Introduction to Boundary Layers
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Module - 04
Lecture - 20
Runge-Kutta Method to Numerically Solve the BL equations applied to a flat plate

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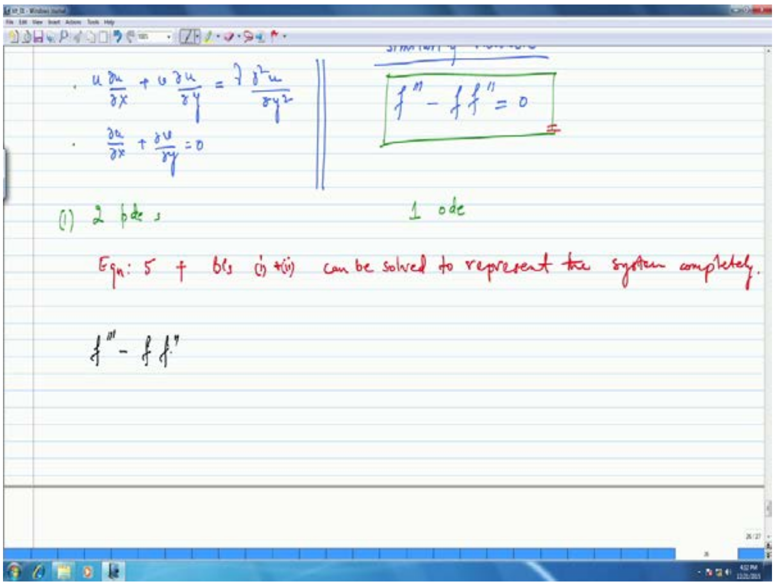

Let me explain that little more, the Runge-Kutta method and let me explain that little more in detail. What I mean? Essentially, we got this equation now. Our equation is basically this.

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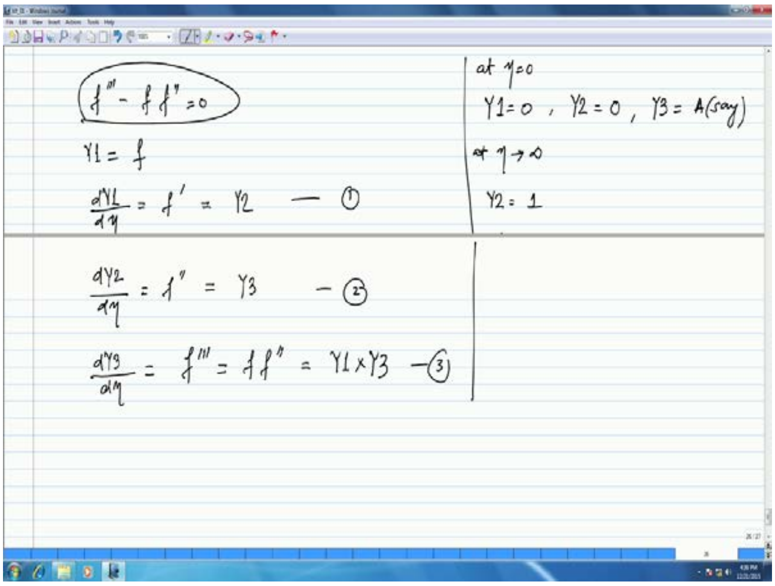

The way we progress here, is in the; in our equations is this. I am going to just write it like this. This is my equation. I am going to just say, that let this y 1 be equal to f. Let this be f then, d y 1 d eta is equal to f dash and let us call that as y 2 and that is this. Then d y 2 d eta, is nothing but double dash and let us call that as y 3 and d y 3 d eta is nothing but f triple dash which we get from this equation if you see to be equal to f f double dash, which is equal to y 1 into y.

It is y 1 into f double dash, which is y 3, y 1 into y 3. This is essentially you know what I get, I mean how I going to start out. Then; this is what I am going to actually look at, see how we will develop the Runge-Kutta method. Now, again let us look at boundary conditions as well. Let us look at the boundary conditions, now at eta is equal to 0. Which is that is basically at the wall. So, f is equal to 0, which means functions of 0, so say y 1 is 0. So, this y 1, y 2, y 3 these are variables which I am using in the computer code.

I am going to write it that way. So, y 1 is equal to 0 and f dash is also equal to 0 which is y 2 is also equal to 0 and f double dash which is y 3; y 3 this is equal to some value. It is some we assume some kind of values, which is equal to a. This is an assumed value. You assume whatever you want. Start with an assumption and then, we iterate of this and at eta far away from the wall. In this case when I say far away, so eta could be 10. I think I spoke about that, then f dash which is equal to y 2 is equal to 1. This is essentially my boundary condition. So, this thing and then I have called it eta infinity in my code, this 1, y 2 underscore i n f. So, you can sort of see that is the name of the variable, so that is how you put it in the code and all of these things are input. Let me just sort of speak about that a little bit, how we sort of input that into the code.

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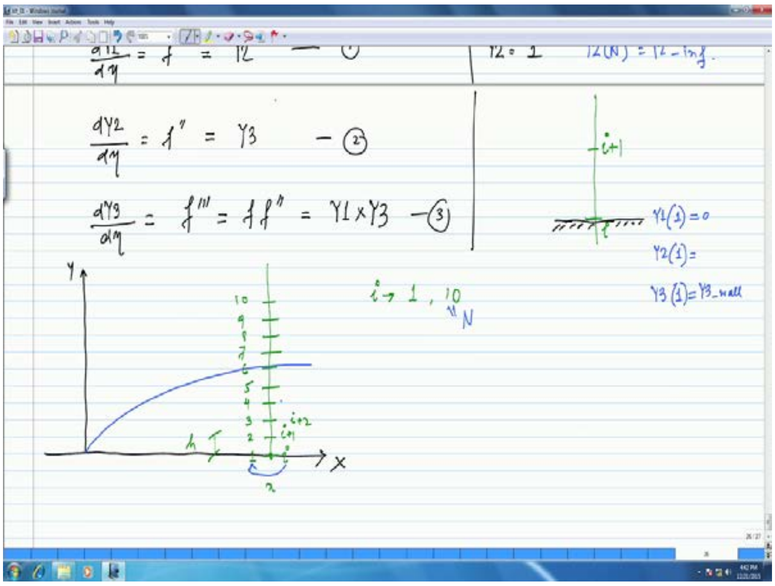

Now, the interesting thing is that, say this is your boundary layer. This is your y; this is your x and says this is your boundary layer. Now, what we will basically do here is at a particular section, if you go like that and we take what we do is I am going to go like this we take several sections like this, and so on and so forth. So, essentially when I come, say let us say this section. Let me call that as; so this is a section 1 in a computer you are not going count start from 0, you start from 1.

This point, rather the location, this is say at some location x is it not. Now, at this point say this is 1, this is 2, this is 3, 4, 5, 6, 7, 8, 9, 10. We will sort of do this. Now, in order to do this, let us say this is at any point. Let us call this as say i. Then you can write this i plus 1, i plus 2 and so on and forth.

Then I can say that this i is basically an index which goes between these sections. Now, this is essentially the height which basically I decide. So, I decide that, and that I am going to call as h. That is really up to you how small you want it to be. So, I do this now

i. Then; therefore, I can say that this, i basically loops from say 1 to 10 because we said that from very far up. What we going to do with 8, 9 say 10 that should be probably. So, you can go further and check and see what the good thing for you to do is and what we have right now. Therefore, if I have something like this is now for example, let say i b at the wall. What I am going to do here is set of just zoom into this z 1.

So, what I have here is therefore, I am zooming into this section. So, this section is i and this section is i plus 1 and this section is my wall. So, if that is, then the boundary conditions that eta is here eta is 0, y 1 is basically 0. So, what I am going to say. The way I write that in the computer code is I say y 1 this becomes an array. This is 1 is equal to 0, because like I said v i is equal to 1, here in this particular case is 0. Again y 2 is 0 and y 3 is something, y 2 again y 2 at 1 is equal to 0 and y 3. So, y 3 at 1 is equal to this value a.

The way I have written it there is I basically call it y 3 at the wall. So, that something I define at the beginning y. So, I define 2 values and I call it y 3 at the wall. So, that you do not have to go and change anything here all the time, this will be at the top and y 3 at for n, and I call that as infinity. This 1 is say, some value say I start with say 0.05 and this is equal to 1. Therefore, I am going to write this as y 3 wall. So, when you change that, all you do is, just change here, do not touch the code. That is how we sort of do that. Now, what I am going to do is again I am going to take this now that I have i and i plus 1.

Now, that here in this particular case, I basically I am going from the wall, i is at the wall, but this essentially you could take anywhere. If this could be or this could be i plus 1, I mean does not does not matter. So, anywhere I am taking this at here we will use, we will be able to use this kind of boundary conditions only here and this 1 y 2 at infinity. So, you could just say, like I said how does; this we can call this as say capital n and this could be also an input value. Then I write that this boundary condition basically as y 2 n is equal to y 2 infinity which also I have as this 1 here. Now, let us come to this Runge-Kutta stuff.

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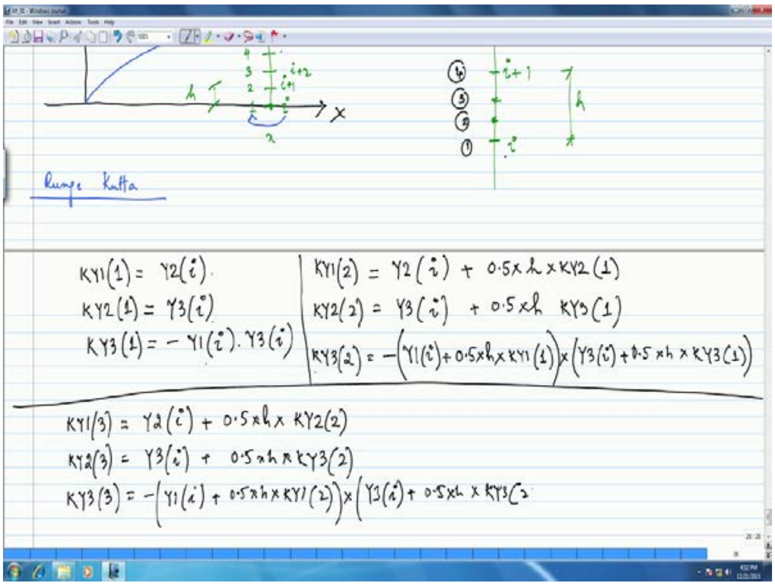

So, this Runge-Kutta stuff. I am going to use basically, again if I sort of zoom in I am going to just zoom in here and this is i and this is i plus 1. Like, I have to explain. So, basically I will take 2 points. I am going to integrate the value. So, basically I am going to integrate my equations. You can see that I have these derivatives here. I am trying to integrate that between i and i plus 1, in this section whose height is h. I am also going to estimate the location of these 2 points. So, this is all between within this height of h. How do I do that?

I am going to just write out the variables that I am using in the computer code. So, the first time I write. So, basically I say this is a name of the variable that I am using. Like I said, this is point 1, 2, 3 and 4. Then I say, k y 1, 1 is equal to y 2 i.

What is y 2? y 2 is nothing but f dash. So, what I am saying is I am doing this variable which is k y 1; 1, this 1 is corresponding to the first points in this incremental lengths, incremental height let us put it that way. So, this is the first location of the incremental height. So, k y 1 this value is equal to y 2 at i and y 2 is nothing but f dash. Then k y 2 at1 is nothing but y3 again at i. So, ky2 is another variable and y3, what is y3? y3 is nothing but f double dash. f double dash then k y 3 at 1 is nothing but minus y 1, y 1 i. This is the syntax this star is this syntax in what you will do. So, basically I am going to multiply this or let us just say I am going to do this y 1 and y 3 i, so what we essentially d1. We got y 2 and we got y 3 and hence we calculated this term, y 1 and y 3 and this y 1 and y 3 from where do I get it; is nothing, but this third equation. We are writing out essentially 1, 2 and 3. These are the equations that I have actually written out if you see in the computer code, so k y 1. Let us just say, if you see, let me go further down little bit just to sort of give you an I just to show you that. So, little further probably sorry about that.

If you see here, essentially what I am saying is. These are the 3 equations 1, 2 and 3 which is what I have written out here if you see, I think I miss the negative sign here. So, let me sort of write there are well that kind of, defensive terms you have depends on how you are defining your variable, you could or could not. Have the negative sign you could, also have a half some people also have a half. It really depends on the how you defining the variable. In this particular case, my equation is really, I guess just. So, I do have like a negative sign here. This is a negative sign I can miss that is equal to this I do have a sign like that. I will probably have this equation here. So, that is how my equation changes.

Now, essentially y 2 this equation 1, 2 and 3 is what I have written out here if you see. I am going to. So, basically I am saying that k y 1 is nothing but, I am writing this derivative is k y 1, 1 which is at this location and that is y 2 then k y 2, 1 is I am getting is y 3 which is this again at this location and k y 3 which is this in 3 is going to minus y 1, y 3. This is something that I write at 1. Now, let us go back then what we will do. So, let me write it here. Then we are going to move to the next point, which is 2.

This is very interesting. Now, all we do here is then we say k y 1 at 2 is equal to y 2 i or you could write here this or you could write k y 1, 1. Which is like the value from earlier step, this plus 0.5 into delta eta, so, basically this is h. This h let me write that as h, h into k y 2, 1. So, k y 2, 1 is already something that you have calculated here. This is what it comes to then k y 2, 2 is y 3 this thing plus 0.5 into h k y 3, 1 and k y 3 is from here in the previous step and then k y 3, 2 is equal to essentially y 1, i plus 0.5 into h into k y 1 k y 1, 1; k y 1, 1 k y 1, 1 into y 3 i plus 0.5 into h into k y 3, 1 you get that. Then again once we have that. So, again we go back 2 point 3. You can see basically, what I am saying is that if this is making estimate at this point 2, this is k y 1, k y 2, k y 3 this essentially valid here. Then k y 1 at the 0.3 which is here is equal to at the 0.3; is also you have to use original values here, so again y 2 i plus 0.5 into h into k y 2.

Now, we just saw the previous location then k y 2 at that point is y 3 plus 0.5 into h k y 3 2 and k y 3 3 is equal to minus y 1 i plus 0.5 into h into k y 1 2 in this case 2 into y 3 I plus 0.5 into h into k y 3 2.

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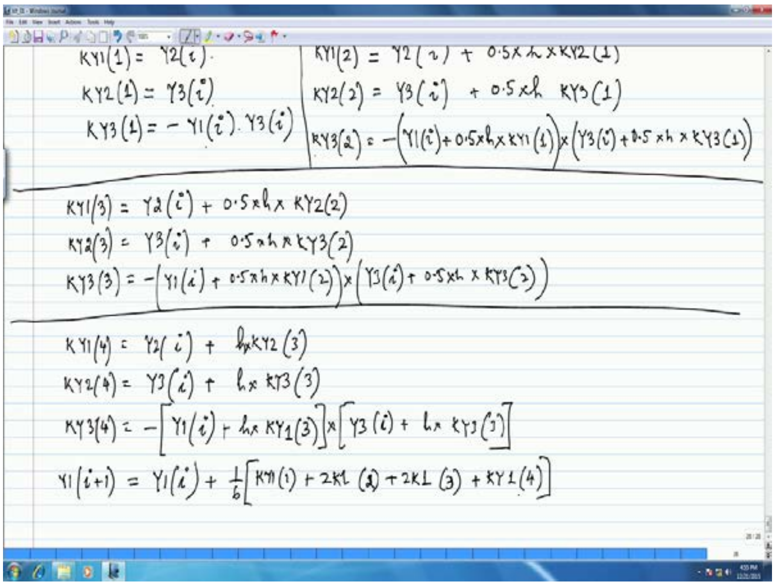

That is at the third point and what do you have on the 4th point. So, again k y 1 at 4 is nothing, but y 2 i right. So, that is delta eta ah k y 2, 3. Let us just put a multiplication there then k y 2 at this point is y 3 i plus h into k y 3 at the previous point and k y 3 4 is minus, so y 1 i plus h into k y 1, 3 into y 3 i plus h into k y 3, 3. Now, the moment you find this, what you see is. So, you get, essentially; now, you got some values here at I how do you get the values at i 1. Now, we have got some k y's and set of like that. Then what do we now, therefore, we say that y 1 at i plus 1 is equal to y 1 at i plus 1 sixth of what 1 sixth of k y 1 at 1 plus 2 k y 2 at 1 plus 2 k y 3 at 1 plus k y 1. This is we will use k y 1 everywhere. This is k y 1 at 2 k y 1 at 3 k y 1 at 4 into the h delta eta. So, you get that. So, then you get y 2 i plus 1 is y 2 i.

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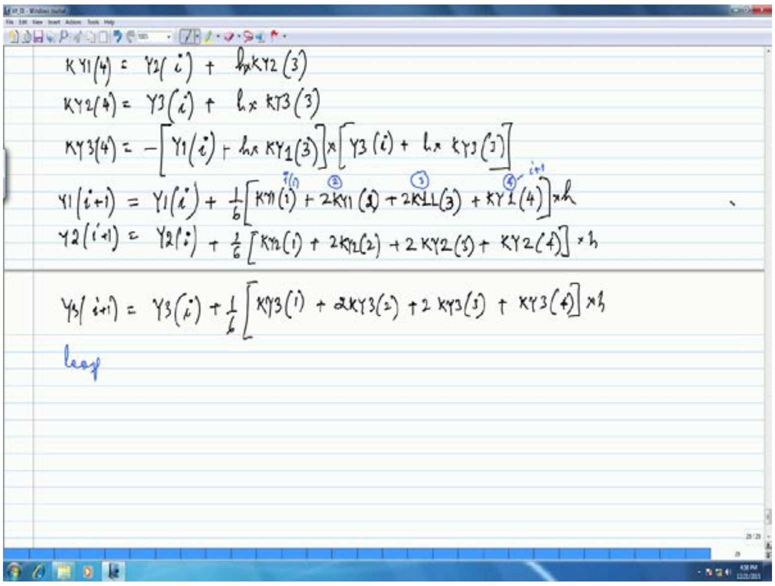

Now, that and so, this is y 2 i plus 1 sixth k y 2 1 plus 2 k sorry about that this is k y 1 k y 1 k y 1 2 k y 2 2 plus 2 k y 2 3 plus k y 2 4 into h and y 3 i plus 1 is y 3 i plus 1 sixth k y 3 1 2. So, I keep missing if you have seen on there. This is k y 2, so k y 3 2 2 k y 3 3 plus k y 3 4 into h.

So essentially, what we doing is the first the k y values at the first location. This is at if you see the i location. We get at the i location or see this is at first location, this is the second location, this is the third location and this is the fourth location which is i plus 1. This is the first location, so that is what we do. So, for y 1 we use k y 1, for y 2 we use k y 2, for y 3 we use k y 3 and that is how we basically get and we integrate from there to the top and finally, when we get to the top. Now, the thing is that we do this now that we do this we are writing all this.

Now, you can put this entire thing in a loop when you say that I go from say 1 to this n which you define. Then you loop this through and you loop this entire through here. So, say loop, I am just writing, this is not how you would write the computer code. Let us say, basically we will. Then you sort of loop this through, you loop this. If you go from i to n, you start from 1 and you see that you will do this entire thing as you go from right as you go from each steps. So, you start out at the wall and then you go towards the free stream and then you use this boundary condition. So, that is exactly what we do in terms of the Runge-Kutta.