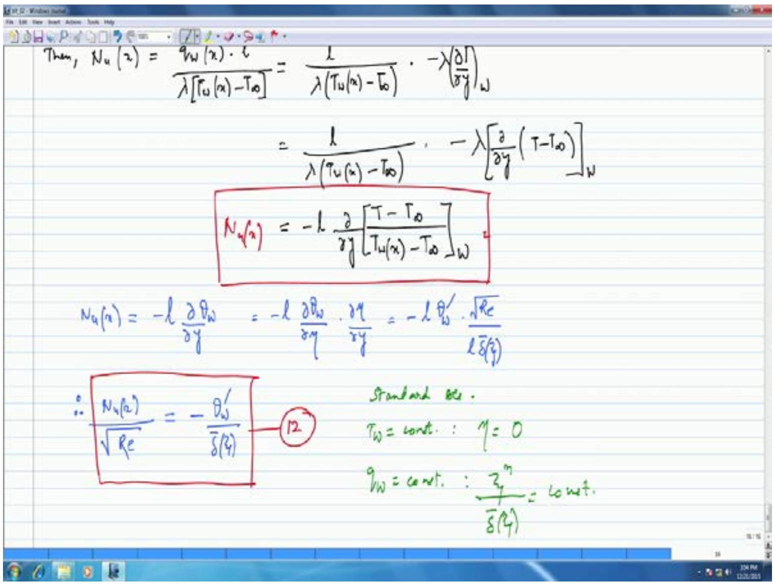
**Introduction to Boundary Layers**
**Dr. Rinku Mukherjee**
**Department of Mechanical Engineering**
**Indian Institute of Technology, Madras**

**Module - 04**
**Lecture - 27**
**The Energy Equation-II**

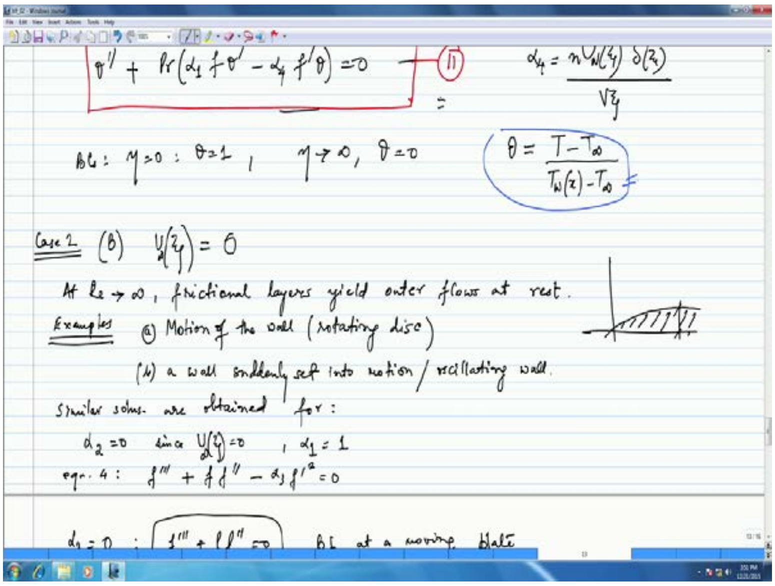
Hi, welcome back. So, we were at the Nusselt Number, trying to find out the nusselt number. We did arrive at this, we were somewhere here.

(Refer Slide Time: 00:23)



We got an expression for the nusselt number; we did math and try to find out the nusselt number. And let us see if we can expand on this a little more, if we can do this a little more. Now, again the nusselt number therefore, I can write x minus l del del y.

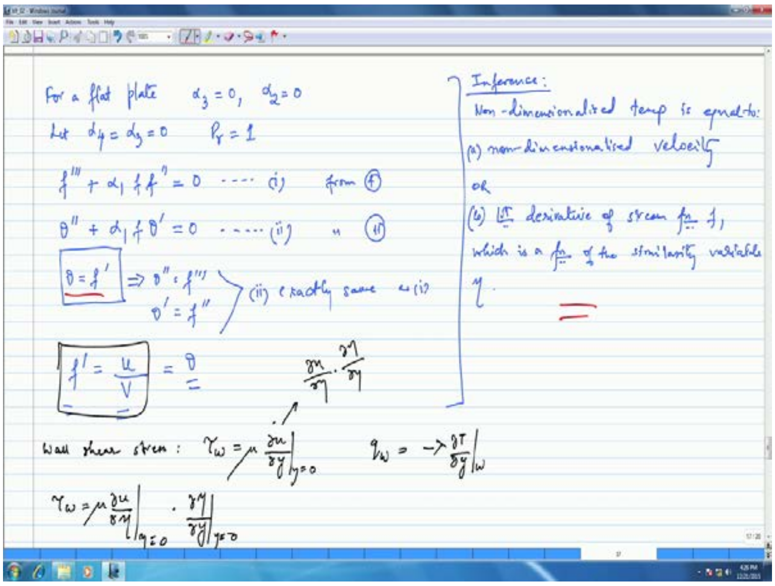
(Refer Slide Time: 00:54)




Now, what did we define theta earlier on? We did defined theta see, we defined theta as T minus T infinity by T w T wall x minus T. So, what we have here in this expression if you see is just that. It is nothing but, theta at the wall. If that is true so then I can write this as minus this del theta at the wall eta del eta del y, so this is equal to minus l, this is theta w this is the derivative, derivative of the temperature difference. So you did take in derivative with respect to eta and del eta del y is nothing but under root of Re by l delta bar (Refer Time: 02:04) of zeta.

Again these two else will sort of cancel out. Therefore, we can write that the nusselts number by the Reynolds number is equal to wall phi zeta. That is the nice expression to use. Of course, then combine this so we are going to call this as 12. I am going to call this as 12. And of course, we will have standard boundary conditions. Of course, T wall is constant which means eta is 0, then q wall is constant, zeta by constant. We are going to just sort of take a step back and look at this a little bit. Now, what we will do is we will compare equations 4 and 11. Now 4 and 11, it is very interesting. Now, that we have kind of you got an idea about nusselt number and all that, so we will see now if we understand something out of it if the equations are telling us you know giving a some more equations. So, what we will do is, look at equations 4 and 11. What was equation 11? Let us go back and look. Basically, this is equation 11 if you look at this value equation this is 11, and this is the equation 4 this has been derived earlier. These are the two equations.

(Refer Slide Time: 05:07)




Now, for a flat plate for example, alpha 3 is 0, alpha 2 is 0. Then you let alpha 4 equal to alpha 3 is 0 and this prandtl number to be equal to 1. Then what happens to the two equations? What happens to equations 4 and 11? If I do this then what I have then equation 4 basically becomes this plus alpha 1 f of f double dash is equal to 0, that is 1 and the equation 11 it becomes theta plus alpha 1 f of theta dash is equal to 0, and this is this way.

Now, let us look at these two equations. Essentially, this is from equation 4 and this is from equation 11, is that right. So now, see equation 2 is exactly, now let us sort of see what the difference between these two equations is? For a second for example, just forget that this f and theta and these things mean anything just think of as to you know just symbols and you know there we have written some differential equation here. So, just see, what is the difference between one into that we have written here, if you want sort of gauge look at this little more intentively. What I can see that these two equations, equation 2 can exactly equal to equation 1 if theta is equal to f dash. What I see is that for we have f triple dash and I have theta double dash. I have alpha 1 f and we have f double dash and I have theta dash. So, if theta is equal to f dash just think about that. So, theta double dash will be f triple dash. This would mean that theta double dash will be f triple dash and theta dash would be f double dash. In that case, if this case 2 will be exactly same as 1.

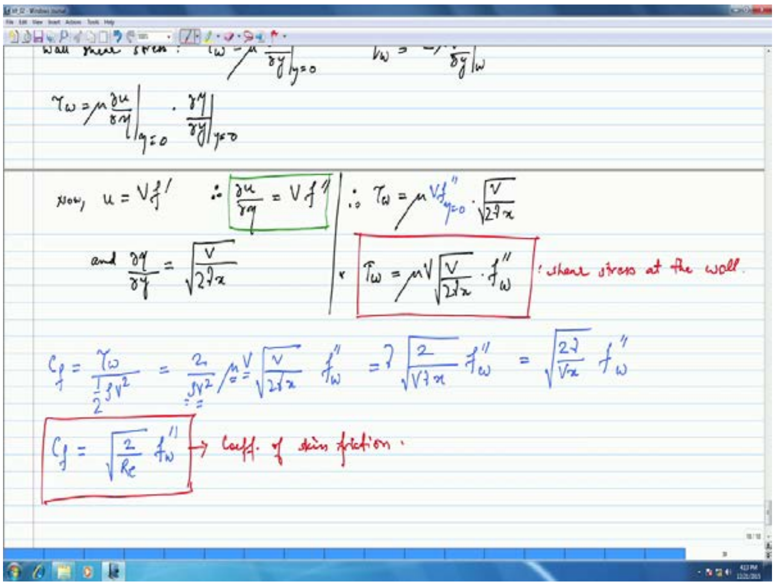
Now, what is that even mean? Does that mean anything, what is the implication of that? So let us see. We are basically saying that theta and f dash, of course they have a physical meaning where we will be know that. Now we saw that, if we set of played around with these values of alpha 1, alpha 2, alpha 3, alpha 4 and the prandtl number we get something like this. And from our studying of these two equations 1 and 2 we get that these two can be exactly same with each other if theta is equal to f dash. So, what is that even mean? Does it even mean anything, and what is the implication of that? Now we know that ok let us use this space here now. So, f dash is nothing but u by v. In this case this is equal to theta. Of course, if I were to, it is very important to sometimes we write down expressions in a math making to mean something only when you sort of read them aloud in full sentences.

So let us sort of write that and what is this even mean, this thing that we just wrote here that non-dimensionalised, this theta is nothing but non-dimensionalised temperature is equivalent to the non-dimensionalised velocity or the first derivative of the similarity stream function f. That is what this things means let me write that down. So, from all of this what I can write is non-dimensionalised temperature difference temperature rise is equal to equivalent to is equal to number one or let just say a; is equal to non-dimensionalised velocity or basically the first derivative of f which is the non-dimensionalised of the similarity stream function. Stream function based on the similar variable.

So first derivative of the stream function f, which is a function of the similarity variable eta, f is a function of eta. What I am doing here is, this is very important for you to do sometimes, so when you look at something like this just sort of jolt down is to what inferences you get. You could write you know inference. For example, I would write what is the implication. Whatever does not matter? What is the inference that the non-dimensionalised temperature is equivalent to non-dimensionalised velocity or the first derivative of the extreme function which is based on the similarity variable?

So now, wall shear stress. Now, wall is this, so therefore, y is 0. Again, therefore, I am going to write tau wall as mu del u del eta at eta is equal to 0 into del eta del y at y is equal to 0. So I am just breaking down this the derivative here with respect to y when I write the wall shear stress in the similarity variables. So essentially, I mean what I am doing here is writing it is del u del eta into del eta del y that is all. Therefore, in this case eta is 0; in this case y is 0. So if I do that.

(Refer Slide Time: 15:10)




For example, u so f dash is basically equal to u by V which we just wrote here, this is something that we know. So therefore, del u del eta, see how interesting this is nothing but f of double dash so del f dash del eta, so that is del V f double dash. And del eta del y well that is equal to V by 2 nu x. That is basically the math, the way we define that. Therefore, tau w is equal to mu del u del eta at eta is equal to 0. Which is say del u del eta at eta is equal to 0, where f double dash is. So, let me write that down. So this and it is V f double dash eta is equal to 0 into delta eta delta y at y is equal to 0 and into V 2 nu x.

Then, therefore or at the wall is equal to mu. So f double dash at eta is equal to 0, what is f double dash? F double dash at the wall; I think you can just go back and look up that when we sort of did this, but all sort of think about what it will be. I am just going to say that this is going to be wall so cross check ok just cross check that. Check that for yourself. So I am going to write V and then write like this V 2 nu x. So I will just say this is f double dash at the wall, let me just write it that way. Think about how this f double dash eta is 0 or f double dash of the wall will be high. Therefore, shear stress at the wall for me if I were to write this down. So this is shear stress at the wall, is that right.

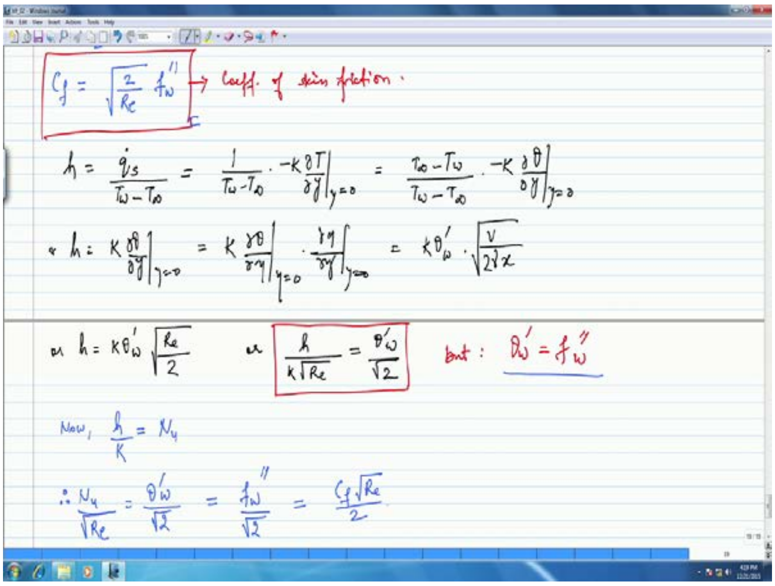
Then our interest is of course in the coefficient of friction, so then we know that this is equal to half rho V square into say yes so shear stress. I hope you remember that coefficient of friction, so that is basically force by dynamic pressure into area. Since we are writing this stress so the force by area is going to this, you do not have that area term in case you are getting confused. Then I will write this at two by rho v square and I will put in the expression for the tau wall that we just wrote out. If I do that so then I get mu V V by 2 nu x into f double dash. If this is two, then this is equal to under root of 2 by V you can do the math a little bit so this V at the numerator cancels one V of this V square.

Then I have got a root V here, so I bring that then this is kind of cancel one root so then I get V here. Then you have nu and then you have x. Then again I will write this as f double dash at the wall. So then, this is giving you further hints is to what if I could write it further. So, we miss something here sorry about that. We have got what we have missed here is we got nu and we got a rho, the mu and the rho so that will be a nu like that. So what we will get here is therefore, root of 2 nu by V x f double dash w. Now, it is makes it clear, so what do we get now? Therefore, this is equal to 2 by Re, hopefully that is not difficult to see here. So therefore, this is nothing but our coefficient of skin friction. So this is nothing but our coefficient of friction.

If you remember the computer code that we wrote, so we were getting outputs for f dash for f double dash and then what are we going to do with that. One is of course, if you can see here that if I get an output for the f dash, so then I should be able to plot u by v. I should be able to do that. So, that is what the solution is going to do. You can plot out the velocity profiles at different x locations depending on what service you looking at, that is what we mean by solution. Then if you continue further I should get a plot of f w, I mean

I should get an output of f double dash at the wall so then I can use that to calculate the coefficient of friction. So that is that. Now, let us do a little further instead of on this and see what we will get. Let us do this little a further.

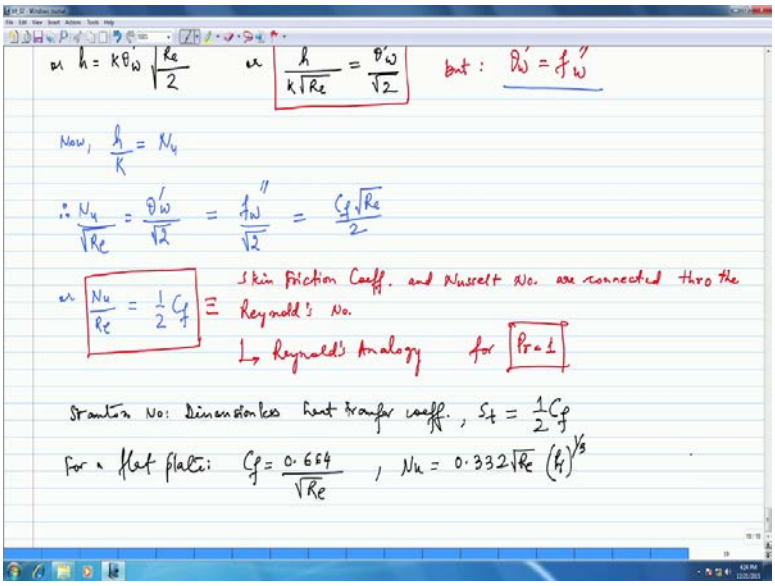
(Refer Slide Time: 24:00)




Since we are on this, now we will write h, we will define h as this. If I do that then what I get is, I will write this as 1 by T w this into that at y is 0. So, I am going to write this at T infinity minus T w and T w minus T infinity minus k del theta del y y is equal to 0. So therefore, basically we get h is equal to this, or say h is equal to k delta theta delta y at y is equal to 0 which again I can write as k delta theta delta eta, eta is equal to 0 into delta eta delta y, y is equal to 0. So which is again equal to k theta dash at the wall because eta is 0 and this again we know, so therefore that is V 2 nu x, or h is equal to k theta dash this into Re by 2 or we can also write h by k under root Re is equal to theta dash at the wall by 2. But, that something we know right that something that what we were sort of talking about is not it.

Now again, h by k is equal to nusselt number. So therefore, nusselt number is equal to or say; nusselt. Now, h by k here in this expression is nusselt number. So therefore I can write; if I replace that by under root of Re is equal to theta dash at the wall by root of 2. This is again equal to f wall double dash because theta dash of the wall is equal to f double dash by root 2, which again if you back and look at the coefficient of friction which we would in terms of f dash so from there if I write, so then that is equal too. So, f double dash by root 2, by root two should be equal too. What we get is C f under root of Re by 2.

(Refer Slide Time: 28:47)

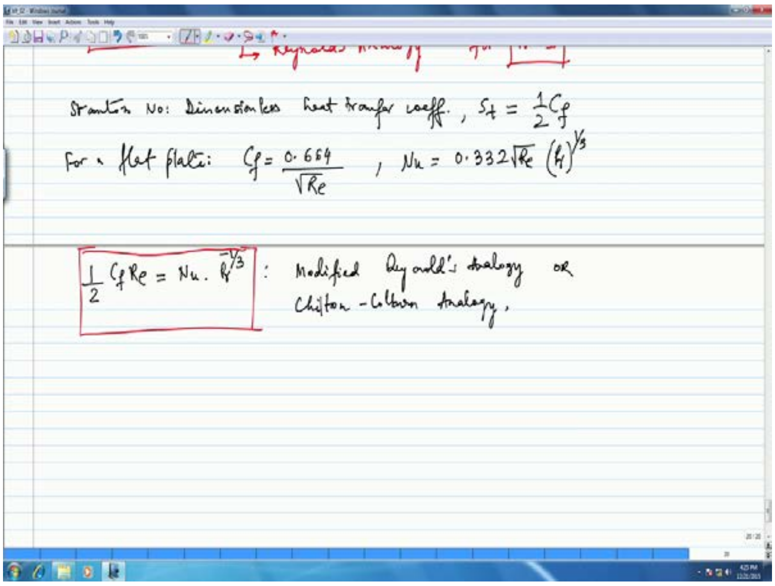



Or nusselt number by Re is equal to half the, this thing. This is quite revolution to us is not it. Therefore, these skin friction coefficient nusselt numbers are connected through the Reynolds number, that is quite interesting is not it. In other words, in order to calculate the heat transfer at the wall knowledge of this skin friction coefficient and Reynolds number are sufficient, you do not need to do anything else, you do not need to kind of go and solve the energy equation separately just this information is sufficient. And of course, it is kind of easier to evaluate C f and coefficient of friction and the Reynolds number. Now, this is Reynolds analogy, but please remember that we were able to get all of this only when you know for prandtl number 1.

Let me instead of write that down. So what this is basically telling us is that skin friction coefficient and the nusselt number are connected through the Reynolds number. So I am not going to write the rest of it. Basically, what you can see here is that in order to calculate the nusselt number all we need to do is calculate C f and Re should be able to get that which is easy to do. This whole thing that the skin friction coefficient and the nusselt number actually connected through the Reynolds number, this is actually the Reynolds Analogy this is basically for prandtl number 1. This is when we were comparing equations 4 and 11. So that is how we came up with. I will just sort of before I close this write up this you know small expressions and then we will close.

So, the stanton number is essentially dimensionless heat transfer. So, let us say that is stanton number is equal to half of the skin friction coefficient. Again for a flat plate of course, we know that C f is equal to 0.664 by Re. So, we know this, therefore, now nusselt number is equal to 0.332, if you see here this expression is nusselts number is equal to Reynolds number into half of C f, which is equal to under root RE and prandtl number to the power one third. This is just to kind of wind this up I will kind of a stop here after that. So I will just write this.

(Refer Slide Time: 33:27)




Now, if I were to write this, so half C f Reynolds number is equal to nusselt number into prandtl number to the power and that is 1 by 3. This is essentially the modified Reynolds Analogy. Well, it is named after people so I will sort of write them.

So, I think will kind of stop there. I think we kind of did an elongated couple of lessons on using the similarity variable to get solutions, and hopefully this gave you some idea. And how different basically doing a basic math not too complicated you should be able to tackle a large number of problem. The categorize is very important to remember that when we are using this for example, this last one when we looked at this, so there are there are conditions here. So when theta is this, so then we wrote this down and y is this that is because for a flat plate alpha 3, alpha 2, alpha 4, alpha 3 prandtl number.

These are things you need to remember but we get some very interesting results. What we first we were doing is trying to get a physical field for the thing now what we did is we kind of did just a math and from the math now we are trying to get a picture of things and we see that we are able to kind of go back and forth. Sometime we start with the math which seems a little bit complicated and then it is you know gives you some idea about the actual physical flow. So, I think we will stop here and take up something in the next module.

Thanks.