Introduction to Boundary Layers
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Module - 03
Lecture - 19
Similarity solutions to the BL equations applied to flat plate-III

Hi welcome back, so what we kind of did in the last couple of modules, is basically we trying to use the boundary layer equations and applied it to a flat plate.

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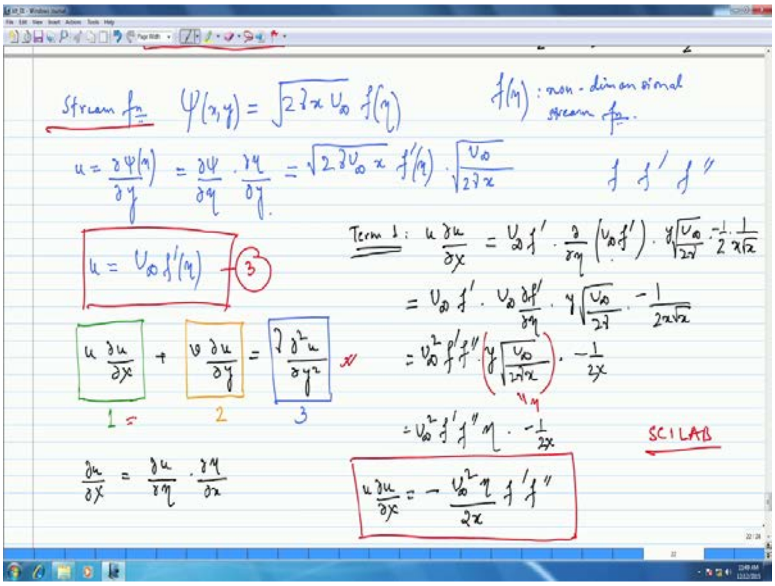

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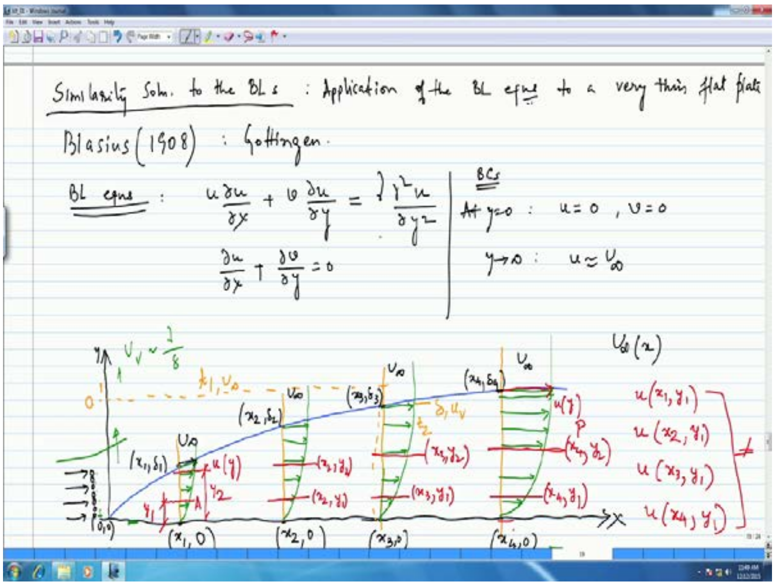

So, where we stopped was; essentially we have the dimensional form of the boundary layer equations, which we are now using.

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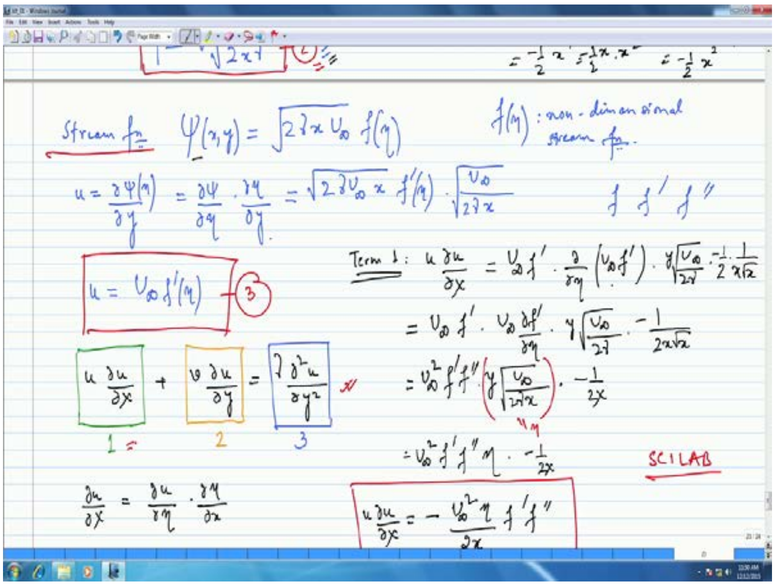

So, this is the basically the dimensional form of the boundary layer equations. So this is the x momentum and this is the continuity equation. And we are basically using a similarity variable, which is this; eta and we are going to use that, with which we have come up with basically, this is my eta given by equation two and we going to use that right given into the equation and we are using definition of stream function, which is given by psi here, so that we can write the velocity u as this.

So, what we did for the first time is that take term 1 of the x momentum equation. Write that in terms of the similarity variable. What we come up with was that, u del, u del x can be represented in this form. So, we got x, u del x basically is a function of x eta f dash and f double dash. And f dash essentially means del f, del eta and del 2 f by del, eta 2 is f double dash. And f is basically, the non dimensional stream function. So, let us continue to do this and you know also work on the second term which, right here, so we are done with the first term here. So, we will do the second term right here, so term one read on with that let us do with term 2.

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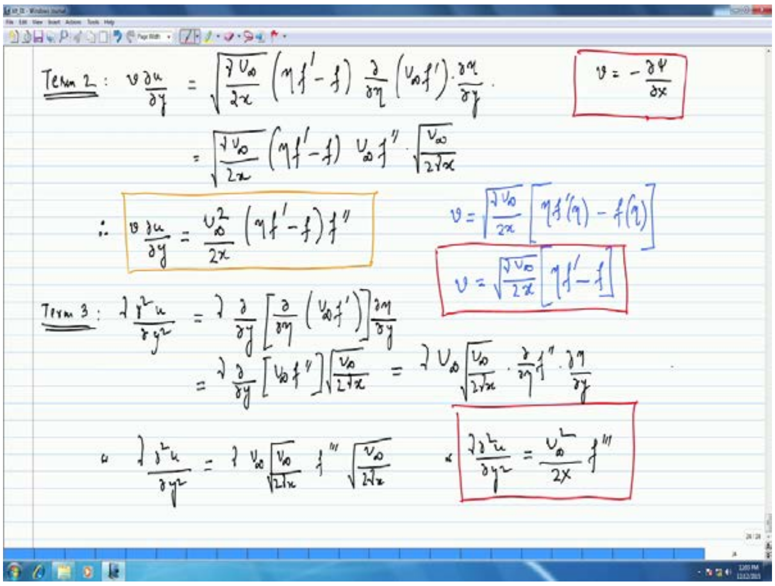

So, what we have here is v; del u; del y, now this is equal to del u del y. So, this is eta; u infinity, 2 x right eta f dash, minus f; del, del eta, u infinity; f dash, del eta, del y. Now, it is important that well; so, I think that you should be able to figure out what I did here, I am not going to elaborate any more. So, u is something that we found out, using the psi like this and so you know what is v? So, v is nothing, but minus del psi del x, so if you do that you should be able to get using this psi function, you should be able to understand what is going on here, please do that yourself and the second thing is u; u is, u infinity f dash eta.

So, basically what we need del u, del eta so again that should be del, del eta. So, that is del, del eta of u infinity f dash and del eta, del y. So, if I do that; so I am going to expand this a little more, if I am going to write this if you see. So, this del eta, del y so let us see what we get here, so this is, del eta, so it is u infinity and f double dash. Now u infinity is a constant take that out, so del l eta f dash is nothing but f double dash into del eta, del y, what is del eta del y, so because eta is this right. So, del eta del l y is nothing, but term inside the root. So, all we get that is u infinity by 2 nu x right. So, if I do that, you can see that therefore, v; del u, del y is equal to u infinity square right by 2 x because the nu cancels out, eta f dash minus f into f double dash. So, that is essentially my second term.

Now let us go to the third term. So, the third term is nu, delta u, del 2 u, del y 2, so if I have to do this third term it is nu, del 2 u, del y 2, now this is again is equal to nu, del, del y. I will kind of request you to do this yourself, I think you can explain yourself. So, del del y is basically del u, del y, do we have a term there del u del y. So, that is nothing, well you know let us write it like this del, del eta of u infinity f dash, del eta, del y. Now then; this is equal to so they get, so then this is equal to nu del del y. So, the terms are the bracket is nothing, but u infinity f double dash and del eta del y is nothing, but u infinity 2 nu x.

Then again this is equal to nu, I will take the u infinity out, u infinity into u infinity 2 nu x, so del del y of f double dash. So, that is del del eta of f double dash into del eta, del y. So, the way I got this term, is basically first I took the u infinity outside the bracket, which comes here. So, I got essentially just, so, I got del y of u infinity, f double dash. So, that balls down to del del eta of f double dash eta del eta, del y. This therefore, or nu, del 2 u, del y 2 is equal to nu u infinity, u infinity 2 nu x, this is f triple dash into u infinity 2 nu x right.

So, therefore, this becomes kind of easy or nu del 2 u, del y 2 is equal to u cancels out. So, u infinity square by 2 x right, by 2 x, f third derivative, let us see if that is what I now get. So, that is my term basically, I think that is what I get, so nu del 2, so this term is basically. So, this u infinity square and nu cancel out to x and after that is fine. So, now I have this in case see this is the important part here v term, I do have the elaborate del like you know the velocity, the working out of this. So, I would suggest do this yourself get some practice, do this yourself and however, I will post this, I am not going to spend time on this right now doing it. So, I since I have it, I will post it for you to kind of look at it if you need any help, but I would really suggest that you know do the map yourself, I mean this is all we have to do. So, remind yourself or revise your calculus knowledge and do that, I think it is useful, so that is what we get the equations, so therefore we got here and so these are the three terms of the velocities and this v term, v because we got the value of u; we got an expression for u, which is u infinity f dash isn't it, but we do not have anything for v. Now this v is central, you should be able to get that from the stream function in this particular case and this actually balls down to this v balls to this. So, v is equal to by 2 x, eta f dash eta minus f of eta or basically you can just say that v is equal to nu u infinity by 2 x, eta f dash minus f, so that is also something which is important to know. How did I get this, well I basically carried out this depreciation, so I would suggest you please do it yourself now. So, that is for v, now we need to convert some of the boundary conditions as well. We should convert these to boundary conditions. So, well you know the boundary conditions where something that right.

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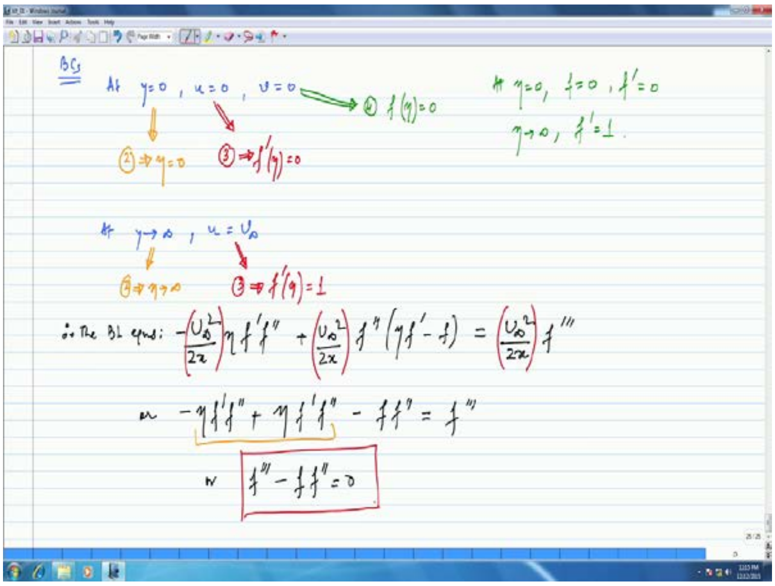

So, we have the boundary conditions, now at y is equal to 0, u is equal to 0 and v is equal to 0. So, what I say convert these also to the similarity variables, so basically now you know what exactly how does this relate because now, I converted my equations to f and eta and x. So, then what do I do now, so therefore, now if you see eta right, the value, this is essentially your eta, so this equation 2. So, eta is a function of x and y and u infinity and viscosity. So, eta is now given like this. So, what you see here is that at y is equal to 0, eta is 0; if you see here from 2 that even at y is equal to 0, eta is 0. So essentially this means that here eta is 0 and if eta is 0, what is u, u is; u infinity, f dash eta. So, now, eta is 0, f dash is also is 0, u is also 0 and f dash eta is also 0.

So, eta is 0 0, u is 0 well, so at y is equal to 0, eta is 0 and u is 0; let me put it this way, now this is where is made up 2. Now let us put it this way what is y is equal to 0 mean, what it means is that from expression 2, what we get is eta is 0. Then what we will do is, we will go to u, and what this means from the term 3 right or this means that f dash eta is 0. So, if you go to expression in 3, u is 0 right, so that essentially means that f dash eta is also 0 and again so we have an expression for v, which is this here.

So, v is also 0, what you get that; is that f is also 0. So, because in this term; in this expression right. So, now, what I essentially have is that v is 0, so let us write this as an expression, so this is 4, so from the fourth expression. So, from 3 what you come to know is this and from 4, what you come to know is that well at v is equal to 0, here y is also equal to 0. So, eta is 0, so what you get from here is that f dash eta is also 0. So, this is how you convert the boundary layer, is a boundary condition at y is equal to 0.

So, essentially what that balls down to is that at eta equal to 0 sorry this is f actually this is f. So, f is equal to 0 and f dash is also equal to 0, so this is my condition; the boundary condition. Then again the second boundary, the another boundary condition is that; at y is at y basically for of right, u is the free stream. So, now, at y is equal to infinity, if y is very large, so what do we get here for eta. If you see a now eta is also very large, if y is very large eta is also very large.

So, what I can say is that this essentially means from 2 that, eta also tends to infinity, eta is also very large and u is equal to u infinity. So, if that is, if you see here from 3; u is equal to u infinity then f dash is equal to 1. So, it actually takes constant value right, therefore, what I get here is that this right from equation 3, it basically means f dash of eta is equal to 1 so, therefore the next condition is that eta tends to infinity then f dash is equal to one that is very interesting isn't it.

So, basically what we are saying is, if you go from eta 0 to very large number, your f is starting value of f or f at eta 0 is 0 and f dash at 0 is 0 and f dash at infinity or final all the you know n value of eta is 1. So, f dash goes from 0 to 1, whereas f is 0 at the starting location. Now, so what you see basically that what we have done, in terms of the equation here. So, if I sort of transform, this equation using these conditions here, we shall set of put that together, let us do that. So, if we you know put this together boundary layer equations.

So let us see, so u, del u, del x. So, u del, u del x is this term here, this term there v del u del y is this term here and is equal to nu, del 2 u, del y 2 which is this term here. So, if I add those 3, so therefore the boundary layer equations, so that becomes. So, if I am going to write it (Refer Time: 24:02) 2 x eta, f dash; f double dash plus u infinity square, 2 x f of double dash eta, f dash minus f is equal to, u infinity square 2, x f triple dash. If you see this, then what you can see is the basically you can get rid of u infinity square by 2 x throughout if you see, so this bits can be canceled out.

So, then what I get from here is minus eta, f dash, f double dash plus. So, I am going to expand this eta f dash, f double dash minus f, f double dash is equal to f of triple dash. So, therefore, you can see here that the first; two terms cancel each other out, so this basically cancels is each other out. So, what we are left with is f triple dash minus f; f double dash is equal to 0, which is so interesting I mean if you what to know look what we from where we started and what we achieved, it is very interesting let me come to this. Now there is another thing still to be done again, I would leave you to do that, work that out for yourself; however, you know I will post the working out of it.

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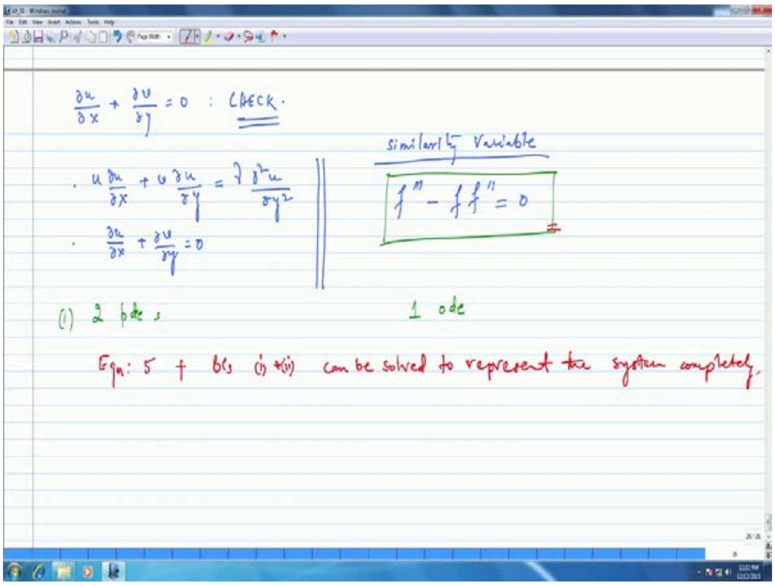

Now because what we have not done so far is the bound is the continuity equation. So, we got del u, del x plus del v, del y is equal to 0. Now, what we need to do is basically you know check that if you; when you replace del u, del x, del v, del y widths you know your f and eta in terms of eta. So, it these two terms basically add up to 0, so you need to check this. So, do this yourself with whatever we have done so far this should be useful, so go ahead and do that.

Now what is interesting is let me actually write that term, wherever we started out where you know the navier stokes equation from there we reached del u, del x plus v, del u, del y is equal to nu, del 2 u, del y 2 and we have del u, del x plus del v, del y is equal to 0. This is my equation, this is the equation and I use the similarity variable eta and then I come up with; I come up with f with this.

So, essentially, if you can see; so, where we landed up. So, first we went with the navier stokes equation and we did a lot of. So, we non-dimensionalised it, we used physics and knowledge of the boundary layer basically boundary layers and then we came up and we were able to write this down the equations in non-dimensional form which is applicable at the boundary layer and after that, we put it back the non dimensional equation we put it back to dimensional form, so that we can you know you know apply it to a flow regime where it goes from laminar to turbulent. So, we wanted use the dimensional form of the velocities, which we did. So, why does the first instance the moment we take you see there is no pressure term. So, mathematically that balls down to that that unknowns are basically the components of velocity u and v and the pressure term is not there.

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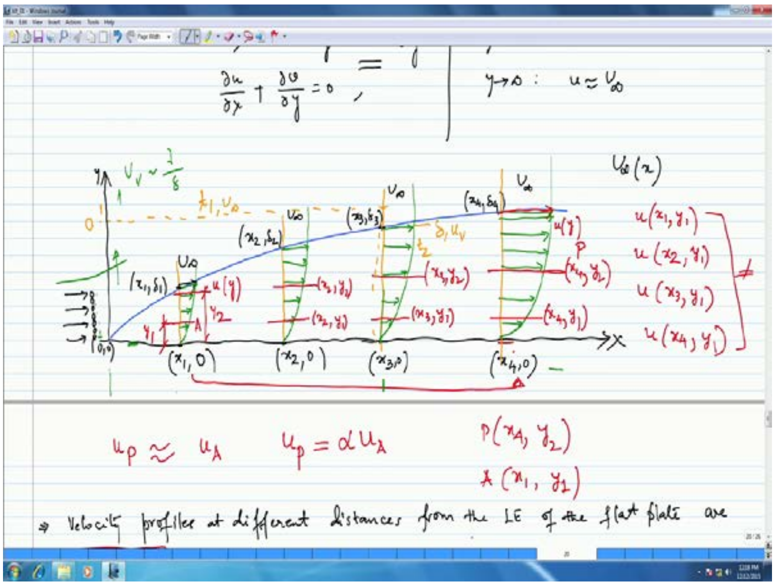

So, we went ahead further and again we use the concept of similarity in the boundary layer. Where we said that the flow at say location x 1, multiplied by a certain scaling factor represents exactly the flow at x is equal to x 4. So, that was the basically we said the flow is similar, so when we say that the flow is similar, we use that information and that is knowledge and then again introduce a similarity variable.

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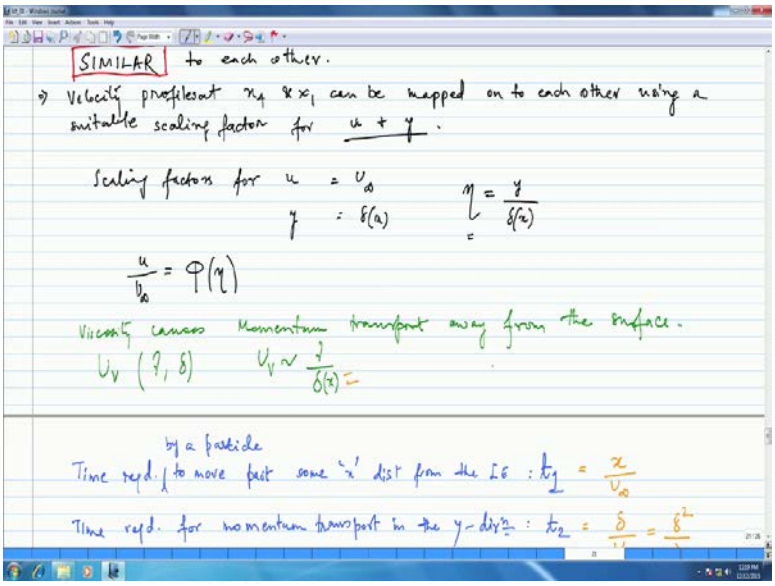

Which is eta here right and then, so eta and then we came up we also defined the stream function and from there we were able to transform our basic equations into using the similarity variables. So, the movement I did that I came up finally, with this equation right with this equation. So, what we have essentially done; is taken number 1, what we have essentially let me instead of write that down. So, what we done is taken 2 pd’s like two partial differential equations and converted them to one ordinary differential equation.

So, that is basically numerically simplicity it is easy to solve that is all. So, therefore, we have taken two pd's and convert it them to one o d e using the similarity variable right and this is of course, it is a non-linear o d e and it is of the third order. So, because you have f, d 3 f d, f d eta 3, now what we will basically do is that, using the boundary conditions that we have. So, let us call this equation as this equation, so this equation let us number it. So, we are going number this as 4. This is 4. So, and also these are my boundary conditions, so I am going to call this as 1 and this is 2.

So, therefore, equation 5 equation, 5 which is basically this equation here plus boundary conditions 1 and 2, can be solved to represent the system completely, to represent the system completely. Now, so we will stop here and what we will do in the next module is go and see, how we do this was basically we go ahead and now you know write this out as a computer code and then we solve it. So, I have done that and again well, let us talk about that in the next module so we will stop here for now.

Thank you.