Introduction to Boundary Layers
Dr. Rinku Mukeerjee
Department Applied Mechanics
Indian Institute of Technology, Madras

Module - 02
Lecture - 22
Similarity solutions to the BL equations (other than flat plate)-I

Hi. So, now that we have got a similarity solution for the to study the Boundary Layer or a flat plate. I think the next obvious thing to do is to see whether we can, take those equations which we developed basically the boundary layer equations. And use it for something which is other than a flat plate. Let us sort of a go ahead and do that. We were basically term this the next couple of modules and this one, similarity solutions to the boundary layer equations, other than the flat plate.

So this is also where involves some bit of a derivations, but I hope that you kind of begin together new ones is as we do the derivations. And there are some little elaborate once which I would probably not do, but I will pause them for you to look at, and of course any questions should be welcome. So, your basic knowledge of calculus is something that we will look forward to. Let us get a hand and do that.

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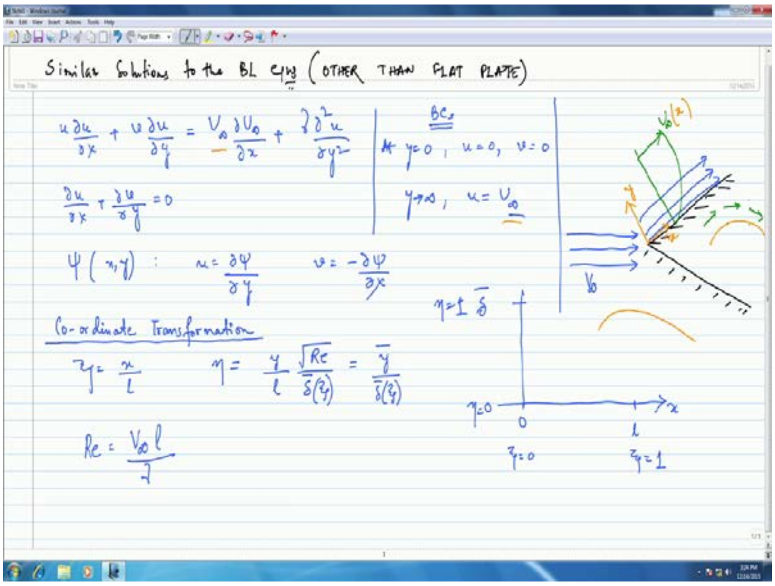

So, what I am going to term this is, similar solutions, to the boundary layer equations;

this is basically other than flat plate. Now, let me sort of again write down the boundary layer equations, which we will solve. So, that is the x momentum equation that contains information about the one momentum equations, so two rolled into one, and this is the continuity equation. And we shall combine this with the boundary conditions, that at y is equal to 0, u is equal to 0, v is equal to zero and y at a far of place. Basically, these are the dimensional form of the boundary layer equations.

Let us begin, so let us say the stream function; so we are going to use stream function which is two d stream function. So that u is equal to that, and v is equal to that. And this case, we also going to use a coordinate transformation, so we are going to write this, in such ways, such that z is x by l, which essentially means that if x goes from 0 to l. So z will basically go from 0 to 1. That eta is equal to y by l, under root of Re. So essentially delta bar is a function of zeta which is essentially nothing but y bar by delta zeta. This is the coordinate system that we will use.

So like I said if I had a coordinate system like that, so this is say, 0 to l and this is 0 to well let us say, delta bar. Now here, so this is x, this is x goes from 0 to l and the same thing zeta is equal to 0 and zeta is equal to 1. This is actually, I am going to write this is y bar. So, y bar it goes from eta right y bar and zeta bar, let us put it this way, so 0 to delta bar this is my eta. When y bar is 0 it is 0. When y bar is delta bar zeta eta is equal to 1 ok. Let us just say here, eta is equal to 0 and eta is equal to 1. So that is what we mean by transforming the co-ordinance, please dont confuse this with the non-dimensional form that we got earlier. That was very different purpose. So that is what, so basically we are going to use this kind of a coordinate system.

And there is yet one more thing which I would like to draw your attention to, because here the way we calculate the reynolds number, it should be V infinity yes, l by nu right rho v d by mu. So, I got this V infinity and I also got this U infinity. Let us not confused two. I am going to take just in a arbitrary kind of a, let me draw it here itself, I am going to take an arbitrary kind of same body. So for example, there is some kind of a body, this is some kind of a body, and you have free stream which comes in.

Now this is reinfinity and we will have flow like that, so on and so forth. Right? And, this is essentially U infinity that, I am talking about. If I had a small access system, so I would say that, so if I had something say like this right here, that and this, so this is a y and this is x then this is a function of x. So, what we use? The reason we kind of differentiating all this, the reason you are writing all this is because now we are really going to talk about you know bodies other than a flat plate, so the geometry will play an important part. So we need to make the distinction about that.

Hence, this U infinity that you have everywhere; that we taking is really the localized velocity and it is kind of the tangent to the surface at that particular point. Obviously, this the surface of the body could be anything else in here is like two plates kind of place certain angel with each other, of course you would have an f for shape like that you know then this fact become different.

So again if I had an airfoil like this, you will still have reinfinity coming in and then at each point for example, you know at each point say you would have something like that and here you would have something like that, here you would have something like that so that would be U infinity. So you can see the that is changing with x. Now, having instead of gotten that, so let us go and just a little bit something, we also going to look at some velocity. So, let me kind of write this down a little bit for your reference so that you know we make no mistakes so far.

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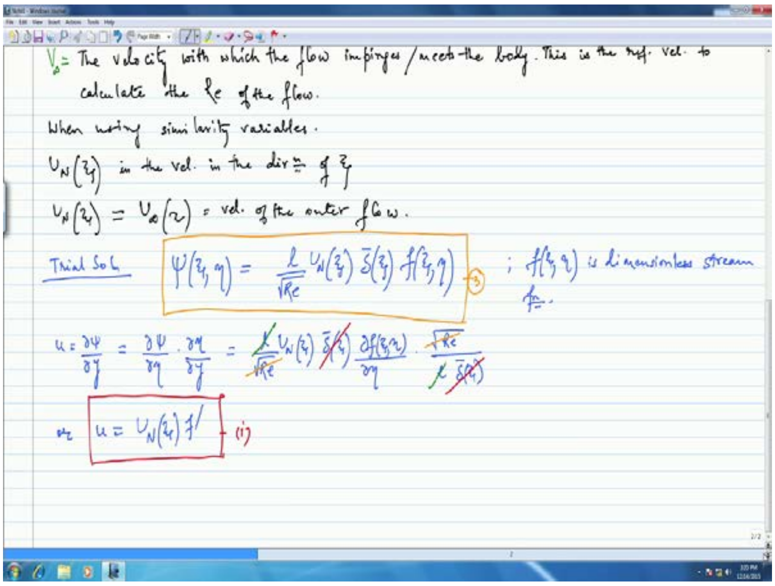

So, V is the velocity, with which the flow impinges or some need, the body. And this is the reference or say this is basically V infinity, I am calling it V infinity. You can basically say a free stream velocity which meets the body and this is the reference velocity to calculate Reynolds number of the flow. So, this is the reference velocity to calculate the Reynolds number of the flow.

Once the flow is passed a surface boundary layer is formed, then U infinity x by U infinity etcetera is used to denote the velocity of the outer flow. Outer flow in the sense that now you have a boundary layer so you have, a velocity profile like that, so that will be thing. So, velocity in the outer flow is basically U infinity x, so that is what is used to denote that. So here, I said small x or y.

In our particular case, when we use similarity variables. So, when using similarity variables, U N zeta is the velocity in the direction of z or basically parallel to the z, like when I wrote down the z. So z basically goes from 0 to 1 and eta goes from 0 to 1. So, U N basically goes from U N 0 to U N 1 so it is in the direction of zeta. And this can be set to be the same as the velocity of the outer flow. Essentially, what I am saying is that U N zeta is equal to U infinity x. Which is, let me just write that. That velocity of the, when I say outer flow I basically mean that now we do have a boundary layer and this is a velocity just outside the boundary level which is essentially is the free stream. Is just that the free stream, you have to take a component now of that free stream of v is not it, because we have a body and we do not know the shape of the body.

So we have to define a new a set of coordinate access which we are doing. Now, the way we all do is the like we did earlier, we just choose a stream function and then we applied that and then we looked at the solution and the feasibility of the solutions whether that solution make sense. Right? That is really up to us. So here too we are going to do that. So therefore, basically what we are going to do now, is say that, we have a, so I have not defined that. So therefore; yes, I did talk about the Reynolds number ok. So that is that.

Having come to that, so we going to now look at a trial solution, what is the trial solution? So we are going to write the stream function in terms of the zeta and eta. Which we are going to write as l by under root Re U N zeta delta bar zeta and again this is the f of zeta and eta is the dimensionless stream function. In this case again is dimensionless stream function. Alright so now, that we have that, let us write down velocity which is del psi and del y which is equal to del psi del eta del eta del y. I think we have been doing sufficient mathematics so far for you to understand that. So then, let me label a few things, let me call this as may be one or something. Let us go here.

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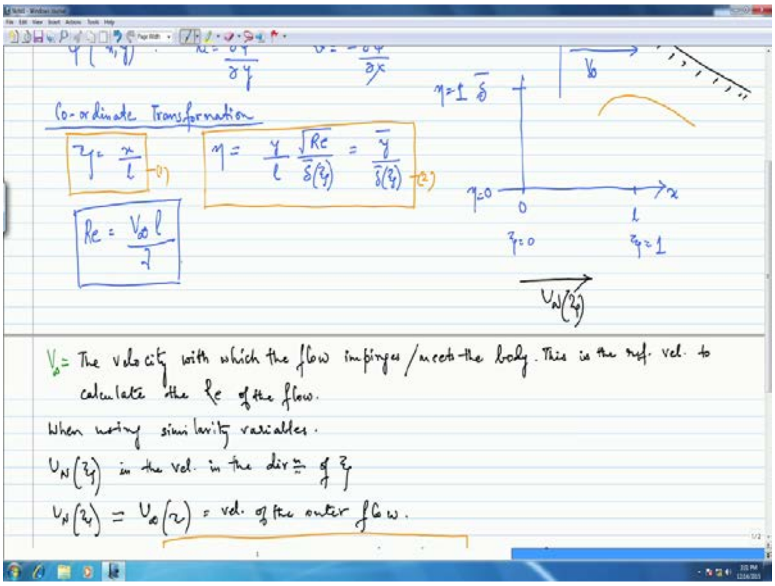

So, this is also something else so we have got. So basically, what we are saying is this actually that is fine, so we will say if we got an eta, we got a z. So let us call this is one, this is two and I am going to call this is three. So, when I say delta psi del psi del eta let us use three and if I do that then what do I get. So, l by under root Re, U N zeta delta zeta, none of that sort of changes, the only thing that really changes now, this is the one which is the function of eta, so we get delta f which is the function of zeta eta, delta eta right into delta eta delta y.

If you go back here, delta eta del 2 y. So, it is under root Re by l into 1 by delta zeta because, that is also again not a function of eta, so we will just write under root of Re by l delta bar zeta that is all. So, under root of Re by l delta bar of zeta. If I do that or u is equal to, so we got this and this that cancels out, I got delta bar delta bar that cancels out, Re Re that cancels out.

What we basically left out, let us do that. So, we got l and we got l that cancels out, then we got Re and Re that cancels out and we have got delta bar and delta bar that cancels out. what we are left out with is U N zeta, f dash and again all the derivatives here are with respect to eta and f in this case is a function of zeta and eta. If I get that, this right so we get relationship for U. Then again you will have a relationship, again probably I will skip a bit of steps and post the entire derivation on the web page so you can take a look at it. I think you should get a hang of the calculus which is being used here. It is important you know one of the things you could do is kind of do this and so that you do not get lost for anything on the way.

In the next step again is, to do it for the v, and the v is essentially minus del psi del x which is minus del psi del zeta del zeta del x. And if you see from here, del zeta del x is basically from 1 if you see is 1 by l so I can write this as minus 1 by l del psi del zeta

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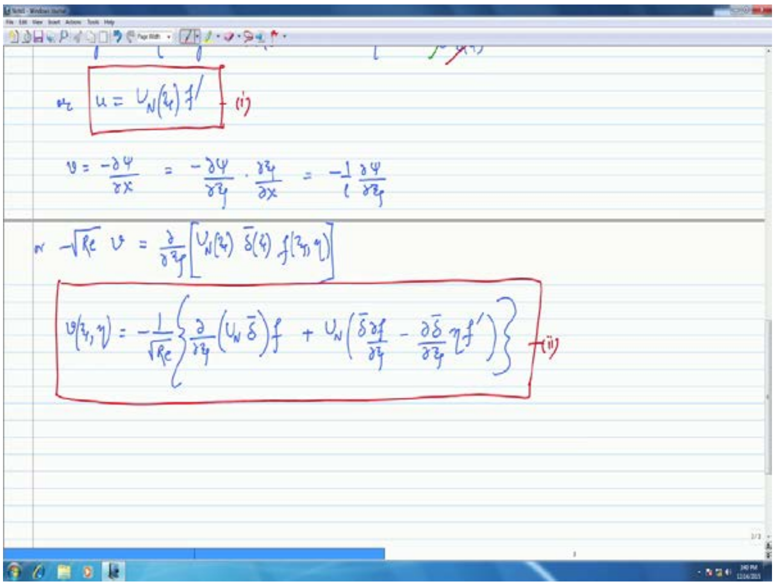

Well, so basically what you know if you use this del psi del zeta, so you know you just del zeta of three. If I do that or what I will do is I will write this out and what I will get is minus under root Re. v that is equal to del del zeta U N zeta delta bar f of zeta eta and that is it. So, minus Re into v is equal to del del zeta of U N zeta delta bar zeta and f of zeta eta. So we get that right. So once we get that now this right hand side let me skip that for now, you can basically I will post this but I think you should also give a try yourself, but I will pause this for your reference and you can take a look at the working out of the right hand side.

After having skipped that so let me write out the final expression, which turns out to be like this. So basically, V which is the function of zeta and eta, that is equal to minus 1 by under root Re del del zeta. U N delta bar into f plus U N, so this is U N this is also U N this is delta bar del f del zeta minus del del bar del zeta eta f dash, and please note that f dash is a derivative with respect to eta localize coordinates. So we get that and I am going to, so like I said, I will pause this solution, the solution of the right hand side the working out of the right hand side. So, what I am going to do now is so now that we have u and v in one and two we are going to use this in the boundary layer equations and, what we get is something like this.

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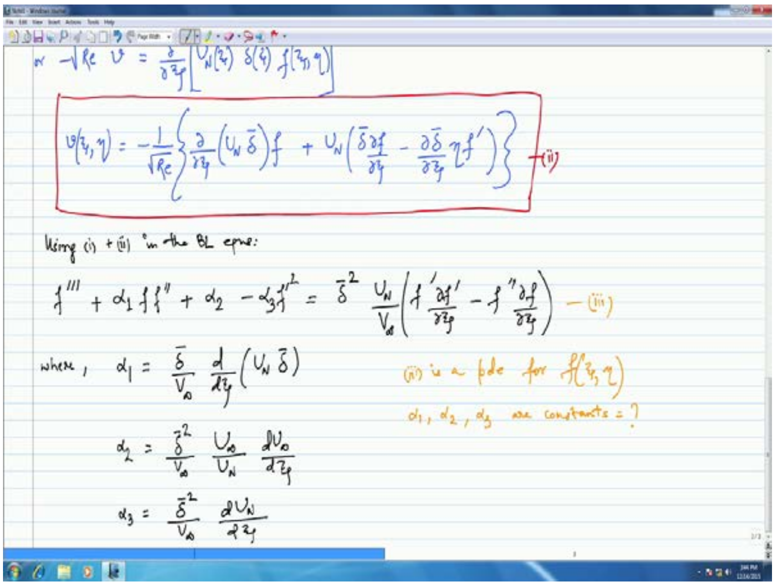

If we use, so basically what we are doing is, we are using one and two in the boundary layer equations. So if we do that then what do we get, what we get is something like this, plus alpha 1 f f double dash plus alpha 2 minus alpha 3 f dash square is equal to delta bar square U V by V this is the total total velocity, f dash this is basically V infinity, f dash del f dash del zeta minus f double dash del f del zeta. Where alpha 1 is equal to delta bar by V infinity d d zeta U N delta bar. Alpha 2 is delta bar square by V infinity U infinity U N d U infinity d zeta and alpha 3 is delta bar square by V infinity d U N d zeta.

This once we get, so I am going to call this equation as three and this equation as you can see three here it is a partial differential equation using f zeta eta. So, three is p d e for f this. Now, what is interesting? What is interesting here is here is, this equation as it work that three is the p d e for f. But I would like you to think over this, then what if alpha 1, alpha 2 and alpha 3 are constants. If alpha 1, alpha 2 and alpha 3 are constants then what happens to this equation. So I think a lot of the times in fluids mechanics you encounter equations, which are quite regress and quite complicated and but, the moment you sort of break this down and you applied to specific cases, specific physical problems things become not it still a bit of solvable at least, you get some inferences. Otherwise, the equation looks the first question that comes from mind how am I even going to solve this, does this even going to mean anything physically.

So, let us come and come back in the next module and look at that, and see that one of the possibilities of a solution form such an equation which we have developed and whether, it is even worth doing all of this. So I will stop here for now and I will meet you in the next one.

Thanks.