Introduction to Boundary Layers
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Module – 03
Lecture – 26
The Energy Equation-I

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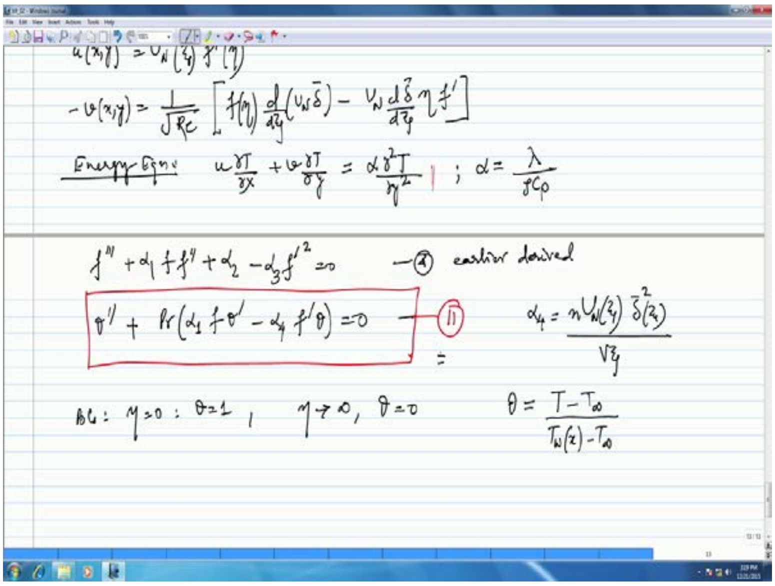

Hi, welcome back. We were doing, trying to find solutions to the boundary layer equations using similarity variable. And we said we going to divide these, we going to sort of categories the set of problems, one is when the outer flow is you know non zero, when the outer flow is 0. So, we kind of done or also kind of touched upon a little bit is to how to use the energy equation.

Last time we did this, so this is the energy equation. Now there is this theta, so theta is nothing but the non-dimensionalized temperature. So, theta is basically, it is nothing but the temperature minus the free stream temperature, so temperature at the wall minus the free stream temperature so that is basically theta that we are talking about.

In the sense that usually we measure temperature as a difference to a reference. So, hence we are kind of taking this as a difference of the temperature from the free stream with respect to the temperature at the wall - the wall temperature as a difference from the free stream. So, this is what theta is all about and we got this equation 11. And you can see that we have got the Prandtl number and the f and f dash and alpha 1, alpha 4 and so on and so forth. And we have got the boundary conditions as well and this really sort of now, cause for this. So this is the case that we kind of dealt with at the end.

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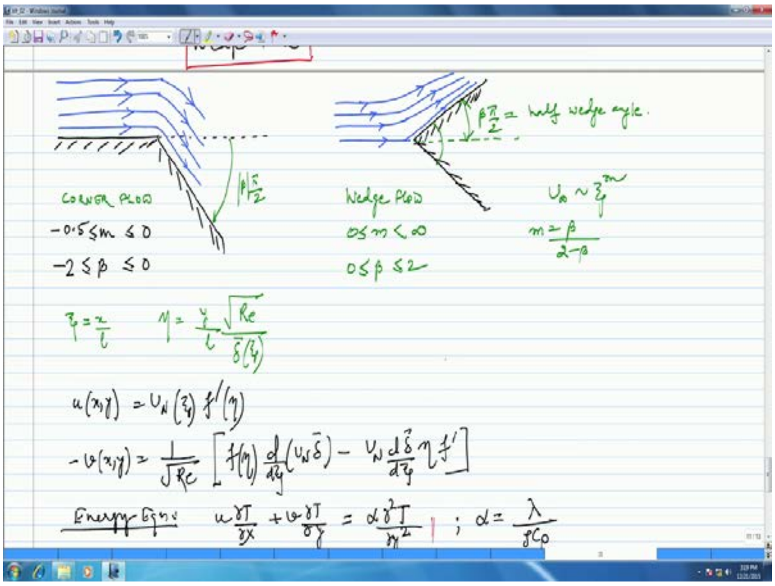

We have talked about wedge flow, corner flow and things like that. So, now let us just look at little bit for the case where the outer flow is 0. So, what is that even sort of means if I would continue from here.

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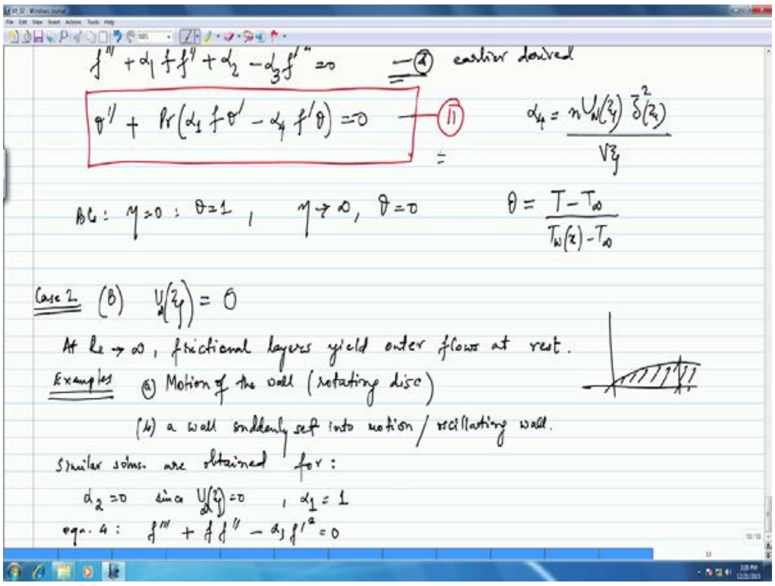

This is a case 2 actually; this is case 2 which is the b part. And that is that u zeta is equal to 0 that is basically the outer flow. Now it is actually kind of little unusual think, I mean, how can you have a boundary layer even form, if we do not have an outer flow. But you know something like at high Reynolds numbers, at very high Reynolds numbers frictional layers yield outer flows, this is not to confuse that you know we do not have a fluid outside the boundary layer, it is just that the flow is at rest.

So, at a very high Reynolds number, actually at very high Reynolds numbers what happens to the boundary layer? The boundary layer also becomes very thin. So, the friction layers will yield outer flows at rest. So, then in this case, if you have this is your say boundary layer, so this will basically cause your frictional this; your viscosity is of course, confined to this zone. This is your boundary layer, it is confined to this zone but it at very high Reynolds numbers and it can actually make this flow to go to rest.

Hence, now frictional layers to give you an example, they could arise due to - so example of say frictional layers, when can you see you know frictional layers? Motion of the wall, so for example; rotating disk, what is the best you can think of. Just think about it. And another is a wall suddenly set into motion or something which is oscillating or an oscillating wall. So, you have break disk and there is a fluid which is attached which is basically in between the disk. So, if you think about it so if you have to rotating disks like this and due to the disk, if the fluorine is in contact with this disk you will end up having the outer flow which is velocity being 0, it is at rest, this is a possibility.

And a wall, of course, I mean if you kind of just suddenly push it, you know motion is not suddenly it is not going to set into motion. And if for example, if it sort of keeps going back and forth, back and forth then too, there is not enough say momentum generated to make a flow happen. That does not happen, so therefore you do have outer flows at rest. So, how would I in this such a case; how would I sort of use the similarity solutions to get and solve our equations. So, in here similar solutions are obtained for now alpha 2 is equal to 0 here, since u is 0 and alpha 1 is 1.

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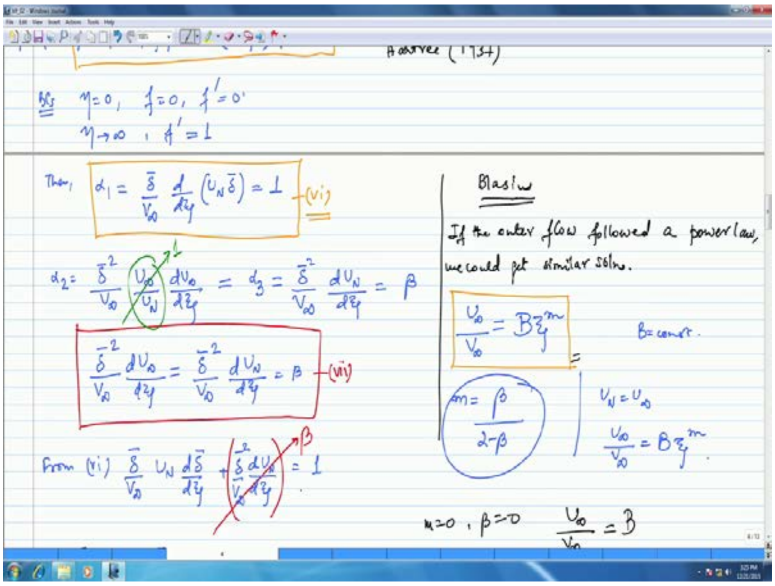

Let us remind ourselves what alpha 1 and alpha 2 are, let me go and remind ourselves. There you go. If you see 6 here, if you see 6 so that is alpha 1 and you can see that we have V D zeta of U N which this value is basically equals to the outside flow in this particular case and that is U N. So, which is u in this case and this is 0. So, since this is 0, so sorry, I mean yeah, so if this bit is U N is that what we talk about, we were talking about alpha 2, what is alpha 2. So, anyway so this bits is this U N is the well, we do not have the u z zeta here, this is the normal component to the velocity so that is 1, we take that as 1. U 2 is this here, u 2 is here because this is equal to let us see where is alpha 2, there we go. So, we have got alpha 2 here, now that goes to 0 because the outer flow is 0, this itself is 0. So, u infinity is 0.

And if I do that, so I guess I should just write this since I did not this kind of course, you may be looking confusion. So, this is what I mean is that when I say outer flow is 0, so basically we mean that the free stream is at rest right - outer flows. So, therefore, this is 0, because this is an alpha 1 is 1. So, then again equation 4, which is you know only derived that becomes. So, if this is the equation 4 again written here, if you see. So, alpha 2 is 0, and alpha 1 is 1. Then what we get here is equation 4 then becomes f triple dash plus f of f double dash minus alpha 3 f dash square. So this is what we get this is what we get.

Therefore, now what is interesting is that this is the type of solution view I have shown you right using the little computed code. So, therefore this can be solved very simply. And in this equation for example, if we take different values of alpha 3, you can solve it, you will get a solution and each of that really results for us certain physical flow.

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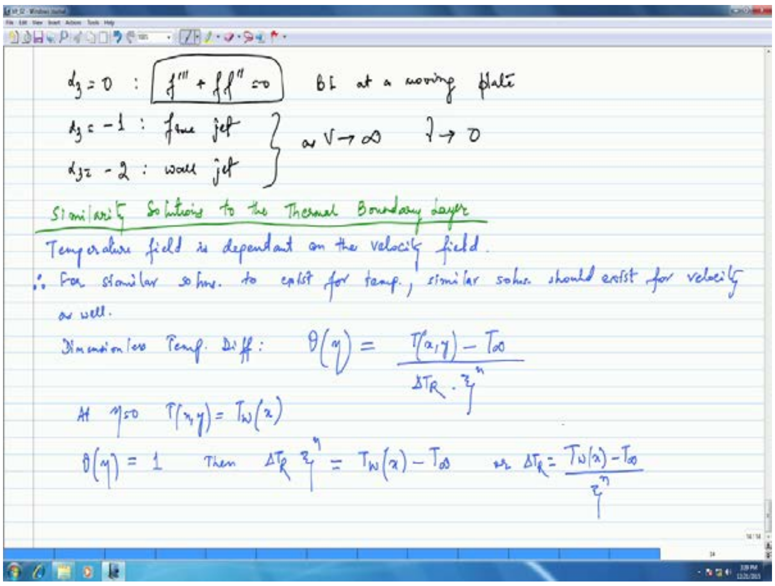

For example, alpha 3 is equal to 0. So, this term actually goes, right? What is the equation reduced to these think. So, this particular thing is actually gone, right that term. So, what we get now is basically this. So, this is something that we have already seen, that is the equation I showed you. So, what is this about this is basically boundary layer to moving plate. So, this is nothing but boundary layer at a moving plate. So, then alpha 3 is equal to minus 1, what we get is a free jet, and alpha 3 is equal to minus 2 is wall jet. Well, now for these two cases, for these two cases actually you know as this becomes very large as basically the outside flow becomes very large then I think that kind of in the sense that the viscosity is kind of being unremind by the speed of the flow.

Thus in the sense that the momentum of the flow is kind of or say yes the momentum of the flow is taking predominance over the viscosity, so that it is kind of intuitive if something like this. So, if you have a very thick fluid, so that is going to flow less. It is common sense, that when it is very thick it is going to be attached more to the surface on which it is moving, it is going to be very thick and resistance to moving, is not it. If it is lighted, it will move.

Now, if however, you know force it with increasing its velocity then it should be able to move. It is like basically the viscosity is nothing but it is applying a resistive force. So, I apply a momentum to counter that and therefore, I mean that if the velocity is pretty large for the same mass of the fluid then I should be able to counteract the resistive with viscous forces. That is what it means, but I unlike it is really intuitive if you think about it. So, I think you know we have kind of covered it more or less. So, the point is that you know if you just take the little computed code that we wrote, and you solved it basically for some this sort of an equation. The equation will have other term if you have alpha 3 is minus 1, minus 2, etcetera if you are able to solve that you should be getting solutions and that should give you idea about free jet, wall jet etcetera.

Now let us now look at this I mean we looked at the temperature etcetera we just took the equation. We took the equation, energy equation when we were able to get something like 11 - this equation. So, we got something like this. And your theta is nothing but the non-dimensionalized temperature. Now, let us see if you can get some similarity solutions to the thermal boundary layer. So, let us call this and say this is similarity solutions to the thermal boundary layer. Now the temperature field is dependent on the velocity field; well, yes I guess, I mean that this is also dependent on the velocity field. So, we are kind of this is the energy equation that we looked at. So, theta is of course, the non dimensional temperature difference. Now, let us to say the temperature field is dependent on the velocity field. Therefore, if that is true then for similar solutions to exist for the temperature the same should exist for the velocity.

Since we were there, so, like we said you know the dimensionless temperature difference of course, so let me complete this. Therefore, for similar solutions I am going to do that, to exist for temperature similar solution should exist for velocity as well. Now so therefore so the dimensionless temperature difference, so let us start like this.

So, dimensionless temperature difference so that is theta, this is a temperature difference, this. So, now, if I have that right, just to see at of course, this is the boundary conditions then at eta is equal to 0, of course, it is equal to the wall, so that now theta eta is equal to 1. But then basically if that is true if that is 1, so basically I am using this expression here. So, zeta to the power n is equal to T w x minus this, right? So, therefore or delta T r is equal to T w x minus T infinity by zeta to the power n. So, I bring basically this a zeta on the right hand side at the denominator. Now, zeta is something that we know zeta is nothing but x by l.

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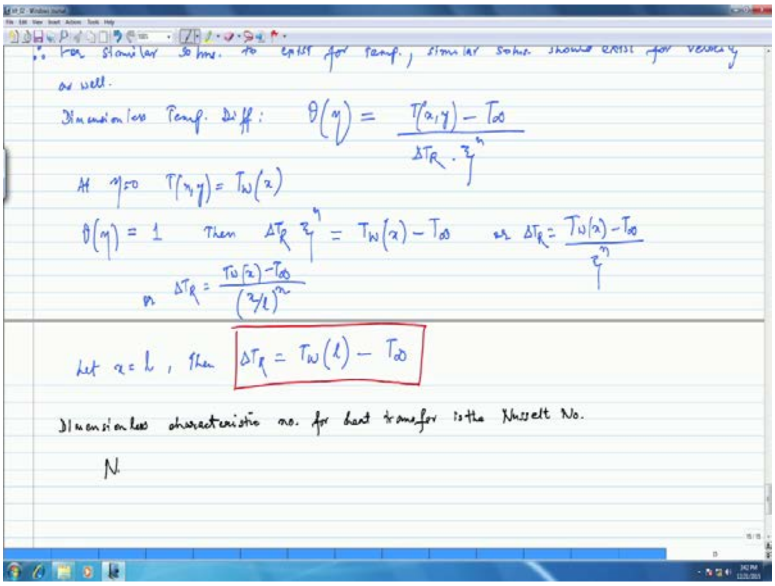

So, or again and or delta T r is T w x minus that x by l to the power n. Now, let us say x be equal to l. So, x is equal to l then delta T r from just the expression up of here. So, the bottom basically becomes 1 right is T w l minus T infinity. So, this is T w, so this is something, we had written back up here right. So, I did not explain that I did not explain that too much. So, this is what we had written. So, T minus T infinity, T w minus T infinity, so that is what we get actually, so T w how would do we get that? So this delta T R is nothing but this, T w. You can keep the l or it does not matter, it is basically at the wall minus T infinity.

Now, there are you know like we have been dealing with the Reynolds number, Reynolds number is something that we use once we define our flows. Now the dimensionless, characteristics number, for heat transfer you must probably know this is a Nusselt number. So, let us see what that is, and what we going to do with it. So, dimensionless, characteristic number, for heat transfer is the Nusselt number; so Nusselt number which is we will call that this.

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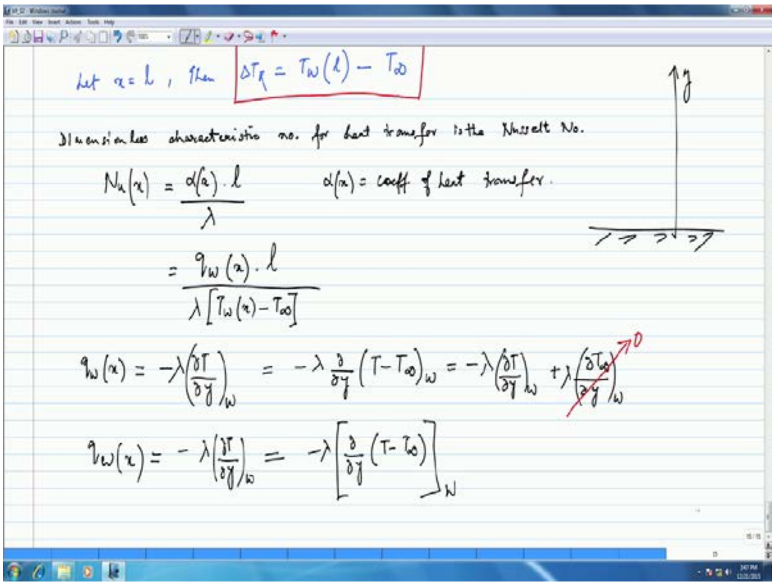

So, the Nusselt number again that will depend on your x is nothing but. And alpha x is nothing but coefficient of heat transfer. So, if that is there, so then how do we sort of you know write this a little more? So, alpha x is nothing but coefficient of heat transfer. Therefore, I can write this; is the coefficient of heat transfer. So, essentially this is the flux, is not it. So, then if that is true, now this is what? This is nothing but, this is basically at the wall; is not it. So, basically lambda you know is lambda is the slope right slope of this curve. So, delta T delta y the way the temperature is changing along y so that is essentially your lambda the slope of that curve is basically your lambda, so which you can see from here. So, this is similar to you know when we put down stress you know viscous stress in the boundary layer. So, then this I can write as minus lambda del y T minus T infinity at the wall.

If I do that then I get this minus lambda del T del y at the wall plus lambda del T infinity del y and this thing at the wall, clearly of course, that is you know that does not change, is not it; that does not change. So, this is going to be 0, do you understand why that is going to be 0, it is just the free stream. The free stream value does not change; free stream value is a constant value. So, at the wall that does not change so we get that 0.

Therefore, q x at the wall is basically equal to minus right, and think about this why do we have a negative sign there, think about that. So, which is basically this q x is giving you an idea as to how the temperature changes as you go away as you along y, as you go along y. So, if this is your wall and this is your y direction, so delta T delta y gives you an idea about the wall and you have a negative lambda either there, so how? So basically what is that negative sign exactly mean think about that; I mean that is a good question to sort of think about it. You can probably get the answer in a book, but I still like you to know dwell and down there a little bit and think about it.

And use a common sense, you know take a little bit of the way to do this, take your tea cup, put your hand at the bottom of the cup, and then go up, go up, do not burn yourself but think about that; how the temperature is and things like that, that is should give you an idea. So, then this is equal to minus lambda del del y T minus T infinity at the wall that is interesting, so we get that. So, basically we get an expression for q w. So, then we should now be able to get an expression for, we will go and complete this so we are able to write it this way.

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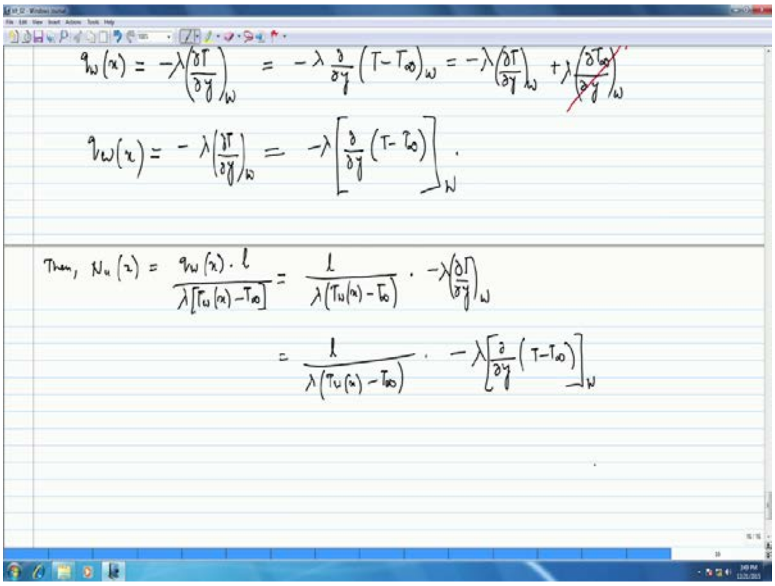

So what happens to the Nusselt number? So, to the Nusselt number therefore, the Nusselt number becomes lambda T w x minus T infinity. Now that is equal to l by lambda this kind of writing it out dot q w x. So, I am going to write that as minus lambda del T del y at the wall, or basically I can write this as you know or this is equal to l by lambda T w minus T infinity into minus lambda del del y which we are getting basically from here now, T minus T infinity at the wall. If we do that, so then it balls down to what? So therefore this balls down to minus l, so basically the lambdas sort of a cancel out. So, I can say it is del del y of T minus T infinity, T w x minus T infinity at the wall.

So, we will kind of stop here, we will continue this and look at the boundary conditions which we will use with this and what we arrive in the end. So, I will see you in the next module.

Thanks for now.