Introduction to Boundary Layers
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Module - 04
Lecture - 24
Similarity solutions to the BL equations (other than flat plate)-III

Hi. So, let us continue with this solution of the boundary layers and boundary layer equations for surfaces other than a flat plate. So, we came up so this place where we were using a power law for the velocity description; if you see here, so we were using this description. And with that we have come up with the solution.

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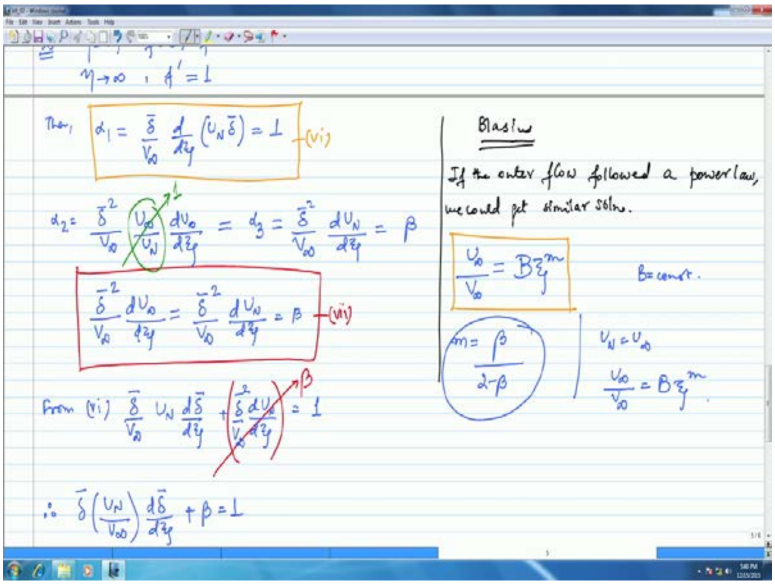

Finally, we stopped where we got a description for you know delta bar which is given by this expression in eight.

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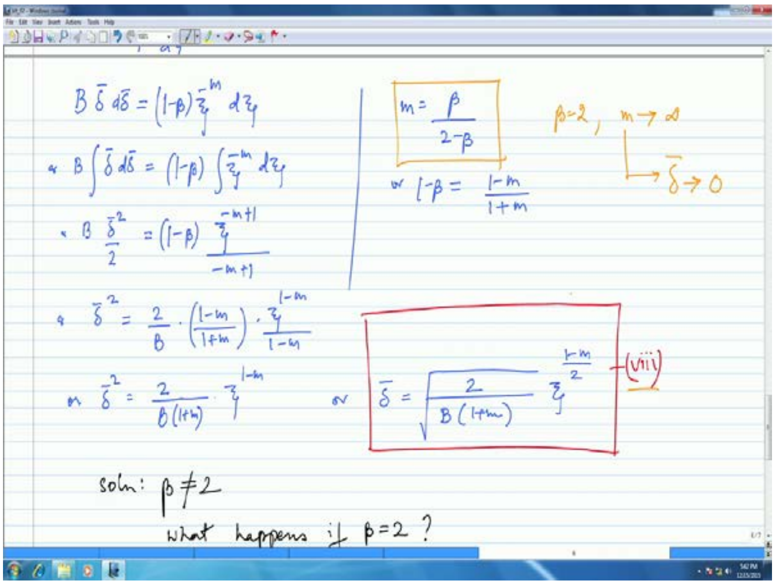

And we said this is a solution for beta not equal to 2 and where I stop was what happens if beta is equal to 2. If you have not sort of a time to think about that, let us go and you know discuss that and then proceed further. So, you can see from the expression here for beta. So, if you see that, if beta is equal to 2 then of course, so if beta is equal to 2, I think m would become very, very large.

So, and if that happens, if m would become extremely large, then if you come and look at this equation right here at eight, m is very, very large and this equation is inverse function of m. So, delta power is an inverse function of m so that means, delta bar will if that happens, so then automatically this would mean that delta bar will tend to 0. And if that happens then this equation does not have any meaning again because all we are trying to do here is to understand the behavior of the boundary layer.

If boundary layer itself is becoming in significant then it does not mean anything. So, we have to restrict a solution to beta is not equal to 2, beta is not equal to 2. Now, what we do if, you know if beta is equal to two then you know we should have a solution for that.

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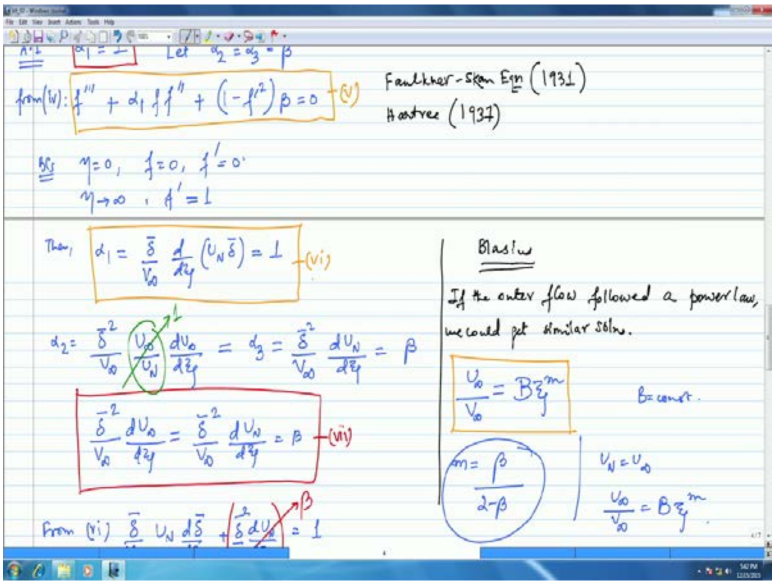

So, in that case basically what we will do is we have you know we going to use this you know equation six here, expression six here. So, let us see what happens what we will do in case.

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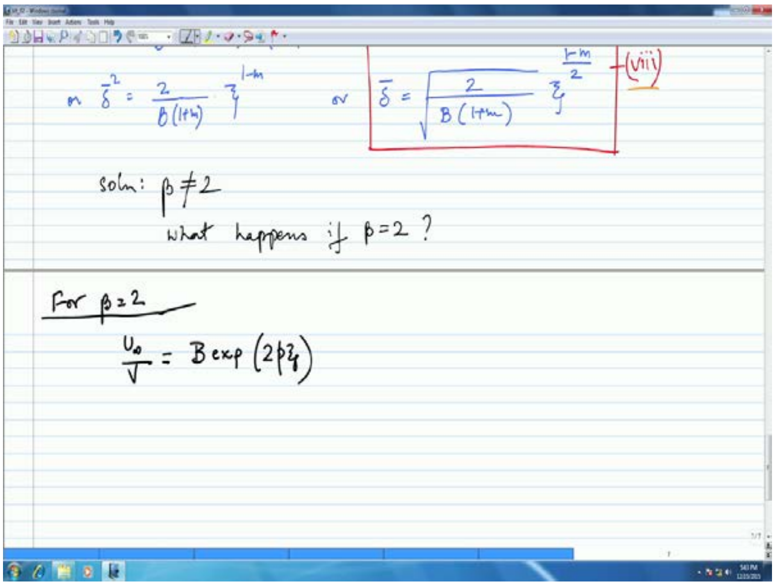

So, let us say that for beta equal to 2, let us say that U infinity by V is equal to B exponential 2 p zeta. Now if we have an expression like that then, we can go here, we can go again to this expression at you know six where we basically say that alpha 1 is 1 we took that, so we will keep that. Now we have got an expression for basically U infinity and this is U N, so and we have the expression given in six.

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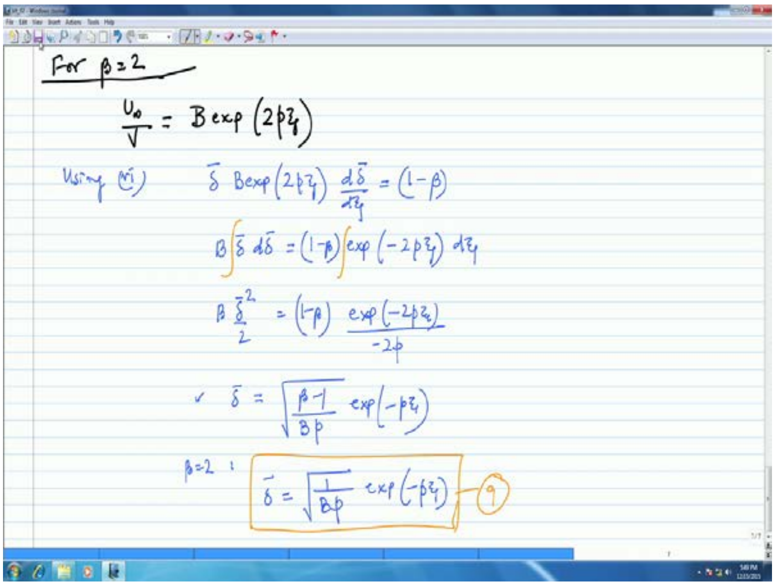

So, using this, so let us just say so using six, if we use that, what we get is delta bar B exponential 2 p z d delta bar d z is equal to 1 minus beta. So, if I would choose sort of, so let me do a couple of steps in then again I think slowly you should get a hang of completing these derivations. I will also post this again on the website. So, I will miss a couple of steps right or let me write it is B delta bar d delta bar is equal to 1 minus beta exponential minus 2 p zeta.

I am just rearranging the terms hopefully this is easy for you to understand. And now what I will do is I will take integral, so I will take the integral this way taking the constants out. So, if I do that then say if I take the integral then what I get is essentially well B delta bar square by 2 is equal to 1 minus beta exponential minus 2 p zeta by minus 2 p. So, then I am going to miss just the step or I am going to just say delta bar is equal to beta minus 1 by B into p exponential minus p zeta. If I do that then like we have said this is for beta is equal to 2.

So, if beta is equal to 2, again beta is equal to 2, therefore, delta bar is equal to this exponential minus p zeta. So, you have a different expression for delta bars and we have to do that, because, Let us call this as nine. So, we need to do this, because otherwise the expression you know what we did earlier. When we take the power law it does not work for beta is equal to 2. So, we need to have something else for this is special case basically. So, we will we will keep that ok. So, now let us look at the variable eta, and how we will treat that. So, if we go there, so let us see what we will do in that case.

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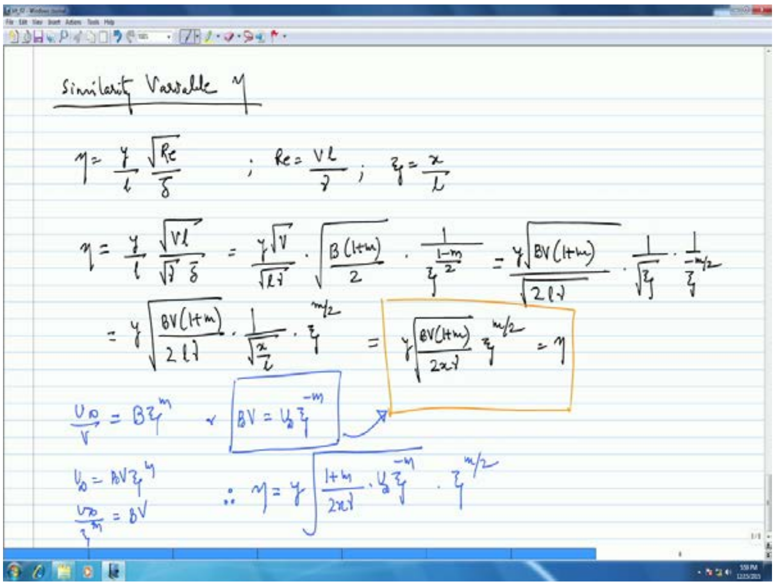

So, let us see how we will treat. So, what we will do is this is the similarity variable sorry about that. So; similarity variable eta. What we are going to do here now, so now eta is equal to y by l under root of R e by delta. Where of course, R e is V l by nu and zeta is equal to x by l. Why we know that so then eta can be written as y by l under root V l under root nu under root not under root this is delta bar.

What is delta bar? Did we define that earlier on, how have we said what is delta bar let us. So, we have our delta bar as what we you know do here, so it is basically the stretched version of your boundary layer displacement thickness. Boundary layer thickness basically. So, if I do that, so then this I can write as y under root of V under root of l and nu into under root.

So, I think, I missed something here. So, this here is my delta bar which I kind of root out. So, how do I write this? So, under root l this, you get that. So, you have an under root l here and in the numerator and you got a l here. So, I am going to dividing by that. So, I get you know, so I put these two insides l and nu together. And I am writing out 1 by delta bar which is nothing but here this m value this one eight, so that is eight. And we are write that out is then B 1 plus m by 2 into 1 by zeta to the power 1 minus m by 2 you got that. So, now, I hope that is clear.

So, then this is equal to so again this is let us write it here. So, this is equal to y and you pull all this inside these two inside. So, I get B V 1 plus m then to that 2 l nu into… So, I got all these things side of this. So, 1 by zeta to the power half into 1 by zeta to the power of minus m by 2. So, if I am do that then this is again equal to y b V 1 plus m by 2 l nu and this one is under root of zeta.

So, I can write this basically is as under root of zeta and zeta is x by l. So, what I can write is under root of x by l right into, I will write the positive power m by 2. So, clearly you can see this l and l will kind of you know cancel out. So, if I do that this is equal to y, so 2 x nu, so this becomes B V 1 plus m 2 x nu and zeta to the power m by 2, so that is my eta. So, I can write my eta in this form. So, this is nothing but my eta, that is my eta.

Now; however, now we will write up the power law U infinity by V is equal to B zeta to the power m or B into V. So, B into V, if you see is equal to U infinity zeta to the power m is that fine? So, I miss a step here. So, all I did is take V here right, I move the let me let me do that step here, to make it just little bit clear. So, basically what I said is U infinity is B V zeta to the power m. So, then I bring the zeta to the power m down here, so that left hand side is U infinity by zeta to the power m which is equal to B V. So, then I can write it in this fashion, so then that basically is B V.

And I can write this in this fashion when I took the power to the top. So, therefore, I get this. So, basically what I am saying is we can replace you know the B V term here by u infinity zeta to the power minus m. So, if I what you do that. So, then we input that in here, you input that in there. So, therefore, eta is equal to y under root it is equal to under root, so 1 plus m by 2 x nu B V is U infinity zeta to the power of minus m. And this is zeta to the power m by 2. So, if I do that so then, I mean this makes it simple hopefully you can see that.

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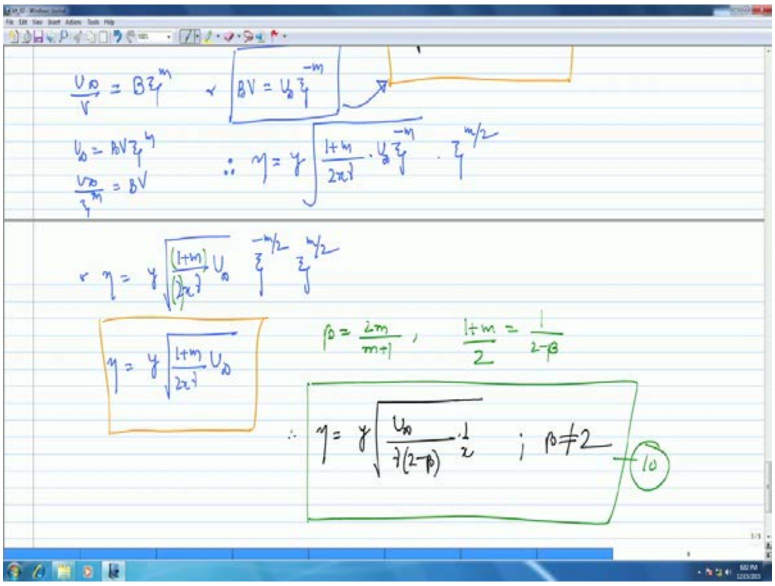

So, we will take zeta outside or eta is y under root 1 plus m by 2 x nu U infinity zeta to the power minus m by 2 zeta to the power m by 2. So, these two cancel out, these two cancel out. And what I am left out with therefore, eta is equal to y 1 plus m by 2 x nu U infinity. I hope you see some you know similarity or pattern in the eta that we have for a flat plate. So, if I do that.

Now beta, if I want to introduce beta in here, so I got m. So, now beta is, two m by m plus 1 right and 1 plus m yeah well this is just for you know writing our purposes is 2 minus beta. Then what happens to my eta, so 1 plus m here right 1 plus m here, this I can write that as 2 by 2 minus beta so basically that. Or 1 plus m by 2 you know, is equal to 1 by 2 minus beta I think that is better. So, 1 plus m by 2 is equal to 1 by 2 minus beta and that is it basically. So, therefore, eta is y this thing so it is U infinity by nu into 2 minus beta into 1 by x that is it. And here this is where beta is not equal to 2. So, this is for that case I mean that is what we were doing so far.

So, then we get an expression for the eta, and let us call that as ten, make sense I mean this is just lot of maths. So, therefore, well you know so now, what are these numbers means? When we do things like this, it is very interesting that I mean we shall be able to get solutions for you know various combinations of beta and things like that. It is interesting that they actually bold on to something, you know physical it is actually physically it means some kind of a flow. For example, having done this, so in this particular case, we were doing alpha one is equal to one. So, we said positive negative and zero that is all we are going to look at the behavior of that.

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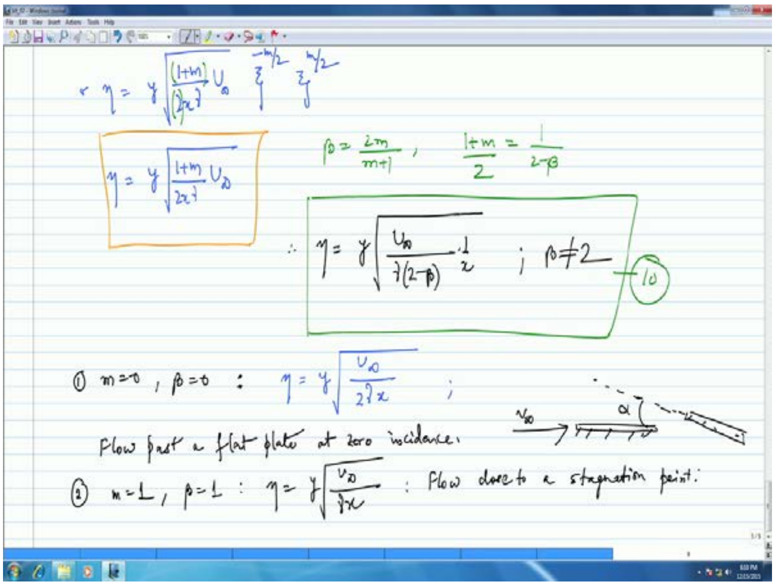

So now, what happens is for if you what to solve this if you were to solve this then m is equal to 0, which means that beta is equal to 0. If you do that and if you solve for this, what did you get. So, because now the only thing is for this eta and you know the corresponding delta bar, so whatever you get from here, so these things are in terms of beta. Let us just remind ourselves.

So, this is for you know beta is equal to 2. So, I mean no, no I mean here, so this is this equation nine is for beta is equal to 2, so that is that. Now so we had a delta star delta bar which is given by eight, you got something like that see you could write this you know 1 plus m by 2 is basically 1 by 1 2 minus beta. So, you could write this in terms of beta. So, you have a delta bar which is in terms of beta. So, and this is your description it obeying a power law alpha 1 is 1 and alpha 2 is equal to alpha 3 and that which is equal to beta that is where we started.

So, now, m and beta is something which is interesting and using that we were able to get some expression for delta bar. Now what happens is, that if for m is equal to 0 and beta is equal to 0 is actually balls down to. So, think about that think about how what this would mean. So, beta is equal to 0 and m is equal to 0.

What is this ball down to you know think about that think about that before I write down what; that means. And let me write down. If this is the case, what happens to our eta let me just sort of write that term, if for this particular case. So, here eta is equal to y under root U infinity 2 nu x, and what is our delta bar let us go and see, we have a delta bar. So, m is also 0, if I have a delta bar what does that mean, so say for m is equal to 0, delta bar is equal to 2 by B zeta to the power half. So, now, this is what we get for delta bar.

Now, if you remember what we so fine, So, you will have to now figure out what the function the constant B will be. Now if you can recall what this eta was looking like, so this basically what is happening here is, that this is falling down to flow pass through a flat plate at zero incidence which we studied at length.

Essentially what we saying is that if you took this kind of an expression, so for certain values of m and beta you would be able to get a solution for the flat plate which is lying in you know parallel to and you know through the free streams. So, basically zero incidence, which means that this is your flat plate and this is your incoming free stream. So, this incidence so in here is basically 0. So, if the flat plate instead was like this, so it is making some angle with the direction of free stream then it would have an incidence. So, that is what we will call it angle of (Refer Time: 28:08).

So, this is what you mean. So, this is something that this is what we started within we spend a lot of time try to understand the flow pass through the flat plate. So, essentially for these comes numerically m is equal to 0, and beta is equal to 0, balls down to flow pass through the flat plate at zero incidence. And the second case that m is equal to 1, and beta is equal to 1, what happens to our eta then eta is y under root U infinity, 2 minus

1 is basically 1, so nu x, this is your eta this is your eta and then the corresponding delta bar. And this is flow close to a stagnation point, it is flow which is close to a stagnation point, so that is what. Now, the thing is I mean we will move into the other two cases that we said we will talk about a alpha 1 is negative, and alpha 1 is positive, alpha 1 is 0.

Now I think is this is a good place to stop. Let me just kind of summaries this and say you know numerically. So, numerically when we trying to look at a solution. So, you know these are values, so if we assign different values numerically, to these you know beta and m, we will get physically different flows. So, this would be able so if you have, if you use this what was the velocity description, our velocity description is this power law m. So, if you see what this exactly means is that say m is equal to 0 that is the flow pass through the flat plate.

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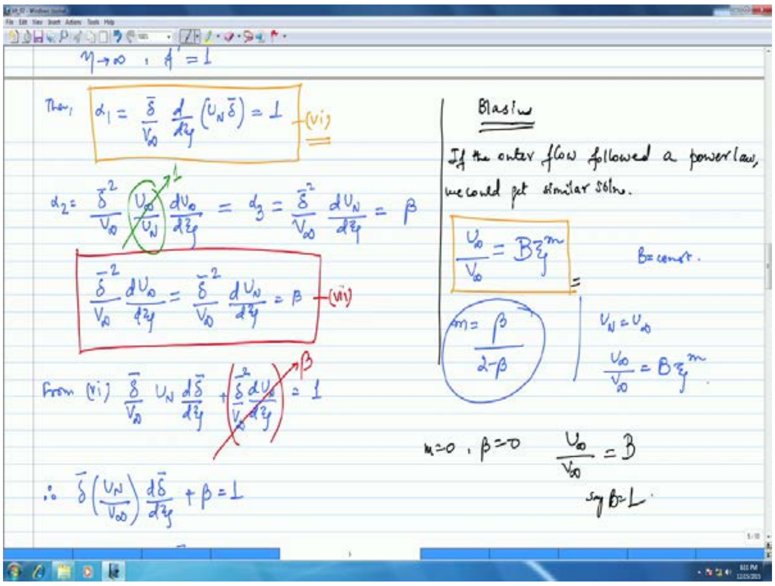

If m is equal to 0, which means beta is equal to 0. Then the power law, from the power law we get this is equal B that is all, and B is a constant, you know you can take that as 1 or you can take that as 2, it does not matter. So, if it is for flat plate, you just have to see what you can just take B as 1. So, if you take, say B is a constant and I take b as 1 then as we wrote down here, if b is 1 then zeta, so what was zeta, zeta is basically x by l, where did we define zeta? So, zeta is our x by l. So, anyway you can look at that, where is our right. So, zeta is x by l. So, I can say if zeta, so x by l.

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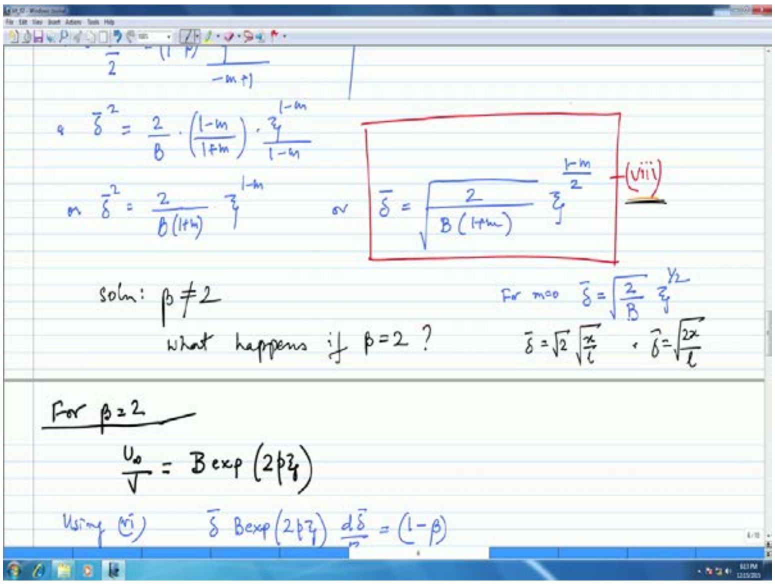

Now if I look at this here, correct. So, B is 1, so delta bar is basically equal to under root of 2 and under root of x by l or delta bar is basically 2 x by l. So, x by l, now you can you know put in the description for delta bar and just get something for delta, you get the expression for delta. So, the whole thing is it if we can get an expression in this fashion then we can change the values numerical values of beta and m, and take sign different values of m.

And this gives you the ability to study different physical flows. I think that is a very powerful tool that you have. So, if you use a description like this, and numerically all you have to do is just change couple of parameters and that actually helps you to study different physical flows. And think of that in terms of if you did not have this tool, if you have to study a flat plate, and if you had to study flow near say in this particular case, we talk about flow near stagnation point, you would have to build that separately and do that. It is kind of you know all you need to do is changes parameters and go ahead and look at it. So, will stop here, and in the next module, we will go and look at the other couple of cases.

Thanks.