Introduction to Boundary Layers
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Module - 01
Lecture - 21
Description of the Numerical Code to Solve the BL equations applied to a flat plate

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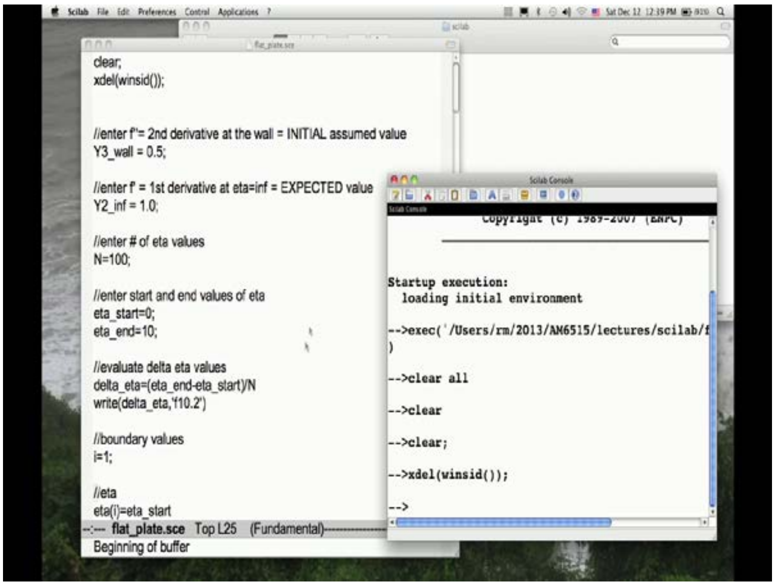

Hi, so now what will do is a very interesting is the equation that we finally got, is in the similarity variable. Now, that we going to solve you know it is like we just said that it is a third order ODE, and we have just one equation to solve, and we will solve one equation and we will gets a picture of the u and v velocities across a boundary layer. So that is essentially the purpose. So basically I will be show you first, first let me show you that what the outcome, you know solving this is and then I will talk a little bit in detail about how we go about doing that, developing this little piece of code.

Now this is something that I will post. This is also available freely over the net I think, and if you feel free to you know use all that information. To do this, you will need a little bit of you know practice or a little bit of understanding of how one does coding. You can do this in whatever form you are you know comfortable. I have done this using a SCI lab like I said to you. So, this is my SCI lab console as you can see. And I am going to use this to basically run you know the code that I have written.

So, before I explain you know the code, there is to how we go about that, and in a numerical method which we used to do this, let me first set of run this and show you that what is the output? What you mean by that? So, basically I take that equation and I use little a computer code to solve it. What is the outcome of that. So, what I am going to do is just, so this is how the code looks like, I will explain that you little bit.

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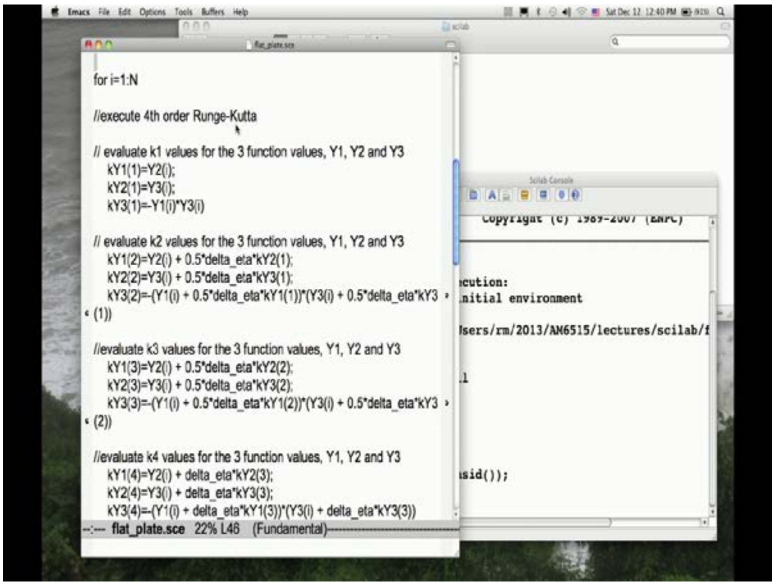

So, what we use is a 4th order Rung- Kutta method, I will come to that in a bit. So, and then we get essentially the output. So, as you can see that, I have an output of eta and you know several other things. So, let us see and then we also plot. So, let me show you what the plot means. All I have to do is here; I will talk briefly about SCI lab also.

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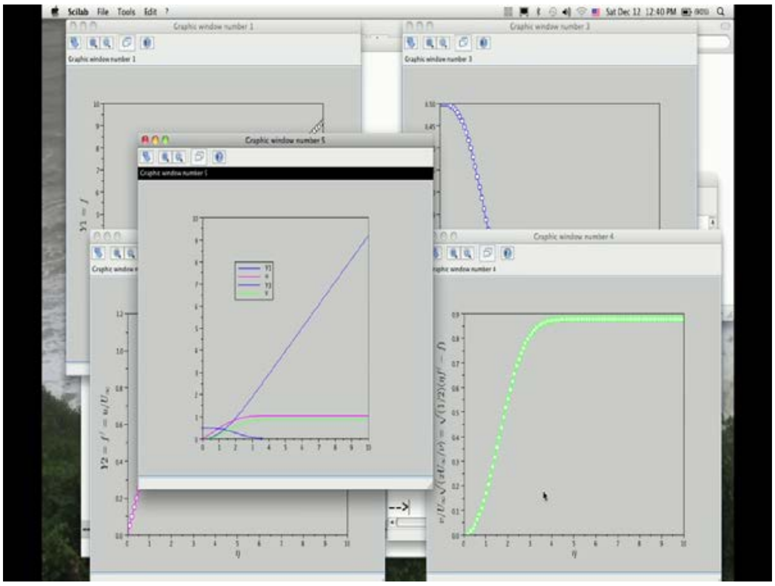

So, this is my output. Now let us see, now the first thing which you will be, little familiar with is this one. So, what we get from the plot.

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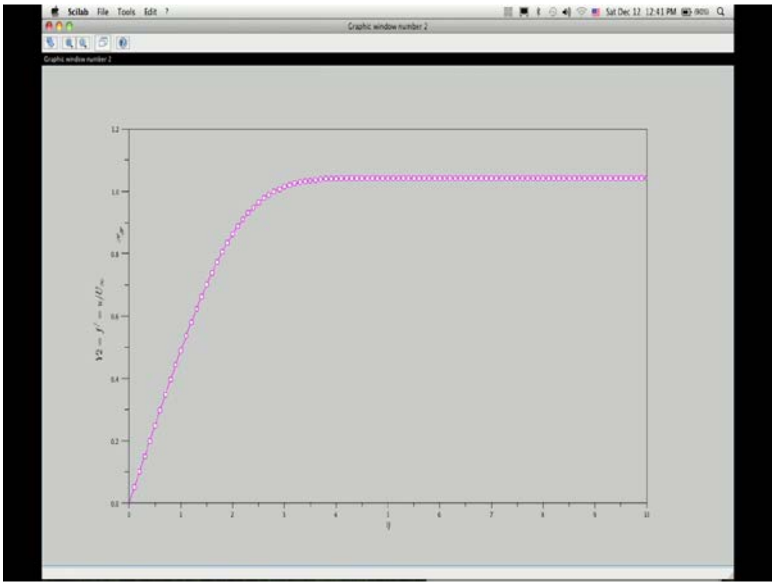

What we get as an output from this plot, is this value called y 2, y 2 comes from the code. So, what I am able to plot out if you look here on this y axis here is f dash and from what we develop so far, f dash is nothing, but u by u infinity. You remember u is equal to u infinity into f dash. So, in the solution here, so what I have done is plotted basically f dash; f dash in the vertical axis and eta on the horizontal axis. So, what I have plotted is, how u by u infinity varies as you know eta. So, basically you can say this is a plot of f dash, versus eta.

Is just that u by u infinity is what is f dash. So, this is the variation of u by u infinity or f dash with eta. So, this is this plot that we get. And what is important to understand here is that, I will not have to calculate anything. I get f dash as direct output of the solution. The numerical code that I write it will give me directly f dash. So, this is one of the process.

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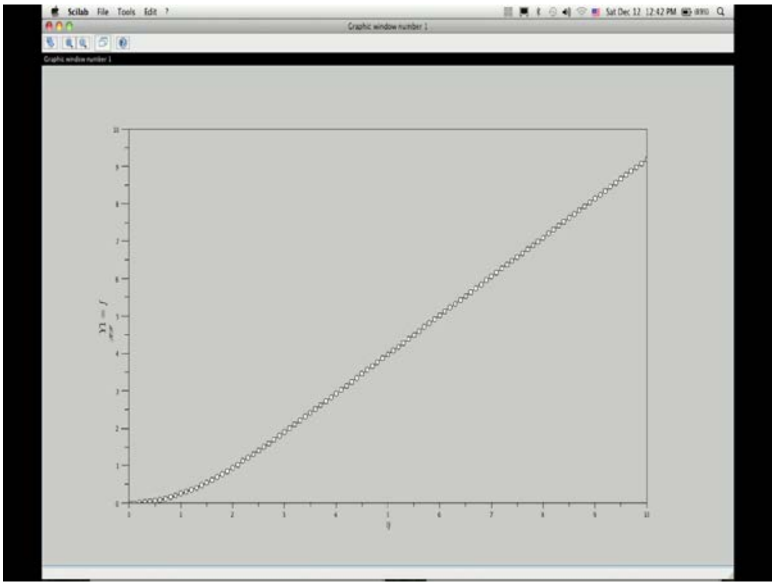

Let see what else we have. We also have this plot. So, we can plot a lot of things. So, you know, I have also plotted f. So, what is f, f was the non-dimensional stream function. So, we got f as a function of eta. So, this is again something that you get.

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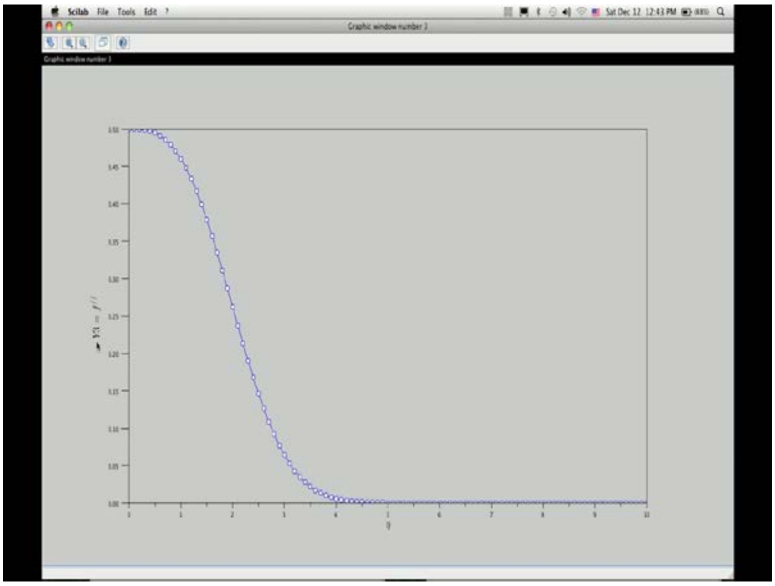

Then, so that is one of the plot, look at this. So, we get y 3 which is nothing but f double dash. So, we also plot f double dash versus eta. So, essentially what we doing is, so this is f dash.

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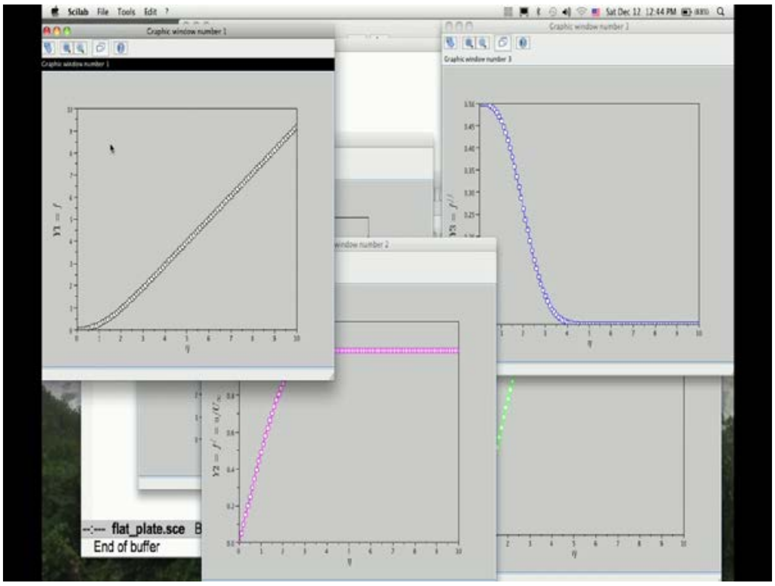

So, this is f. So, this you know in the window one if you can see, so this is a window one at the top you can see window one. So, this is f versus eta then, this window is basically showing you let see if I can do this is the same time. And so this window shows you f versus eta, window 2 shows you f dash versus eta, and then this window shows you f double dash versus eta. And what is important you understand here? That these are basically direct output from the code that we write. So, having done that let us sort of minimize those.

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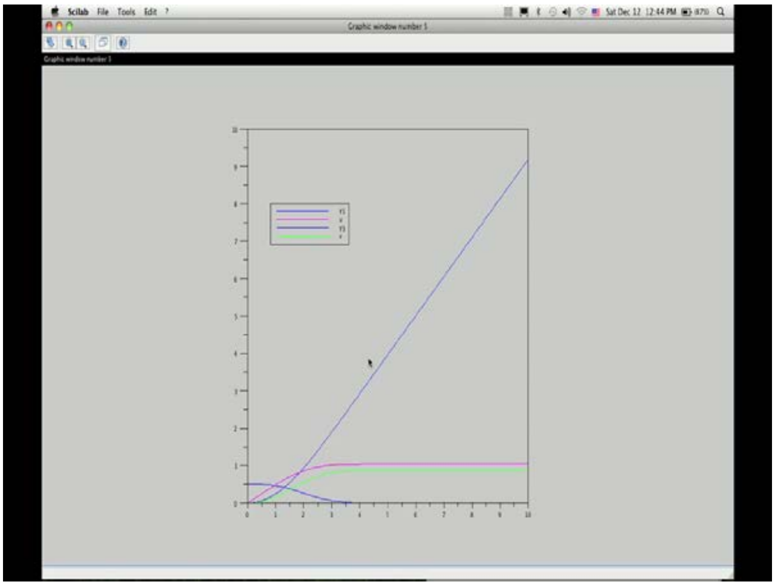

Then look at this. So, if I plot this, you know all together. So, we have set of several plots here. So, I have plotted basically all of them together. We have got the pink one is essentially the u, u velocity. Then the green is v velocity. And there are basically you know blues, so one is this y 1 the other is y 3. So, y 1 I think is f. So, y 1 is f and y 3 is f triple dash, so we get that.

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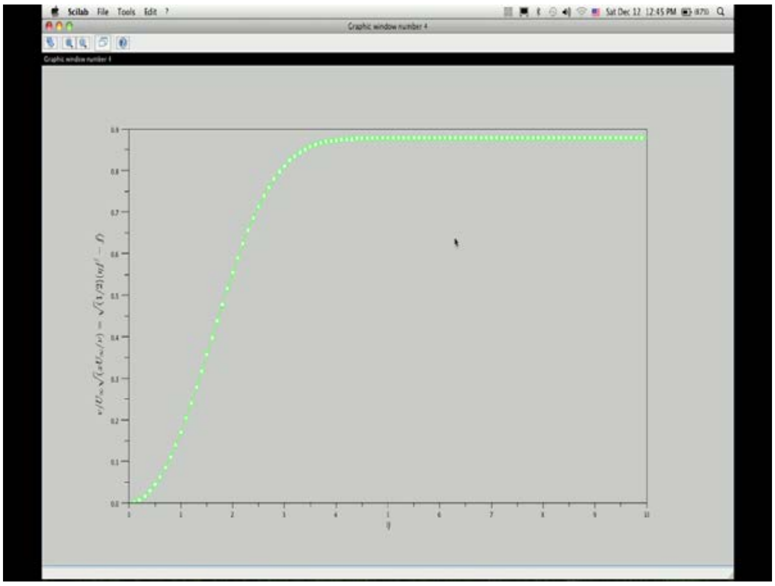

And, if you look at this one, so what we get here if you see now, so we get this term here, u by u infinity under root of x infinity by nu. Is equal to half of eta f dash minus f. So, this is your eta, this is your eta. So, now the interesting thing is that, so this is what we get as a result of the you know as an output from the code. Now, what you should pay attention to here is because this is the plot that you would be really interested in. What you should understand here is that eta is similarity forever. Now, eta itself has x and y in it. Now if you see this plot here. So, if you see this plot here, for example, say for eta value of 1, the value of f dash is 1; for eta value of 6, the value of f dash is 1. When will value of f dash be 1.

So; now, what I have beginning to do is kind of read, you know the plot I think that is for important read the plot; otherwise is a just set of lines or markers. I mean they need to mean something physically then, that is the purpose of writing all this code. Now so at eta is equal to 6 again f dash is 1. What does f dash 1 mean? That u is equal to u infinity meaning that u is equal to the free stream. Now, what is interesting is that eta, what is the value how do we write eta. So, eta is a function of both x and y. Remember it is y under root of u infinity by 2 nu x. Now, if eta is equal to 5. So what is that mean?

Now take a certain value of that x. If you take a certain value of x write then you will get the value of eta is 5, the value of eta is 5. What should be the corresponding value of y. You know that for eta is equal to 5 the f dash is equal to 1.

So, what I am saying is in the eta, there you have a x and you have a y. Now you know the value of eta which is 5, so put that as 5 and you got x and y. So, I am saying fix x, so let x be you know 1, value of x be 1. Then what should be the value of y, to get a eta of 5. Similarly, change x to 6, and eta is 6 and change x to say another value, then f dash is still 1, the corresponding f dash is still 1. So, find out the value of y, so that is very interesting, and now that should be give you a feel for the velocity profile across a boundary layer, which is what we would doing. So, basically whole this plot that we you know route out etcetera. Now we should be able to get all of that from this. Now I will elaborate that on that a little bit more, that what you get out of this, what I mean how do you set of represent this and so on and so forth.

Now, let me come back to the code a little bit. Let me sort of give you a little over view of how we sort of do that. So, what I essentially did here is take several values of eta. I have written here. So, I take a start value of eta to be 0, and eta the end value is 10, and we take the y 3 which is f triple dash. So, these are kind of you know like boundary conditions you might say. So, we are going to take this as you know, in this particular code I have written this, so f dash f double dash is basically the second derivative at the wall. Which is the initial assumed value. And we call that as y 3 wall which we take is 0.5, this is an initial assumed value as you can see.

Then first derivative at eta, so now, f dash this is our boundary condition, which we are using. So, first derivative at eta is equal to infinity; at eta is equal to infinity, f dash was suppose to be 1. So, we have got that, so that is the expected value. So, that is all basically we know at the wall u is 0 and v is 0. So, eta values, so start values of eta. So when I say here for example, evaluate delta eta. So, I take I go from zero to 10 in steps. So, I divide that in entire range to 0 to 10 into 100. So, I go from 0 to 10 in steps of 10 by 100 basically. So, those are little steps.

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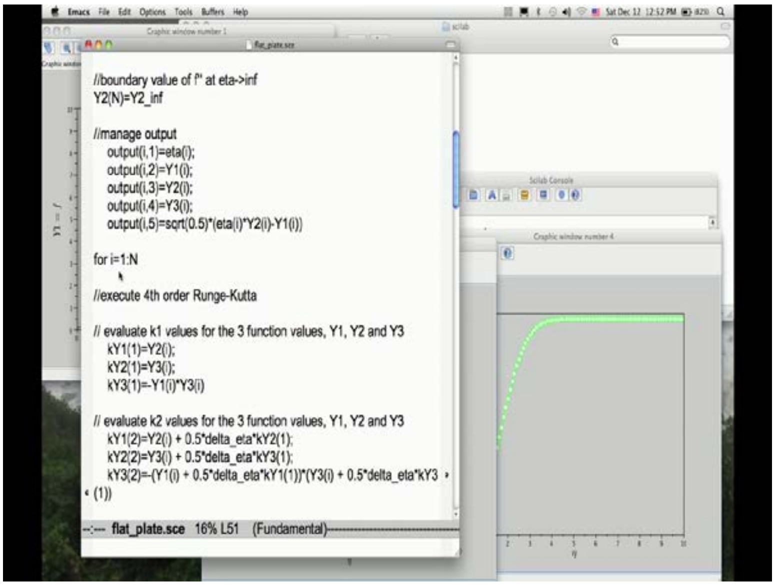

Now, the Runge Kutta method is essentially, it is a method of you know integration; numerical integration that we when we go from one set of boundary here to other.

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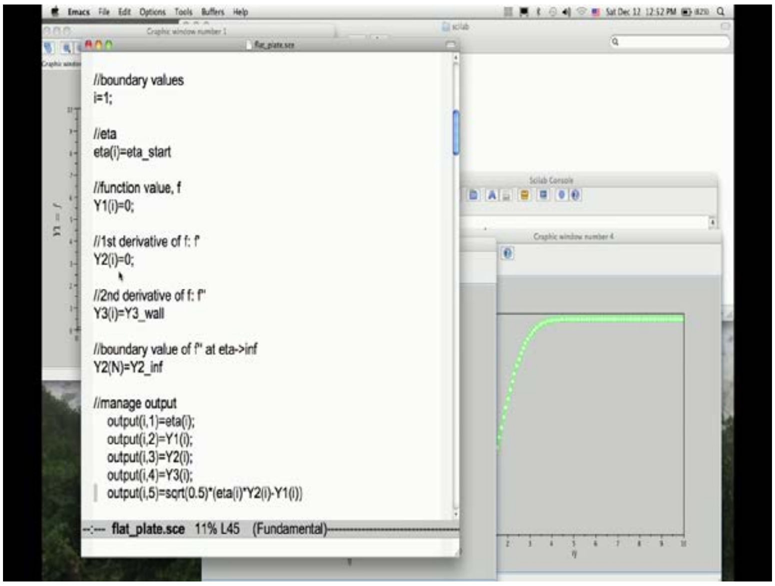

So, here physically this also means that I am going you know a kind of in this particular case all I know is that, what I know here is that the y 2. So, the y 2 at the wall sorry at the free stream or the edge of the boundary layer is going to be 1 or infinity, whatever we have written here – f, f star; I know that value. So, here what we have basically doing is that I know that value and I am going to come and so that is the value I know. So, kind of start from that value, you know y 3 is again something that you know I am not aware of. So, I use that and kind of go from the known value to the other end.

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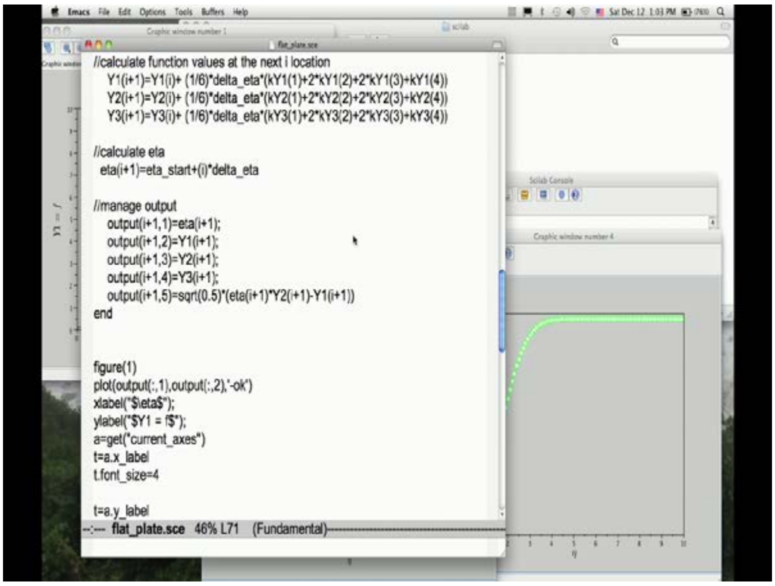

Now, here what we basically doing is we will go from the wall up to the, So, the wall values and then that I do not know, so I assume it. Now once I assume that, I use that value and I interact forward away from the wall into the boundary layer and I go to the edge. Now there I know one value, which is the Y 2 value there, so that is the value that I know, but do I reach that value with assumed value of Y 3 which I started? Not really. So, I what need do is go and come back you know and do this again with a different starting value at the wall. So, therefore, this is an iterative integration procedure we do that, and that exactly what we will go ahead and do that.

So, in this particular case, and also we take you know the several values of eta. So, Runge Kutta is so that is how we by basically implement in the Runge Kutta method. So, this is again, it is a very standard method and I think that is well I think you can sort of see that here, for example, now we start out here for example, so I say k Y 1. So, it is a

4th order Runge Kutta. So, I basically, what I go from1eta to the other or a to the other. I take 2 more points in the middle; I take the 2 edge points, the start point the end point, and I also take 2 more points in the middle, so that is why it is a 4th order Runge Kutta. You could have a second order, third order as well. So, now what I do here. So, first is I write, you know this is a value that I have write. I mean, I am using this variable k Y 1 Y 2, k Y 1 is Y 2.

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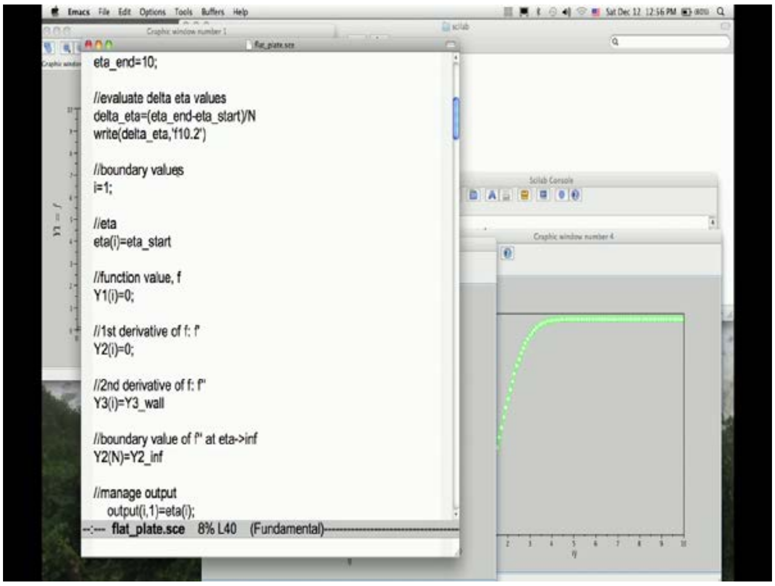

So, now these are values. So, now, Y 2 first derivative of f; these are the boundary conditions. So, the first derivative of f, which is at Y is equal to 0, f dash is equal to 0. So, f dash is what I am calling is Y 2. So, k Y 1 is equal to Y 2. Then. k Y 2 is equal to Y 3, and this k Y 3, I am writing here is as you can see is minus Y 1 into Y 3, so basically these 2 values. Now this is where I implement the equation which is f triple that mean the equation that we are integrating. In this particular case all, you know our boundary layer equation using a similarity variable. So, this actually comes from there. So, basically I am looking at f dash, so I get this from there.

And then again this is something which is the method. When I come to the second point here this is something which is essentially the part of the Runge Kutta solution. So, what I am doing is that I do this and you can see k Y 1 to that I have the original Y 2. To that, what I do is, I add this; I add half, delta eta k Y 2 1. So, this k Y 2 1 which comes from this step above here. Again k Y, so this is Y 3 which is high previous to that again I add half into delta eta, delta eta is the space in which I am carrying of the integration. So into k y 3 1 which comes from here. And here k Y 3 2, so 2, then it goes from minus Y 1 y plus 0.5 delta eta k Y. So again, this here, I got Y 1 and Y 3. So, here what I get is, so I change both; basically, you know with delta. So, I say Y 1 plus 0.5 delta eta k Y 1 1 into Y 3 plus 0.5 delta eta k Y 3 1. So, these are the essentially the 2 points.

So, this is something and then once I get this, I again come to this point, where I say k Y 1 3 which is nothing but Y 2 plus 0.5 delta eta k Y 2 2. So, k Y 2 2 is which I get from here, which I get from the previous step. So, k Y 2 2 then k Y 2 3 is Y 3 plus 0.5 delta eta k Y 3 2 and k Y 3 3 is simply Y 1 plus 0.5 delta eta k Y 1 2 plus Y 3 is k Y 2. And finally, we basically take the 4th point, which is k Y 1, Y 2 and so on and so forth.

So, then basically, what we do is, calculate the function values at the next location. So, what I am saying is, we take the surface, let us say this is my surface. So, I start from here and this is my edge of the boundary layer. So, when I go from here to here. So, I will not go here like directly. So, I am going to go in steps. So, in this case, I have taken 100 small divisions. So, what I just showed you 1, 2, 3, 4 is that when I take a small little division here. So, in this division, I am integrating my function which is what?

So, basically the equation that we derived in using similarity variable, I take that and I solve it in this little distance from the wall, so that is my domain you can say that. So, go from here to here, and I take 4 points there, 1 is at the lowest boundary, 1 is the top most boundary, and 2 in the middle and hence it becomes a 4th order Runge Kutta method. So, you go from here to there, and then you go, so therefore, you find out the edge and again go to the next step. So, again go to the next little you know gap and then and so on and so forth, till you reach the edge of the boundary layer, so that is what this code is doing.

So then this is again this is part of the boundary layer I mean, the Runge Kutta solution. So, when I calculate the functional values at the next location, so all I do is I take Y 1. So, basically what Y 1 here is, Y 1 is f dash, Y 2 is f double dash a sorry a Y 1 is essentially f, Y 2 is f dash and Y 3 is f double dash, so that is what these things mean. So, Y 1 at i plus 1, so what is i, so if you take i, you know at say this is the your surface. So, i is basically an index. So, I go from here. So, the next step. So, I go to the next little step, I go to the next little step and so on and so forth. So, I call this 1, 2, 3, 4, 5. So, what I am calculating is, so I went from the boundary, surface boundary and I go to the next one. So, when I go to the end of this little a step here.

So, basically what I am doing is I am solving this discretely. So, then i write i plus 1, so then i and one-sixth of delta eta and all these values that we calculated at the 4 points. So, k Y 1; 1, 2, 3, 4. Similarly, for y 2, so I do the same; and for Y 3 do the same. And then all I have done is; so once you do that, so basically what you have now is value of. So, therefore, as you go up, you know at a certain location as you go up, you have the values of f, f dash and f double dash. This is something you have by solution of that equation.

Now how does that help you? The reason it helps me is because my horizontal component of velocity, let me see for I have that here. Because if I know f dash, it basically means I know the value of velocity u. because f dash is nothing, but u by u infinity. Therefore, this does gives me value of the velocity. So, if you see this is eta, so like I said from whatever I can see here that from around say 3.5 or you know to be more you know safe, say for example, from eta is equal to 4, f dash is 1. So, f dash is 1 meaning that it is free stream. So, if it is free stream, so that means well you kind of you know list out near the edge of the boundary layer near the edge of the boundary layer. So, this could actually mean edge of the boundary layer.

Now what did does this plot what is this not because now this is in terms of the similarity variable, but what is it now the next question for me for example, what I would like to see is the picture that we drew of the boundary layer. So, where we have a velocity profile at several x sections, how I get that from here. So, that is what I said, so there what you can basically do, you know the value of eta and you know the value of f dash. Which in turn means give that you, also know the value of u. And you can also calculate value of v you can also do that, so you got f. So, v is a little more of complicated sort of an equation, but this is what it was. So, this is what this is right hand side if you see, right hand side equation here is under root of half, into eta f dash minus f, this was the expression for v. This is the expression for v. So, we get this and this kind of you know, a balls down to something like this u by infinity into x u infinity by nu. So, and this is my eta at the horizontal axis.

So, now that is what I said. So, now, in from this to draw your inference, for example, let us go back to this plot. So, draw your inference, all you have to do is, you know take the values of eta from here, the corresponding values of f dash from here, and plot it for different x and y locations. So, there is something you should be able to get from here. So, now, essentially eta is equal to y, under root of u infinity by 2 nu x. Now if eta value is 5 that is what I going to change.

And I am also going to fix the value of x, which is 1. If I do that, what should be the value of Y, we should be able to find out. So, similarly if I then change X, there is a unique solution for Y, you know for the same eta. So, that is something that you do or for example, you fix Y, and you keep changing X. Then for the same eta then what you get. So, that is how you should be able to then you know that information is basically going to be available from this plot. So, essentially then it will give you the kind of velocity profiles, you used to same in terms of a boundary layer. So, that is essentially you know what we get from for example, this kind of a plot.

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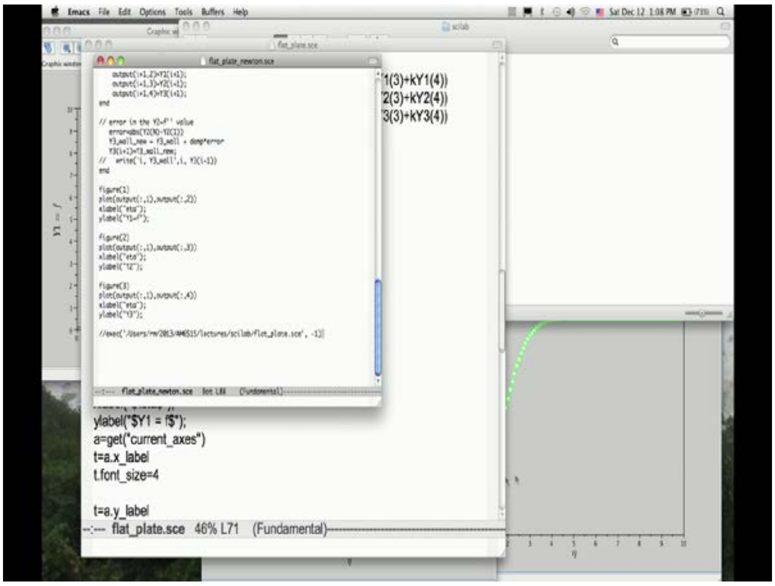

Now, let me see this is I guess. So, now, let me see if this is something different, which I could show you or it is one on the same thing, yeah, actually it is not much different. So, that is probably going to be another you know a part of your assignment actually. So, it is probably going to be a part of your assignment where I can actually sort of talk about that little bit ok.

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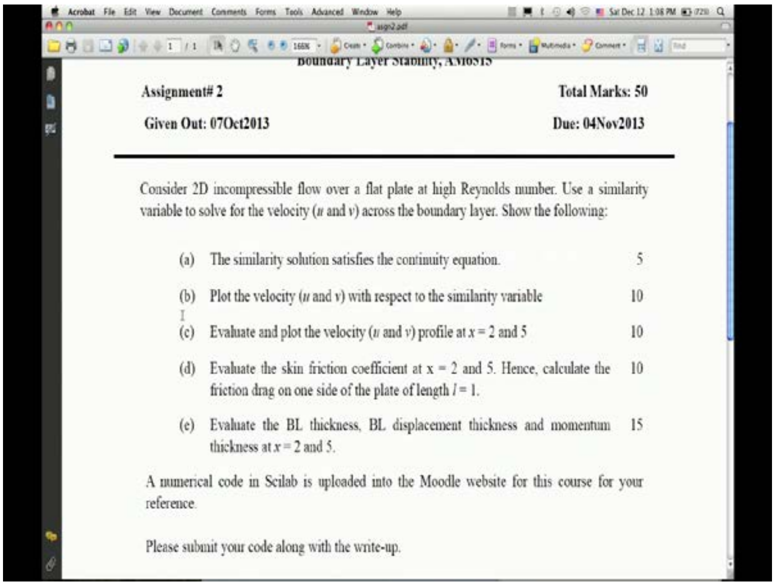
Boundary Layer Stability, AM5515

Assignment# 2 Total Marks: 50

Given Out: 07Oct2013 Due: 04Nov2013

Consider 2D incompressible flow over a flat plate at high Reynolds number. Use a similarity variable to solve for the velocity (u and v) across the boundary layer. Show the following:

(a) The similarity solution satisfies the continuity equation. 5

(b) Plot the velocity (u and v) with respect to the similarity variable 10

(c) Evaluate and plot the velocity (u and v) profile at $x = 2$ and 5 10

(d) Evaluate the skin friction coefficient at x = 2 and 5. Hence, calculate the friction drag on one side of the plate of length $l = 1$. 10

(e) Evaluate the BL thickness, BL displacement thickness and momentum thickness at $x = 2$ and 5. 15

A numerical code in Scilab is uploaded into the Moodle website for this course for your reference.

Please submit your code along with the write-up.

I can actually yeah. So, this is probably going to be a little bit of a problem which I will post for you when you do this. So, one thing is, so let me just mention here that one thing is that you learn to code and things like that, but this is not a coding class. And this coding is not to exhausted either it is it is simple it is nothing to complicated and the syntax that we see here is essentially for sci lab. So, this is type of stuff that you want to use for a sci lab. So, this is nothing to exhausted.

So, what I will be expecting from you is, not to really become a very great code or anything which is should do if you are interested I mean of course, you should do that. But what I will be interested to see whether, once when you get the plots, once when you the moments you are able to see a plot like this, we did all the equations and then we are able to plot all this or get resolves like this, then are you able to interpret, the various parameters of the boundary layer. I think that that is more important whether you are able to understand physics from these graphs. That will be my focus. So, I think I will not go ahead and you know give you further information by Runge Kutta method and all that that is really for another class. So, this is a code that will post for you I do not think that is a problem you can run this in SCI lab and you can also develop your own using mat lab or any other software that you come comfortable with.

So, if you are able to code well, wonderful; if not, it is. Because what I will be doing here is giving you these plots, I will give you these plots and then ask you to do exactly what I just said. That like draw velocity profile, based on this kind of an information. So, that you are able to interpret. So, because the ultimately the reason we did all of these things is to be able to solve numerically, for the velocity profile is not it. So, I think that is what is important.

So, I think we will stop here. Take this up for next time. So, I think next time what we going to do is now we just dealt with a which is dealt with a flat plate. Then next thing is to do only flat plates, so what else you know not fairly, we do not do just flat plates, there are other things as well, but flat plate is a good place to start. So, I think that is why we did that things get can get a little more complicated, so that is what so fine; I was stop here and will pick this up again in the next module.

So thank you.