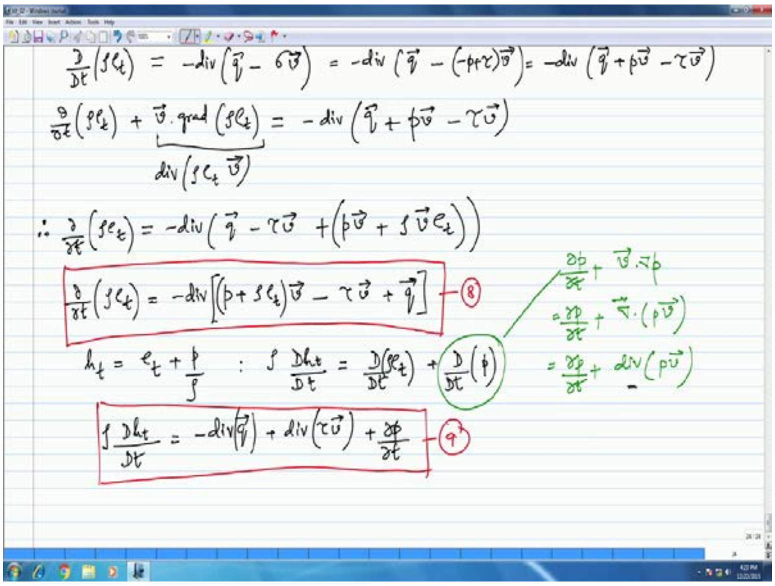

Now, we also know the total in p is also connected to the energy pressure and density, right. If I do that, then rho d t is equal to d d t of that plus d d t of p. So, I took the rho and multiplied throughout. I will need a rho here this is what it is now. If you look at now this term here, the last term which is essentially total derivative of the pressure. What I will get here is this term actually. So, what I get is this is going to boil down to del t plus grad p. What I can get is del p del t plus grad and therefore, we can get this as divergence of p v. This is nothing and total derivative is nothing, but divergence of p v.

Therefore, I can write this as this term equation I can write that. So, that is something and we can also we have seen here in this equation in equation 8 that del del t of rho e t, we can also write that as a divergence given by equation 8 here. So, if I am actually missing particular step, but one thing you can do that there, rho is equal to divergence q plus divergence tau v plus del p del t if you see from equation 8. We have got this term.

The second term is divergence of tau v which is what we got here. This term is there minus divergence of q which we got here. We also get minus divergence of p v, which will cancel out with this divergence of p v which we get from this term. The next one is minus divergence of rho e t v which is minus divergence of rho e t into v which is what we are kind of using here. Let us sort of mark that and we are going to mark that here.

Now let us look at this equation 7 is here. If you see equation 7, I can basically write I am going to separate this you know part, the internal energy and the kinetic and potential energy. If I want to separate that, then I can write. So, then I can write.

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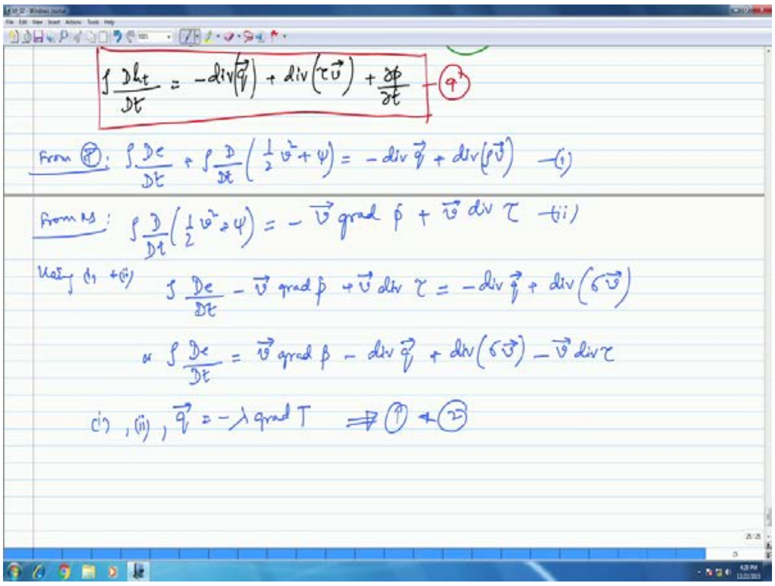

Let us just say from 7, what we can write is rho d e d t plus rho d d t of half v square plus psi which is equal to minus divergence of q. Let us look at this equation here. So, minus divergence of q plus divergence of rho v that. Now, let us call that as 1, I mean from the Navier stoke equation, rho d d t is equal to grad p plus divergence of tau. Now, I am going to call this as 2 and like I said the detailed derivations is something that I will post for you, so that you can have a feel for it. Now, using 1 and 2, what we will get? Let me just set of write that and close this. If I do this, then what I get is rho d e d t, which is sorry minus grad p plus that divergence tau is equal to minus divergence q plus divergence this or rho d e d t is equal to gradient p minus divergence q minus v divergence tau divergence tau.

Now, if we use 1 and 2, we get this expression. Then we use the expression q which is equal to minus thermal conductivity grad temperature. If we get this, we will get equations 1 and 2. If we use 1, 2 and this relation, what we essentially get is equations 1 and 2. So, essentially once we come here, we basically realize that it is possible to solve for the temperature field, provided the velocity field is known. So, based on that what we will do is, we will write down the equation, used in the similarity variable, so that we can also solve for the temperature field of a given boundary layer.

We will stop here and do a little bit of this in the next couple of modules, and see that what kind of things that we looked at, what kind of things we will get and how the equation shape up, and is there like physical thing that some physical inferences that we can actually make from the equations that we will we are kind of near that, and if at all we can make a simple enough calculation, simple solution to get the temperature flow field in, for a boundary layer case. So, we will stop here.

Thanks.