**Introduction to Boundary Layers**
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**Module – 01**
**Lecture – 25**
**Similarity solutions to the BL equations (other than flat plate)-IV**

Hi, welcome. Again, we are going to start with this equation 4 basically, and let us use this.

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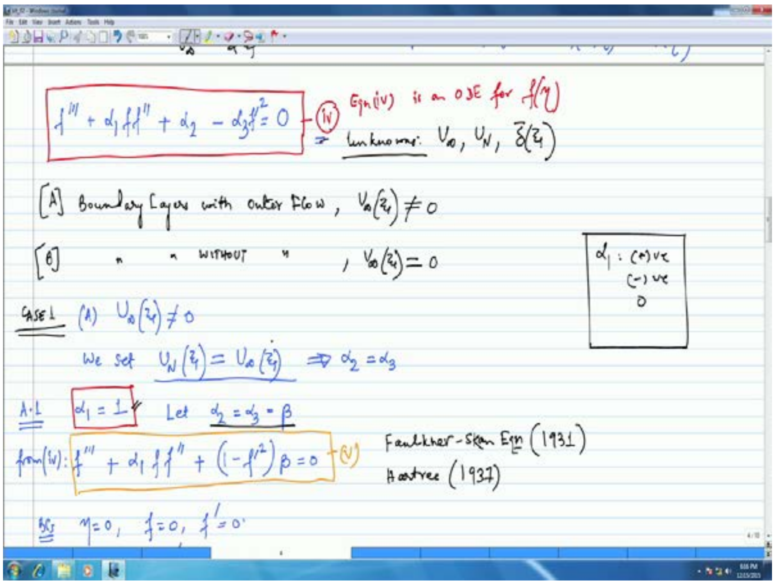

We said that we are going to look at this equation for different values of alpha 1. So we have looked at positive where we took the alpha 1 is equal to 1 without loss of generality that we said. Now, what we will do is we will set it to negative and again we will put that to alpha 1 is equal to minus 1 and we will say let, the thing that we have here alpha 2 is equal to alpha 3 we will said that to be minus beta, then what happens to equation four? If I do that, what happens to equation 4? If I do that, let us come here.

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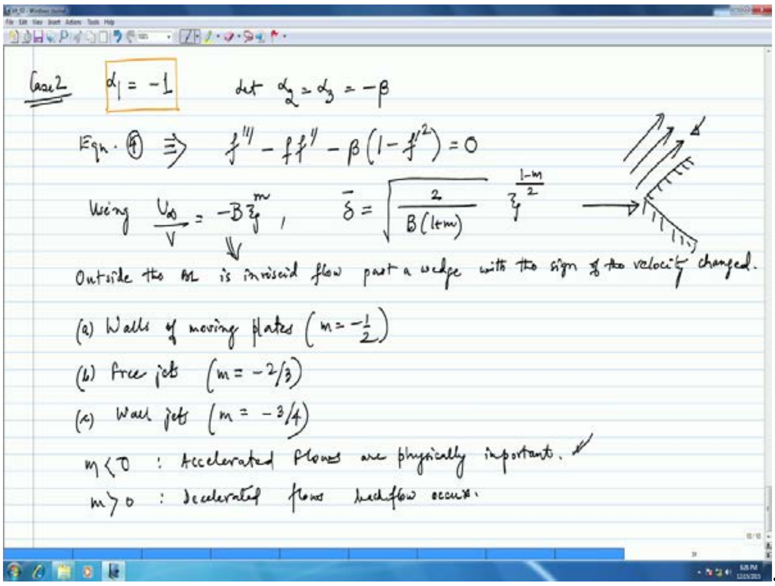


So this is essentially, well the case 2 I think. Here, alpha 1 is equal to minus 1. Alpha 1 is minus 1, then let alpha 2 equal to alpha 3 be equal to minus beta. Then, equation 4 that gives us what, that basically becomes f triple dash minus f f of double dash minus beta 1 minus f dash square is equal to 0 and if I use the power law, so U infinity by V is equal to minus B zeta to the power m. If I use that delta bar, so it is a same procedure I have not sort of work this out again to get the expression for delta bar. If you think you should do it please do it, otherwise it would be just repeating the working out, so I have kind of done it once to drive from the point, not going to repeat it, please feel free to practice this if you need.

So, delta bar then comes out to be 2 by B 1 plus m, zeta to the power 1 minus m by 2. Now again, here what kind of stuff are we going to see, So all we are doing here is assigning different values to alpha 1, alpha 2, alpha 3 and seeing what we get. Here again, outside the boundary layer is inviscid flow, and I mean the kind of flow that we were sort of looking at so when we define the first case that it is basically if we had a wedge like this, we said we will have wedge like that and we had flow impinging this way and you had flow moving this way.

In this case however, the sign of velocity is changed if you see this, so I would have velocity in this direction. So, outside the boundary layer is inviscid flow. Past wedge, basically what I am saying is what this means, past wedge with, I am sorry about that, with the sign of the velocity changed. So outside the boundary layer is inviscid flow past wedge with the sign of the velocity changed.

Again here I will give you the particular cases, what will arise depending on the kind of valence of the better then we can use. Number one, is walls of moving plates, so m is equal to minus half. So if you have m equal to minus half it basically you should be able study flow walls of moving plates. Then you got free jets for which m is equal to minus 2 by 3, this is the case were we said that alpha is going to be negative. We see that m seems to be negative as well, so we taking all negative values. Then with the wall jets where m is equal to minus 3 by 4 and well, flows through nozzles with the counter walls also.

Now, if you see m less than 0, what that essentially means is that; accelerated flows. If m is negative, so then what we kind of understand is that it is basically accelerated flows. Of course, these are physically important, whether it is flow past something like this, flow past f oils and flow past cylinders. Accelerated flows are physically important, and will kind of as we learn slowly as we move on towards the end I think of these sorts of lectures we will also talk about separation. So, I think we will kind of touch upon that, regarding accelerated flows and things like that because are physically important what happens if flows are not accelerated, which is like for m positive. I talk about that, so I will just set up write it here it is like a note, it is if you find it will confusing but hopefully we will touch back on this towards the end of the lecture.

So, for m positive there is decelerated flows, back flow occurs. I guess what we trying to say here is that in a accelerated flows keep the direction of flow, it allows the flow to keep going let us put it that way. Decelerated flows kind of pulls back the flow, and back flow you probably familiar where we talk about back flow and it is really when flow separates. So, these are flows which are important, if we can keep flows not separating it is good for us. So that is that. So if we have alpha negative this is what we come up with. Going to the next part, where it is alpha 1 is basically?

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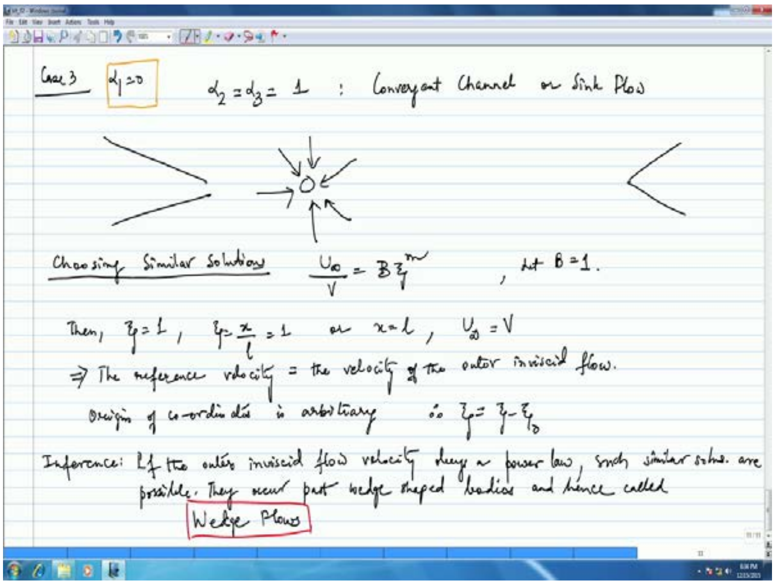


This is case 3, where we said that alpha 1 is equal to 0. If alpha 1 is equal to 0, and then alpha 2 is equal to alpha 3. So, alpha 1 is 0 and alpha 2 is equal to alpha 3 is equal to 1. This actually boils down to convergent channel or sink flow. So, you could have convergent like channel and sink flow is basically flow moving into a sink something like that, so that is that basically. So you get the idea that if I were to do this then how will be. If we were to take different values of m and beta and things like that we should be able to look at various types of flows.

In the sense that here again like I said, it is upon you that you need to choose judiciously, you need to choose the kind of the numerical values in order to study the physical phenomena that you trying to. So, question you will you one needs little bit of experience with that. Let see a couple of examples of that. What I will term this part is, how do I choose the similar solution? How do I exactly go about this? What I am going term this part is choosing similar solution. Let us use the power law, so the way we will use the power law is this.

Now, let us choose B; the constant B in order to set the reference velocity. And so let B be equal to 1. Then for zeta is equal to 1, that is zeta is equal to x by l equal to 1 or x is equal to l. Here, B is equal to 1 and zeta is equal to 1 which means zeta is equal to x by l which is equal to 1 or x is equal to l. So, U infinity is equal to V. If we do this, what is this kind of mean? Basically, we are talking a reference velocity; this is equal to the outer inviscid flow that is all.

What this means is that the reference velocity is equal to the velocity of the outer inviscid flow. Of course, the origin of the coordinates can be chosen arbitrarily mean zeta naught may not have to be 0 or 1 or whatever, so let us say it is zeta naught and eta naught let the origin be that. We can choose origin of coordinates is arbitrary therefore, zeta is equal to zeta minus zeta naught. The inference here is that if the outer inviscid velocity. Of course, here we are basically assuming that the inviscid flow outside obeys a power law. If the outer inviscid velocity obeys a power law, now such similar flows, so we can actually devise means of using that the velocity and we can assume similar flows, and such kind of potential flows occurs across wedge shaped bodies, this is a wedge. When I say that, what is that even mean. Let we just give an example of that. Then therefore, these are called wedge flows.

So let me just write that here, so this final inferences that if the outer inviscid flow velocity obeys a power law, such similar solutions are possible. They occur past wedge shaped bodies and hence called Wedge Flows. So let me just kind of briefly elaborate in that little bit, what exactly do we mean by wedge flows? So now for example something like this.

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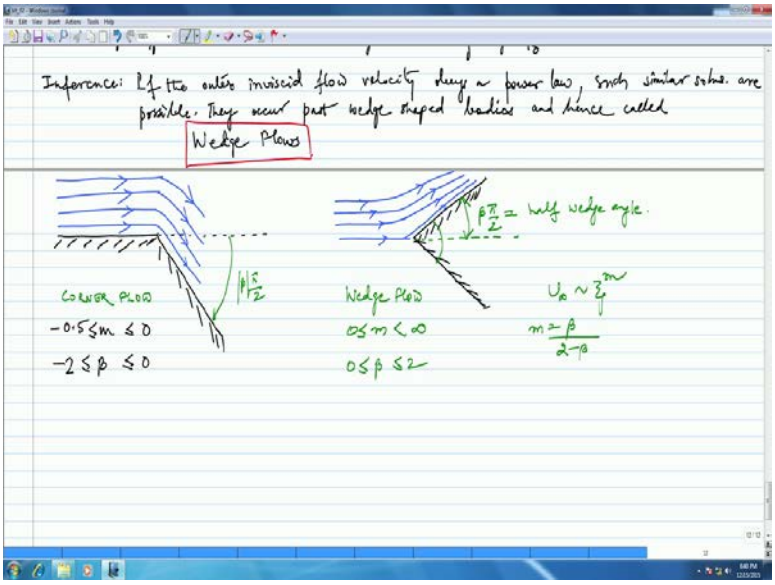


Now, these are type of corners flows will encounter. You have a flow, so it comes in it is a free stream which comes like that, then what does it do it? It turns, it turns, it turns and, so this is mode of beta pi by 2. This is actually corner flow. So let me write that down, this is actually a corner flow, here m is between; it is negative and is less than minus 0.5 and beta lies between again minus 2, so that will automatically come if you are looking at m. If you restrict your solution to these, you should be able to find the solution for a corner flow.

Then again the wedge flow that we have been talking about so for, this is a wedge flow, so we have got this here and again if we have flow coming in and it will go like that and it will go like that. Well, actually this should be little more rounded and this should be a little more. For example, let us do this, this is the stuff and then, and also this one and well these should be parallel actually of you know please bare with me. This will be kind of parallel of each other. Here, this stuff beta pi by 2 and this here is basically wedge flow. This is wedge flow and m here is positive, it has large value and beta this and so we know that. So, now m is of course beta by 2 minus beta and the wedge actually. So, beta pi by 2 is half wedge angle. What this is called is half wedge angle. This is the wedge angle this whole thing, so that is the wedge angle say theta, so this is half of that. So it is called half wedge angle.

Now, so far we have sort of talked about this. We have been talking about velocity a lot; I mean in fact we have just talked about just velocity. Now, what we said is that in the velocity profile similar solutions exist if the outer flow follows a power law. Now, the wall temperature also follows a power law and hence similar solutions exist. If you going to look at that, so wall temperature also will follow power law and similar solutions exist. So if I were to do that, so here what we will do is kind of use our definition of the velocity and the reference velocity and the outer velocity as a power law and use that in the energy equation. If I use that in an energy equation, so what shall we get? What is that boil down to? So that is what we going to talk about.

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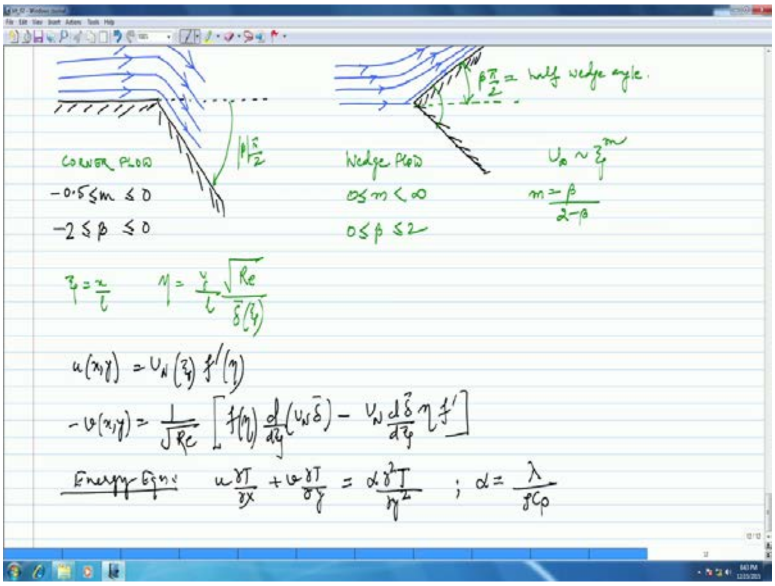


Let us begin that little bit probably we will need some more time to complete that, but let say so we will use this trail solution again. We will use zeta is equal to x by l, eta is y by l under root of Re by delta bar zeta. Then of course, we have this is u x of y is equal to U n zeta f dash eta. Then minus v x, y is equal to 1 by under root of Re f of eta d d zeta of u n delta bar minus U n d delta bar d zeta eta f dash.

Now, if I use the momentum equation and the energy equation. Since, I have not written the energy equation, let me write that down. I am not going to write of the like x momentum equation that you know already. So the energy equation is essentially u del T del x plus v del T del y is equal to alpha del 2 T del y 2; and is alpha is equal to lambda by rho into c p. This is my energy equation and if I were to use this definition here, using alpha 1, alpha 2, alpha 3, etcetera in the momentum equation this is what we will get from I mean I am just repeating this here what we just got earlier.

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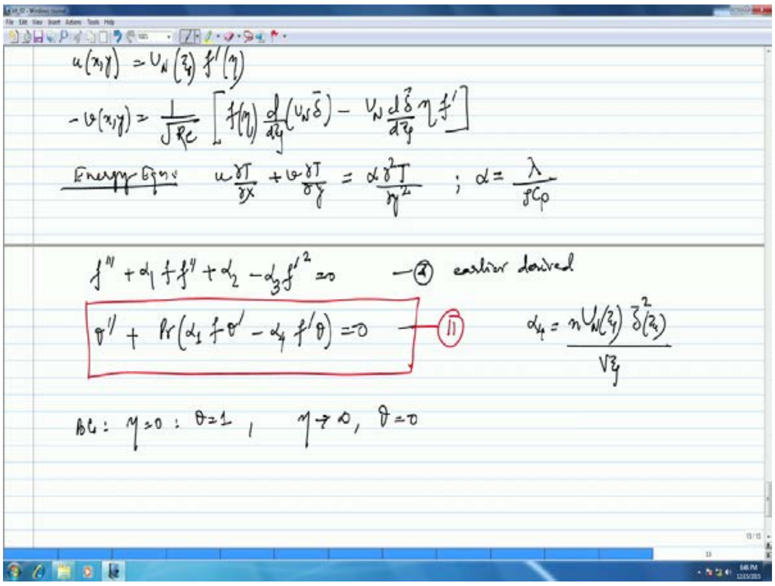

This basically equation four plus alpha 1 f of f double dash plus alpha 2 minus alpha 3 f dash of square is equal to 0, so this is earlier derived. This something we have done earlier.

Now, here if I do this again for the energy equation what I actually get is theta double dash plus prandtl number alpha 1 f of theta dash minus alpha 4 f dash of theta is equal to 0. So, this is the equation that we get and let us call this as 11. Now, alpha 1, alpha 2 and alpha 3 are as we have described earlier, and alpha 4 is n U N zeta delta bar square V zeta and boundary conditions at eta is equal to 0, theta is equal to 1, eta tends to infinity, theta is equal to 0. So basically, theta goes from 1 to; so theta at is 1 and at the edge of the boundary or far away from the boundary layer theta is 0.

So, I will come back in the next module and elaborate little bit on this to; I think some things will need to be explained a little bit I will do that in the next class. So, let us stop here. We have sort of compartmentalized a lot of the solutions were large as you can see, then it is the possibilities of several which is a good thing. We started out where we said that the outer flow is not 0, so we were doing solution that way. So outer flow is not 0 and hence we had to find out that what are the possibilities there and we will do the next one where you know we will have the outer flow as 0. So what kind of, I think it must across your mind that what is that even mean that you do not have an outer flow and you have a boundary layer.

We will just kind of discuss that in the next class and well we have not talked about the thermal boundary layer, we will, but here just we will kind of see numerically if I you know apply these expressions to the energy equation what kind of stuff we get and what are we talking about. So, I think we will do that a little bit to get started and slowly when we move on to the thermal boundary layer then of course formally will talk about it. I will stop here and I will meet you in the next lecture.

Thank you.