Introduction to Boundary Layers

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Module – 01
Lecture – 28
Similarity solutions to thermal BL-I

Hello, welcome back. So, we kind of started talking little bit about the Energy Equation. We kind of in next headed to Thermal Boundary Layers. So, typically when we talk about boundary layers, we are usually talking about how the velocity profile changes across the boundary layer and how do we define a boundary layer in terms of how the velocity profile looks like, is 0 at the boundary and then reaches free stream at some distance away from it. How do we sort of define that? Those are things that we have been concerned about so far.

There is something also called a Thermal Boundary Layer, because we also need to take into account the fact that there will be a certain transfer of say, temperature or there will be difference of temperature and from going away from the plate, and it is or you can say this is basically thermal energy which is being transferred across the boundary layer and I think we kind of touched upon the facts that if there is the temperature field, is actually connected to the velocity field, which therefore means that if we can solve for the velocity field we can also solve for the temperature field. We kind of did a little bit of math and derivations in the couple of the modules earlier.

Let us sort of look at that and I will jolt down point wises to what exactly we are looking at and then go ahead and do some math. There is some detailed derivations and fall to which I would probably not do here and I will put those online for your reference and you should be able to check that. So let us call this today.

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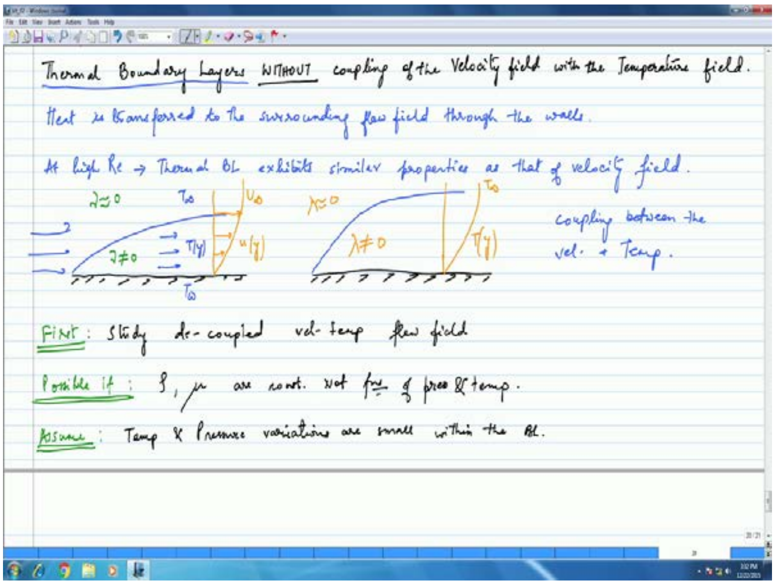

We are basically are talking about Thermal Boundary Layers. Well, I am going to head this and hopefully you will understand what I mean as we go along. This is without coupling of the velocity field with the temperature field, couple of things, so we will basically extend the study of boundary layers in terms of velocity to also include temperature and how do we do that? We shall assume that temperatures or say, well heat. Heat is transferred to the surroundings, surroundings or which basically means the surrounding flow field, let us put it that way; surrounding flow field through the walls. So, basically heat is transferred to the surrounding flow field through the walls. So basically, there is a momentum transport across the boundary layer, there is also heat which is being transferred.

Due to the momentum transport, we have a velocity boundary layer, velocity profile; similarly we will have a temperature profile. Therefore, the velocity profile is boundary layer as we know it. So this one where heat is transferred, and we have a temperature difference across the boundary layer that is basically what we call as the Thermal Boundary Layer and at high reyonlds numbers this will exhibit the same properties as that of the velocity field, question is why, I think about that and what is that exactly mean?

Although, we will kind of talk about this little later on, but let us sort of write this down if I mean so at high Reynolds number if thermal boundary layer exhibits similar properties as that of the velocity field. So, what kind of, let me just elaborate on there a little bit. For example, this is a flat plate that we are looking at and, say this is our velocity profile. In this particular case, what we have said so far is that in here nu is not 0 and outside of the boundary layer you can neglect the coefficient of viscosity.

Now, the thermal boundary layer of course, will have a different delta. Delta with thermal boundary layer will be different, so similar to this if I draw a similar picture the values will be probably different. This is a velocity field across the boundary layer so it will be like we said similar but, without commenting on it I am going to just draw in that way. So this is essentially and there is going to be a temperature profile, we will look at that. There will be say, temperature profile and this is the, so we look at that. We will see how this will look like, is this correct profile or not. As of now let just to say that the lambda which is thermal conductivity, this is not in significant here and lambda can be ignored which is away from the thermal boundary layer.

Now, the point is that you can see there is a velocity and there is a temperature profile. So, physically there will be mutual comparing between the two, do not you think, because you have the way to look at that is basically you have flow coming in this way, you have flow moving in this way and this flow also has a temperature profile which kind of with basically saying this is temperature tau wall, this is something here and something (Refer Time: 09:51) somewhere there. So there is a certain at this kind of a differential temperature, temperature being different at different heights can only be possible if there is a certain heat exchange. So, heat is also a fluid in as you can flow. Therefore, there is a coupling between the two. So, there is a coupling between the velocity and temperature.

The heading says that we going to study this without coupling of the velocity field and the temperature field, I guess that is to just say that we are going to deal with less math and probably comparatively easier math, when we kind of do it without the coupling. So that is the only objective. Firstly, what we will do? We will study basically a decoupled flow field even if the velocity and temperature are not coupled. So, we will study decoupled velocity temperature flow field.

The next thing is, now physically of course, you can see that we are kind of making an assumption that velocity and temperature. Now, the question would be why would, of course I said instinctively like even why intrusion you can think that I have velocity and why would that be effected by the temperature. Now you see again the coefficient of viscosity, the nu here that itself is the function of temperature and pressure. If there is a temperature difference even that is going to change and if the value of mu changes mu is now with respect to temperature then automatically your velocity profile is also going to change. So, if we going to say that study the decoupled velocity flow field if you are going to say that then this is possible only if we consider that the density rho viscosity mu are constant, and they are not functions of velocity and temperature residual and pressure and temperature.

Now, this kind of again throws a soft card because you are saying that these are going to be constant, if these are going to be constant how is this going to be constant, because you are saying there is a temperature difference, so in the sense there is a variable temperature. So if the temperature is varying, then how can kinematic viscosity remain constant? Well, this we will assume this so for that, what we will assume is that temperature and pressure variations are small with that something you can assume. So basically, we are going to say that the temperature and pressure variations are very small within the boundary layer. Therefore, based on that we can say that the kinematic viscosity is constant not a function of pressure and temperature and hence we can study this sort of behavior of the temperature field and the velocity field as a decoupled problem, we do not have to combine the two.

Of course, we have already started doing this and we have talked about the energy equation and all that. So, let us sort of just as starting point overview, let us look at the equation that we going to look at. So, let me name the; It is like this.

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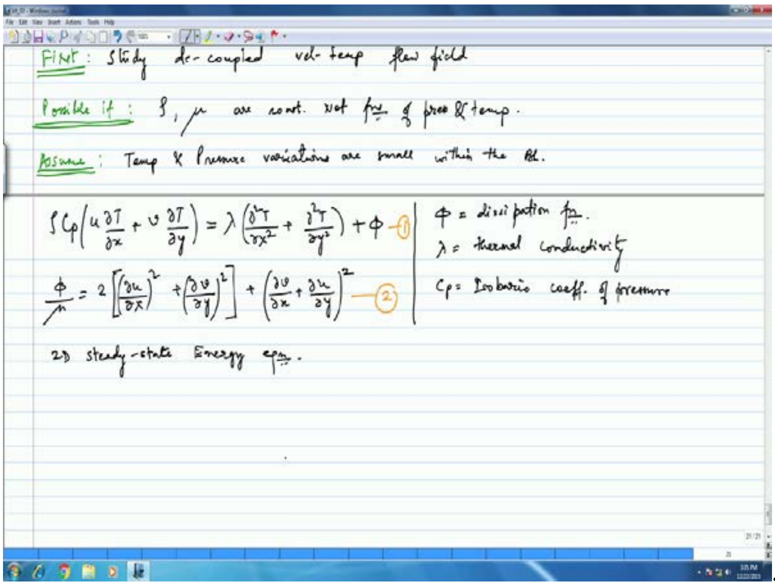

Rho C p u del T del x plus v is equal to lambda del 2 T del x 2 plus del 2 T del y 2 plus phi and phi by nu is equal to 2 plus del v del x plus del u del y square. Here, phi is the dissipation function, lambda is thermal conductivity, C p is coefficient of pressure. This equation that I have written here is, I am going to call this way. So this equation is essentially 1 and this is 2. This is essentially the 2D study state you can see. So, this is essentially 2D. I hope you have notice it is study state energy equation. This is how things look like. So you can very well see that here how the temperature is basically connected with the velocity and how these terms kinds of are all existing at the same place.

Now, let us start from a little back and let us go ahead. Let us go sort of understand what is going on here.

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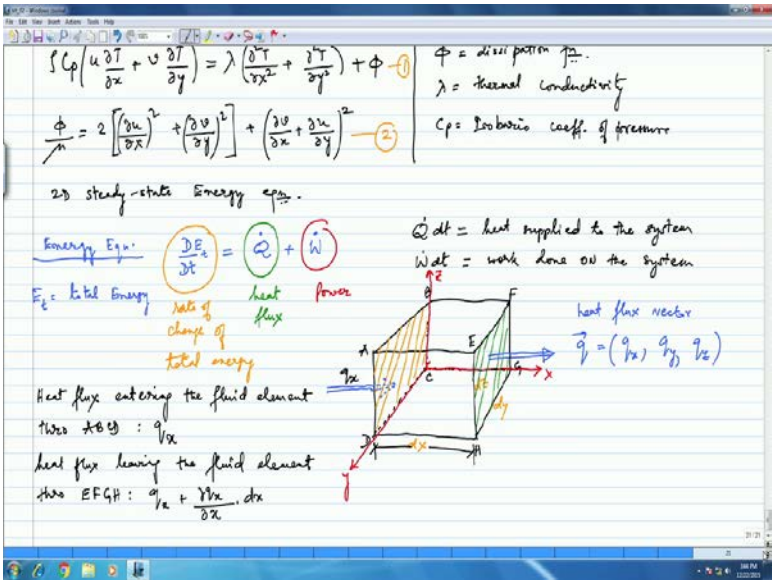

The energy equation if I were to write like this, where this is equal to total energy. The symbol is basically capital E stands for energy and the small t stand for total. So, DE Dt if E is equal to that should be simple, you should be able to understand that. So, what is the left hand side here? This term, it is rate of change of total energy, this is heat flux, and this is power. So they are sort of rate of change of work, to speak.

So, that is essentially my energy equation and of course, the way I write this so I could actually write Q dot dt, this is equal to heat supplied to the system, and it should be work done on the system. This is work done on the system. Now, we will draw a simple little cubic fluid element which is kind of familiar with we been doing this for a while now. So say I sort of do that, there you go. So, A, B, C, D, E, F, G and H and the sides for example, say height that is Dt, this is dx and this is dy, is that right. This length, this sides and of course you have flow which is moving faster, is that right. So now, the heat flux vector is q, which is equal to qx, qy and qz and or you can basically say this is and the unit is of joules per meter square in second.

So this is what it is. So, qx, qy and qz in the x direction, so if I have not drawn the axis system here, but I hope you can instead of understand that. So, axis system is somewhere like this. So, this is your y, is that right. So, once you have that the heat flux entering the fluid element through the face A B C D. So, we have got A, we have got this face that we talking about. So let us see if I can show that a little bit. This is the face I am talking about, so what I can write here is, that heat flux entering the fluid element through A B C D is qx, you understand that, that is what basically we are doing. So what is entering here is essentially qx. And what is leaving through the face E F G H.

What is leaving through that? Let say, basically I am talking about this and heat flux leaving the fluid element through E F G H is qx plus rate at which it changes along x into the total length traveled in the x direction which is dx as you can see. This is the distance which the heat needs to travel, so this should be dx. This is in the x direction, so you bring the similar analogy to the y direction z direction so on and so forth. So having done that, therefore, net heat flux which is supplied to the fluid element so all we have to do is, this is so net heat basically not the element is essentially, which is supplied to the fluid element.

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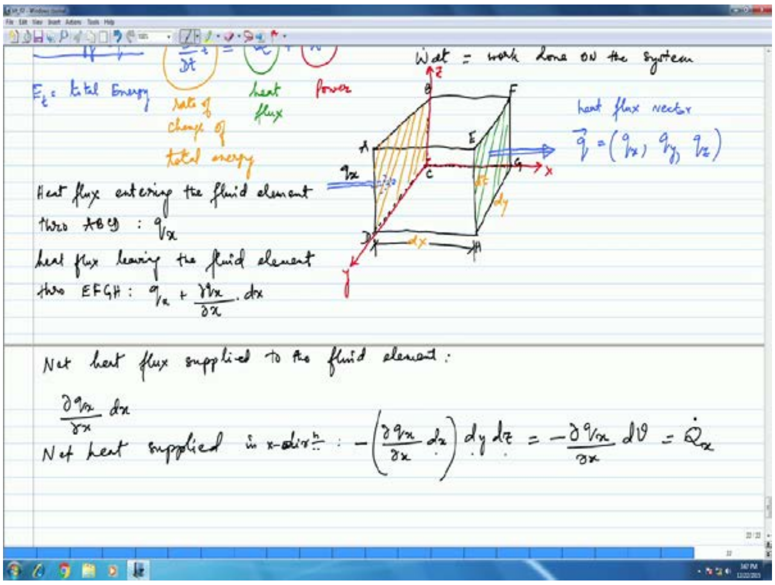

Net heat flux, so what enters is qx and what exits is this. So, net heat flux which is supplied is to the fluid element is; so that heat flux supplied to the fluid element. Therefore, what is the total heat which is supplied in the x direction.? Therefore, if I were to do that, so nets this is the heat flux and net heat supplied x direction is so del qx del x is and I have got up multiply this by the area though which is coming up which is d y into d z. That is dy dz, so which basically means dx, dy, dz that is nothing but the volume. So it is nothing, but dqx del x d, this is the volume. So, let us call this as Q x.

Now, when I kind of say this that heat is being supplied to the system, even there I mean we are missing science somewhere here. When you do the math of course, so you are basically going to do qx minus qx minus this, q x minus, this whole thing, so you will get a minus sign here which is fine but what I would like to ask you here is that, explain this conceptually. Mathematics wise yes, you will get a minus sign there, but I would really want you to explain this conceptually. So think about that. Therefore, write this d v so this is in the x direction as you can see. Hence, total supply therefore, I denote this by with the subscript x as you can see so I denote this that the q dot with the subscript x and you have qx and del qx del x and dv is the volume of the fluid element. So therefore, total supply of heat.

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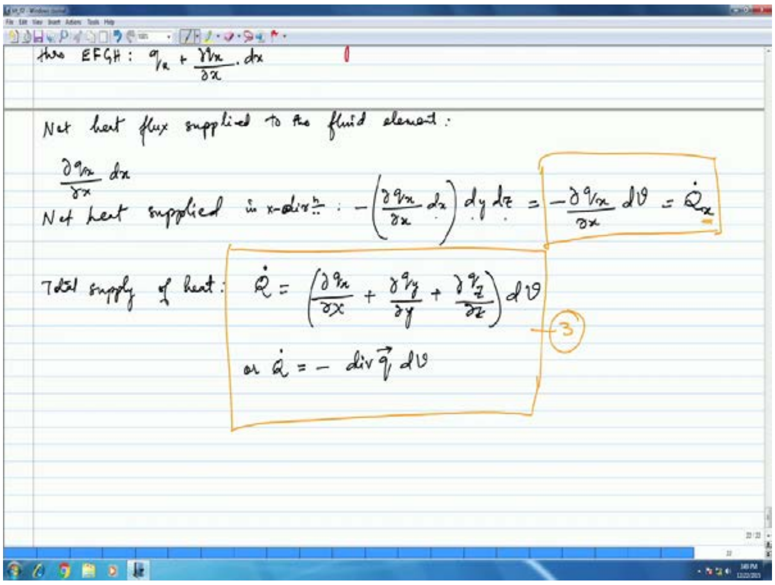

Total supply of heat is essentially Q dot which is equal to del qx del x plus del qy del y plus del qz del z volume and as we had written this is the heat flux vector, and this what I see is essentially the divergence of that vector is not it. Therefore, I can also write this as Q dot is essentially equal to minus divergence of q into the volume, is that right. So what we going to do is keep this for reference and this is this. As you can see we got some expression for Q dot as you can see here. We got an expression for Q dot in terms of the total volume and the flux vector. Let me stop here and we will start with the energy and look at how, what we will do with that. So, right now I will stop here.

Thanks.