Introduction to Boundary Layers
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Module - 01
Lecture - 02
Review of fundamentals of fluid mechanics-II

So, now let us look at this.

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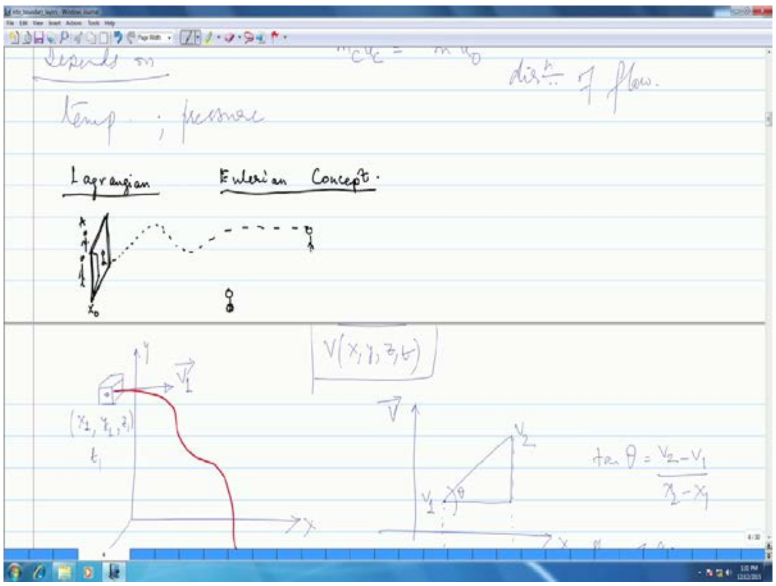

Every time you study a fluid, we use two concepts. So, one is this; the Lagrangian Concept and the Eulerian Concept. Now, what is, what are these two things mean actually? Now, based on this what we, I am going to try and explain to you, how, what you are looking in terms of the velocity. Now, how do I explain this? Now, the lagrangian concept, for example, you have a movie theater or say a class room or room basically, where for people are streaming in for this meeting. So, say you have a door; you have a door like this and you have people walking in. All sorts of people are walking in through this door and there is somebody here, this person is sitting here; it could be a teacher or somebody, who is sitting there, who is watching.

Now, if this somebody is constantly just watching the door, is just watching the door and just noticing the number of students, who walk in and what is happening in every second and how many people are walking, in which direction they are going. But, he is all focused on this location; x naught. That is all he is watching. So, then this is essentially the eulerian concept. On the other hand, say this person looks at this person called A and A walks in. A comes here and then A follows this path and goes here and moves here and goes somewhere here finally and this person is constantly watching all these. He is; what he is doing is, he is following this person A, start to finish and this is lagrangian concept.

So, if this is lagrangian concept, so based on that, we are going to look at a velocity. Based on this, we are going to look at the velocity of a particle, something like this. So, where it is? So, how can you calculate or how can you evaluate, a motion based on this kind of a concept and total derivative; the concept of a total derivative is something which is important from this point of view. Both of this concept is depending on your specific need.

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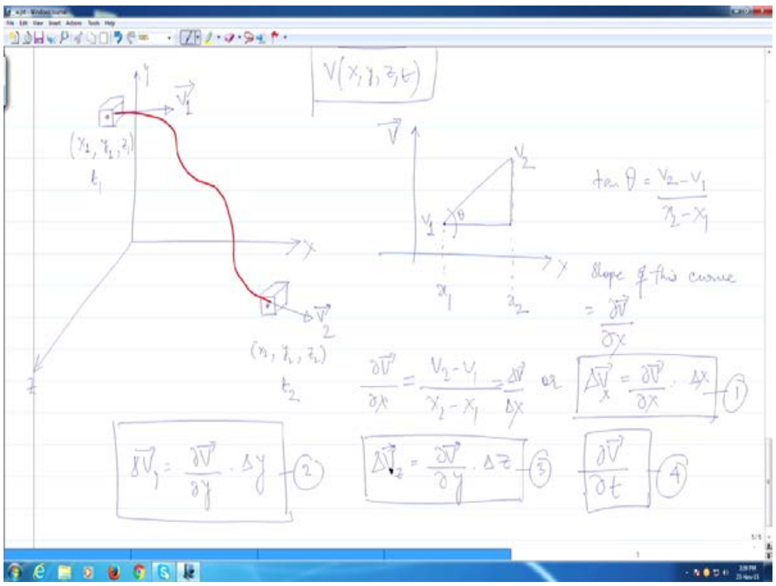

Let us look at this. This is a very interesting thing, so now; so we can call this y and z. So, if we are going to do that. Now, say we have a fluid element, which is here and it moves to some point here. We are not exactly concerned about the path. So, it could be basically any path, it could be anything really. So, let us just say that the velocity here is V 1 and velocity here is V 2 and this location is x, actually x 1, y 1, z 1, it that 2, ok sorry, let us erase that. So, this is essentially x 1, y 1, z 1 and this location is x 2, y 2, z 2 and this time is t 1 and this time is t 2. I could have actually put this t 1 alongside this, which essentially means that the velocity is the functional space and time, so I can write that as x, y, z and t. So, essentially what I am saying is that we are looking at the fluid element, which is moving from say position, let us just say position 1 to 2. Position 1 to 2, in the way where the velocity depends on both space and time. Now, if so, what it is? Now, what will we do at, is just look at the x component for a change. So, what we are saying is we are going to just look at x and this is nothing but v.

Now, so it is moving from x 1 to x 2. So, this is essentially v, this is nothing but, this is x 1 and this is x 2 and it is V 1 here and here it is V 2. So, if I choose this, so here the value is V 1, at x 2 the value is V 2. So, this is essentially the change of a velocity with respect to x. as we go with respective x, then how the velocity changes? So, in here, let us just say that this is theta. So, all we are going to do is find out the slope of this curve. So, tan of theta is V 2 minus V 1 by x 2 minus x 1.

So, slope of this curve is what? Is basically d V and d x. Note, I write a partial here because this velocity will depend on y and z as well as t. So, I write a partial. It depends on x as well as y and z and t. So, slope of this particular curve where we are looking at the change in velocity only with respect to x, is del v del x. So, then what we will write here? So, therefore what we can write is, it is a simple math. So, del V del x is V 2 minus V 1 by x 2 minus x 1 or can I write delta V, delta V is equal to del V del x into delta x. Now, if you see basically what I am saying is I will write this as delta V by delta x. If I do that, then I can write delta V to be equal to del V del x into delta x. So similarly, this basically you get for the extraction.

Similarly, we can write it in the y direction and z direction. So, what we say is; so this is essentially direction in the, so this I can say, so let us put a subscript say x. so, then we can; so let us write it here. So, delta V y is nothing but del V del y into delta y. So, that is for y direction and delta V in z direction is del V del y del z. This is, let this called as; so the change in the velocity in the x direction, change of velocity in y direction, change of velocity in the z direction and similarly is also the change of velocity with time is basically del V del t. That is with that, therefore what we see is that as the particles moves from the position 1 to the position 2, the velocity is a function about space and time. So, therefore, I can write down the changes in space, so that that is defined by x, y and z coordinates as this and the change with respect to time by this. Therefore, let us now; the total change in the velocity.

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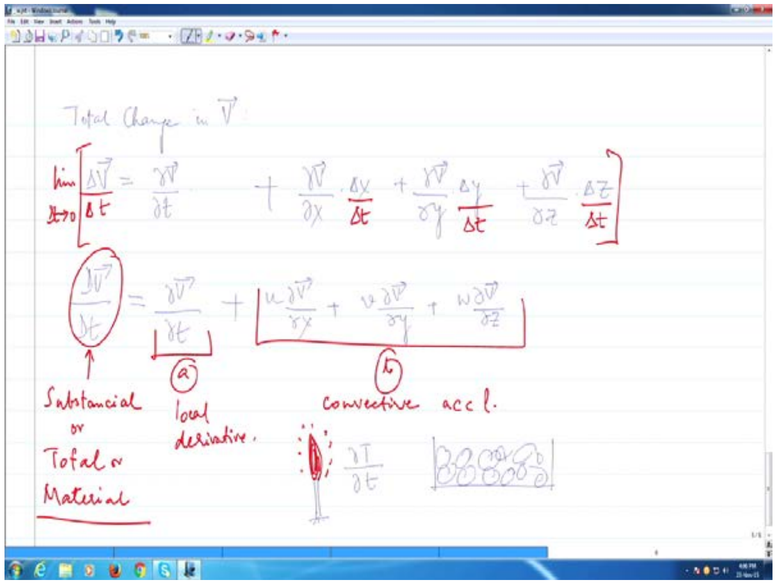

So, if we have to do that, so what is the total change in a velocity? So this is nothing but the total change in V. So that, I will write as, say, the total change in V, so that is del V del t, del V del t is over happening of a time delta t. Then, plus the total change into delta x plus del V del y delta y plus del V del z delta z. If do that, then what I am going to do is to take the delta t out. Divide throughout by delta t. Then, what we get here is you can see. Let me just do that and I am going to; just a second, so let us; what I am going to do is erase the t from here and then to save my time little bit and I got to divide by delta t delta t. So, if I do that and then what I am going to take is limits. I am going to take limits and I am going to say that limit delta t tends to 0 and that is for the entire case.

So if that is true, then what happens? So, what that essentially means that, this is acceleration. But, we can write this. If you see, if you look at this, this delta V by delta t has quite a few components. It is not very familiar to watch, if known earlier. So, this we are going to actually write this as D V D t. If you have not come across this earlier, so this is what I did. So, you write this in the total change in the velocity. If I do that, and then that is equal to del V del t plus; now what is delta x delta t limit tends to zero? That is, by definition the x component of velocity. So, I will write that as del V del x plus V del V del y plus w del V del z. This is what it is and here essentially, so this bit is called a substantial derivative or total derivatives. So, this is nothing, this is essentially it is called several things; substantial derivative or total derivative. Now, this is what?

So, this is essentially and in this particular case, we are looking at the total change in the; we are looking at total change in velocity. So, this bit is the local acceleration. Local acceleration means the instantaneous acceleration. This is the essentially local derivative and this is the convective derivative. Why is this happening? This is happening because of the fluid. This sort of a thing arrives only in a fluid. This sort of a change will happen only because it is a fluid because the particle, the velocity is changing as the particle is moving in space. So, there is acceleration. This is more from the probably more something like the acceleration is rate of change of velocity; rate of change of velocity, which is instantaneous. So that, as you change the time, the velocity changes. But, this if you look at there is no time here, except the fact that during that passage of the time, the fluid particle itself also changing its location and because of that there is this convective acceleration. So, it is convicting also come from there. So, this is nothing but convective derivative in this case, acceleration.

Now, a very simple example a sort of, so I started off with this in the sense to explain what is substantial derivative or a total derivative. Or, it is also called material derivative. Now, I will give you very small example. You could have a candle, you take your finger, right, and you move it very swiftly across the flame, you move it. So, this is my candle and move, this is the flame and I take my finger move it very quickly across the flame. So, you feel that pinch of heat that is nothing but a local derivative of the temperature.

So, if you, for example, say you have; this is your candle flame, this is your, this is my art work. I am sure you can draw better than me. This is my candle. If I take my finger and move it very quickly across the flame, there is that proper change in, little bit of change you feel that it will heat and that is nothing but this change of temperature over a small instant of time and then, what I do is I take my finger and dip it in to a big box full of ice. I dip my finger into it. So, here basically when the heat that I feel that slowly goes so, heat therefore flows. The heat flows from the hot source, which is your finger, this point of time to the cold source, which in the case is this pack of ice or box of ice. So this, here is the convective acceleration. So this, here is essentially a convective acceleration.

So, this we could actually, so that is it. Now, the reason I started with this because I was going to go and talk about the basic. It is something called a Reynolds transport theorem. I am going to talk about that. So, I sort of introduced this substantial derivative.

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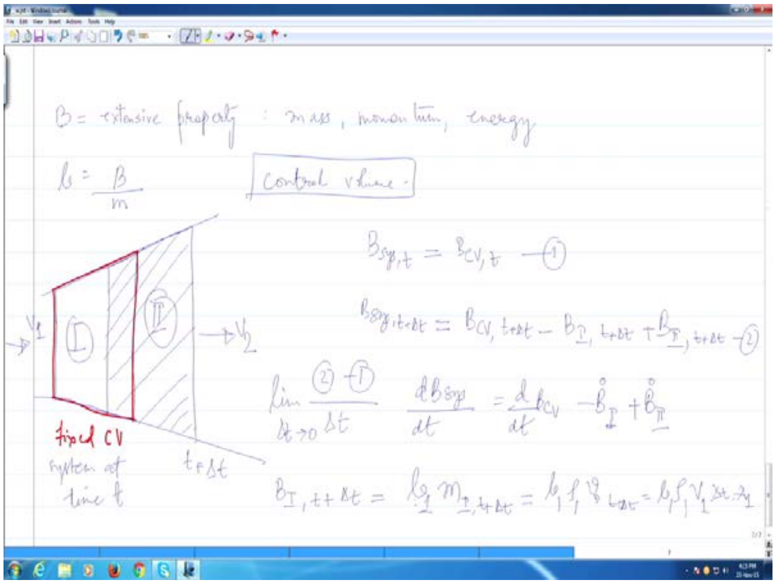

Now, let us; so, for any fluid flow, let us say, B. We will denote, which is any extensive property. What is that? It is an extensive property. I hope this is, so it is extensive property. So, like mass momentum energy, these are any extensive property and an intensive property is nothing but B per unit mass. So, B per unit mass, this is something that we have. Now, I hope that you understand that what a control volume is, so yes, so I am not going to go into details of what a control volume is. So, in case you need to remind yourself, please do. I am not going into kind of the basics of that that is the control volume.

Now, so I am going to do this very simple. Now, I am going to talk about a system and the system moves from one position to the other. So, a system basically would consist of a large number of system would consider of the fluid particles, that we just talked about. We talked about a little small particle, which moves from one point to the other.

But, most of the time for studies in fluids, think of an ocean where you are trying to study the force around; the force generated by a ship. You are not going to be concerned about how one fluid particle is moving from here to there. But, what we are going to be concerned about is essentially a big, you are going to consider a big mass of fluid and you are going to concern about those particles of the fluid all the time, as though how they are behaving because of the movement of ship and that exactly, and so which mass of fluid you are going to consider? Which group of fluid particle you are going to consider is, what the control volume is. But, any way do remind yourselves what that is.

So, let us look at this. So, we are going to consider, say, control volume, like that we are going to consider a control volume and this is fixed. This is a fixed control volume then my system comes in. Now, the system coincides, my system basically coincides with the fixed control volume at time t. So, this is; so this is the system at time t, someone draw over this. So, basically this is my system, my system actually coincides with the fixed control volume at time t. The system however moves, it moves and there is still some overlap, however it moves.

So, my system is here, this is my system at time t plus delta t. So, let me call this area, this unshaded part, let me denote that as 1 and this part, shaded part, between the red line and this one, let me call that as 2. So, incoming is basically, it comes in the velocity V 1 goes of the velocity V 2. This is the system, therefore; so, the extensive property of the system at time t is equal to d, extensive property control volume because it matches that.

So, then of the system at the time t plus delta t is what? Is, if you look at this, basically I am trying to look at this area is nothing but control; this whole control volume, minus 1 plus 2. The control volume is the red part minus the unshaded part 1 plus 2 is this extra part. So, it is control volume. At what control volume at t plus delta t, it does not matter, t plus delta t, it is fixed basically does not matter, minus 1 minus B 1 at t plus delta t plus B 2 at t plus delta t. So, if we do this, then I am going to skip couple of steps because of this derivation. I am going to do that. So, what I am going to do is 2 minus 1 by delta t. I do that and then, also take the limit that delta t tends to 0. So, if I do that, then what I basically get is d d t of the system is nothing but d d t of B control volume minus rate of change of the extensive property in 1 and 2. Now, I would this, like we will just write this.

Now, if you look at this. This is the extensive property over 1 at time t plus delta t. This I can write as b. Let me call this as b 1 and say m i t plus delta t. So, it is nothing but because extensive property is intensive property into the mass, is not it that is by definition, what we wrote earlier. Now, this mass thing, I can break this down. So, I can write this at; this is 1 Rho 1. Then, velocity 1 t plus delta t, I can further break this down and bring it into area, so b 1 Rho 1 v 1 delta t into A 1.

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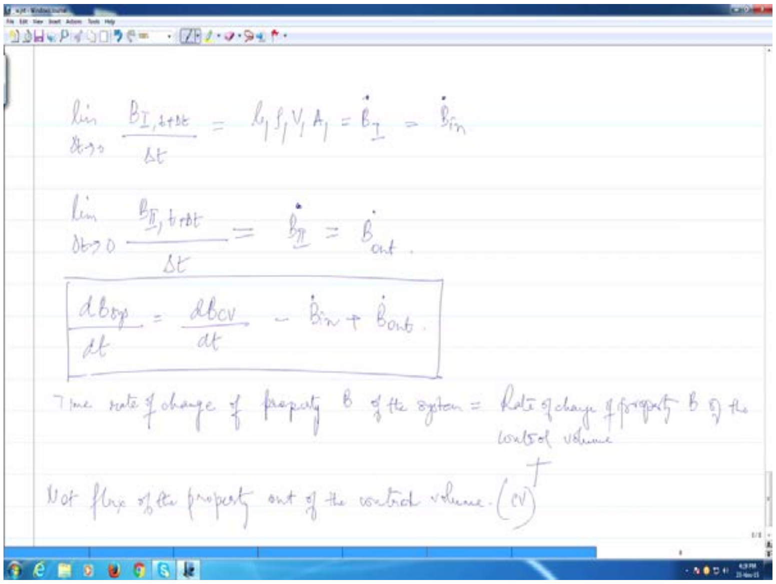

So, therefore if I do that; if I do that, then limit delta t tends to 0. Now, B 1 t plus delta t by this, is equal to what? Is equal to nothing but, B 1 Rho 1 v 1 A 1. So, this is this. So, this is nothing but incoming, incoming extensive property; the rate of incoming extensive property. Similarly, limit delta t. This is, this delta t by t. So, I am going to write that, this is nothing but rate of extensive property, which is going out. So therefore what we can basically say is that d system, right, is equal to control volume d t minus B in rate plus B out. So, let us keep that, therefore the rate of change of property B; the rate of change of property B of the system is equal to time rate of the change of d of the control volume, plus the net flux, if you see, the net flux of B out of the control volume of the mass, which is crossing the control surface. So, let me just write down a little bit. So time, rate of change of property B of the system, right, is equal to rate of change of; is equal to rate of change of property B of the control volume, plus net flux of the property out of the control volume and this is CV basically; control volume.

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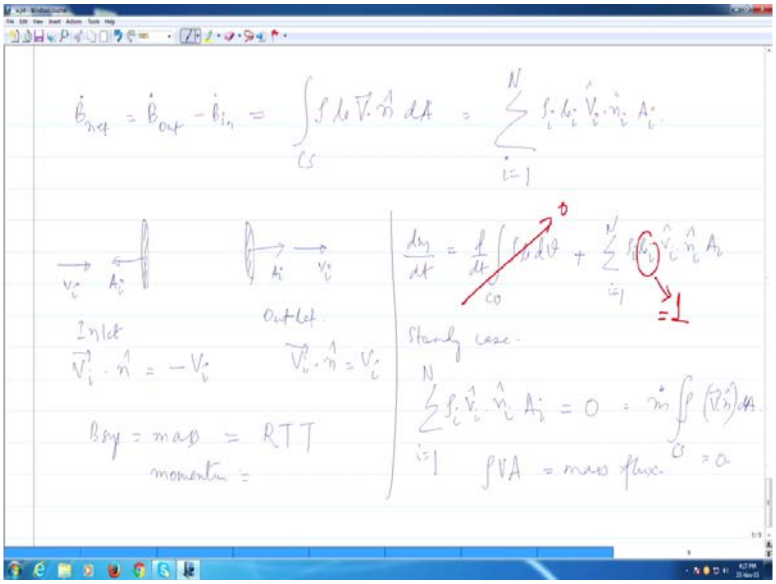

So, I will go just a little bit. So, this has a very interesting form which can use. Actually it has a very, inlets and outlets; you can essentially sort of use that. So, this B net let me write this out, it is nothing but B out minus B in, that is the rate and this I can write out as from the control surface Rho b v dot n d a and this n is a essentially the directed vector out of the area, it is any area, basically. So, this is it and I can actually write this term. I can write this out to be the number of inlets and outlets. So this one, I can actually break it down I can sum it. I is equal to 1 to say n, so Rho i b i v i n i A i. So, let me just do that. This is A i and this is v i, this is an inlet. This is A i and this is v I, this is an outlet. So, therefore what we are basically saying is that for inlet v dot, actually for inlet v i, dot n is equal to this and for outlet, v i dot n is v i. Verify this yourself, convince yourself that what I am saying is right here. Do not just believe me just like that. So therefore, it will automatically take care of it. So, for inlets this becomes negative; for outlets this will become positive and that is how the directed normal is all about.

Now, having said that essentially, therefore we can write the; we return the Reynolds transfer theorem. Let us see what we wrote back. I will just go back to check that or yes, so therefore this; this is what we came up with the Reynolds transport theorem. Now, all we have to do is if we use in this, in this case if the B system, the thing that I consider is mass, if I do consider mass, then what I get out of the Reynolds transport theorem, this is named after Osborne Reynolds. So, if you do that and what you get is essentially the conservation of mass. Then, you can add to it the conservation of linear momentum. So, if B system is momentum, then what you get out of it? Out of the Reynolds transport theorem is essentially, what you get out of it is essentially the conservation of momentum. So, I think you can sort of do this by yourself.

So, for example, let me just quickly do this and we will close. So, for example, we are going to use this, if you are going to use that and the total flux is something that you are going to replace by the summation that I said. This is basically total flux and total flux and out of inlets and outlets and the remaining will remain same. So then, so essentially d b system d t is d b control volume d t. So, if I do that and let us say this system is mass, then if I do this is, is d m d t and this is equal to d d t Rho d Rho b d v. This is volume by the way of the control volume, plus summation of i is equal to 1 to n. So, Rho i b i v i dot n i A i. If I do that now for a steady state case, so, if for a steady case, what happens? If I have a steady case, this will go to 0.

Now, since in this case m, so therefore we are talking about this. So, this is equal to 1, it is equal to 1. So, therefore what we get out of here? What you get out of here is very interesting. What we get essentially is i is equal 1 to n, rho i v i n i A i is equal to 0. So, we sort of get that and we can in this case we could actually, say if the number of; now, you see here. Now, this Rho v and A plus or minus this becomes minus for inlet plus for outlet, this Rho v A, this is nothing but mass flux. We know that already. So, therefore what it is basically is saying that the total mass flux is here. So, you can actually sort of you can even write this down in this way. So, this is nothing but mass flux and that again across the control surface. So, Rho v dot n, v A and that is equal to 0. That is what essentially is what we get.

So, now I think we will kind of stop here. This is the brief understanding that I thought I could brush up and we will start basically from the understanding. Start to begin with concepts of boundary layers right from the next module. So, we will stop here and pick it up directly with the boundary layer.

Thank you.