Introduction to Boundary Layers

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Module – 01
Lecture – 17
Similarity Solutions to the BL Equations Applied to a Flat Plate-I

Hi, welcome back. So, we made quite a bit of (Refer Time: 00:23) I think, in terms of trying to get a numerical picture of the boundary layers. So, we started out with Navier stokes equation and we came up with a set of equations from using the consideration that is going to be applied to a boundary layer, and we came up with the governing equations for that and we used knowledge of the physics of the boundary layer as well as numerical nature of the equations, so that we can basically get a set of equations which will solve then you know it will give us information about a boundary layer.

So, now let us do that exactly. Let us try to you know get ahead and get a solution for that using those equations and I think the best place to start is to apply those equations to the flat plate, and this basically is essentially done by, usually done by Blasius 1908 as part of this PhD phases. So, let us get ahead and see what this is all about. The boundary layer equations, when we apply for flat plate, very thin flat plate.

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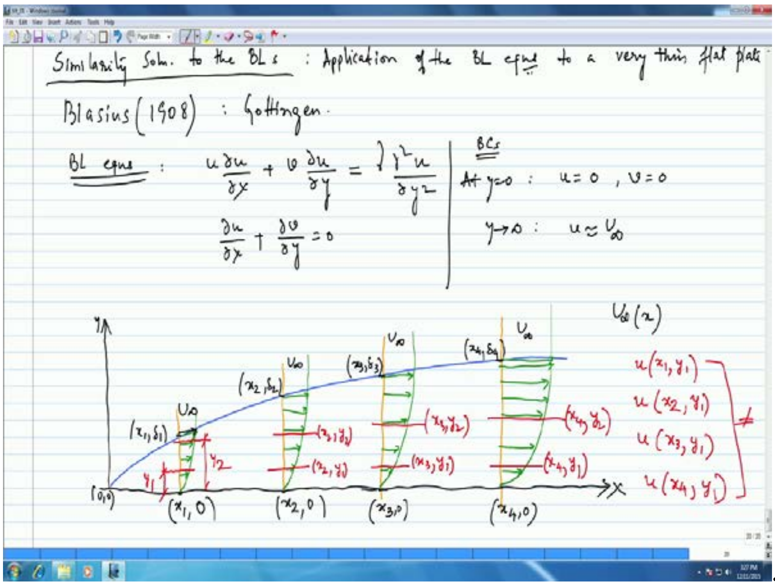

Now, the topic as it is now I would head this, the heading should be something like this. It is basically, similarity solution to the boundary layers. Now, this word similarity is actually technical word here. So, we will instead of slowly understand what that means. So, essentially what we doing now is application. Yes, what we are doing is application of the boundary layer equations, to a very thin flat plate. Is not it flat plate?

So, like you said this was basically done by Blasius, in 1908 in Gottingen. So, this is the work which led to Blasius PhD. So, let start with, so this is boundary layer equations. Now, these are again the dimensionalized form of the boundary layer equations in which you know finally went back to dimensional boundary layer equations after we derive the non-dimensionalized zone. So, that is boundary layer equations. Therefore, basically u del u del x plus v del u del y is equal to nu del 2 u del y 2. That is basically the momentum equation which also accounts for this momentum equation, the extraction. Yes, basically extraction per rho. So, it also takes into account the momentum equation in the wide direction because it contains the information about the pressure distribution, and then of course this is the continuity. This is subjected to the boundary conditions that add y is equal to 0, u is 0 and v is 0. So, basically these two are our unknown, and at y far away u is equal to free stream.

So, we now basically start from here. Now, what is like I said you know the similarity, you know solutions and all that. So, let us kind of what I am talking about exactly, now let us look at the boundary layer over a flat plate. I am going to do a better exaggerated boundary layer here. For some reason I cannot draw a straight line anymore with this thing but anyway. So, please consider that to be a straight line which hopefully not asking too much. So, this is basically my boundary layer. This is my boundary layer. Now, what I am going to do is look at four locations. I am going to look at four locations. So, let us do that first.

So, say one location is here, another location is here, another location is here and another location is here and I am also going to draw the velocity profile. So, when we do that and let us now these; so this one is basically I am going to call this as x 1, this location. This is x 2, this is x 3 and this is x 4; now, this is that. Now, as you can see of course that the boundary I mean there will be lot of things to be noticed here, and I think hopefully if you are not doing that already that every time you know you see up a plot or you see a diagram, you know begin to take the initiative and begin to or learn to be very attentive and scrutinize thing to pick out influences.

For example, the moment you look at this, you know what should be the thought which should come to you. First thing was that we got four locations. So, how does the velocity profile look like? It seems like the velocity profile at x 4. X is kind of you know the velocity here seems to be at the same height say for example, y is much more than what it is here and of course, you know the boundary layer is increasing as we go with x 4.

So, like we have said I guess that, therefore let us say that the boundary layer thickness is delta 1 at this location. So, basically this is delta 1. So, similarly this is delta 1. So, I am writing up the coordinates here so to speak or I can kind of say that yes this point is x 1, 0 actually and this is my origin there. We write it. So, let me not do that. I am going to erase that and say this one is x 1 0, this is x 2 0, x 3 0, x 4 0. So, that is this location, this location and this location. This location is basically x 1 delta 1, this location is x 2 delta 2 and this is x 3 delta 3, this is x 4 delta 4. So, basically delta 1, delta 2 into that is the boundary layer thickness, going from here and of course, this is the distribution of the velocity.

One more thing I mean like we you know spoken about our course is that you know this is that velocity and this is of course, u infinity. This is u infinity, this is u infinity and this is u infinity; that is how it is. So, of course what one should understand is that u infinity is the function of x. So, you could over this is u infinity. I could say this is u infinity 1, you know u infinity 2. So, the u infinity at x 1 need not be the same as u infinity at x 2. I hope you get that. So, u infinity is a function of x even that could change now. So, kind of thing about the possibilities of that is well you know and if that is not, so if u infinity is a function of x, then how does the velocity profile look like and so on and so forth.

So, in this particular case now let us sort of look at this. So, what I do is take this. Let us consider these two you know distances. From these two distances you know x axis, now let us consider this location. Now, this location if I consider this location, if you look at this and say this location is basically this height is y 1, and we will also consider another height which is y 2 and let say that height is this. So, this height is y 2, this height is y 2. So, one could basically say this is x 2, y 1 and this is x 2 y 2 and so on and so forth.

Now, the question to ask here is that what you think of the velocity at x 1, y 1 and the velocity at x 2 y 1, velocity at x 3 y 1, velocity at x 4 y 1? So, now, if you were actually look at this, what I mean by this is x 3, y 1, this is x 3, y 2 and x 4, y 1 and this is x 4, y 2. So, what I am trying to look at is that how does the velocity you know how does the velocity at x 1 0, at the location x 1 at a height of y 1, how is that related or what is the difference or the similarity if at all anything between these velocities that at the same height, but different x locations. So, you can see this is same height y 1 throughout, but different x locations. If I were to look at just the plate, at x is equal to 0, then all the velocities are 0. All these velocities are 0 when y 1 is equal to 0, but when y 1 is not 0 and its finite value, then what happens to that? What do you say that how is allowed that the velocity at say y 1, how does the velocity at x 1 y 1, compare with velocity at x 2 y 1, at x 3 y 1 and x 4 y 1. What do you think? How does that even compare if you want to pay a little bit attention to this.

Now, another way of looking at this, another way of looking at this is that well you know I mean yes how does this look like. Now, another way of looking at this which I think I have said this in earlier when we were doing the delta required or the boundary layer thickness required at x 1 you know is less to reach the free stream velocity compared to the thickness required at x 4 to reach the free stream velocity which means that those at x 1 and x 4, the boundary layer is such that the velocity does reach the free stream velocity, but at different heights. So, that sort of gives hint to what is going on here. So, that is basically at the edge of the boundary layer, meaning that I need a larger height at x 4 to get u infinity and then, in a smaller height at x 1 to get u infinity.

So, what is that basically you know tell as in terms of if I were go along x at different height along y, then how do the velocities look like? Now, if you for example look here, now let us just say that the velocity at x 1 y 1 is not the same as the velocity at x 4 y 1, at least that much I can say. So, these are not same at least they are not same that much I can say, as we go along x. So, therefore, I can probably say therefore from that on that account that these two are not same. For example, if I look at the velocity say at the height y 2 which is at the larger height of y 1, which is a larger height of y 1, I think probably say that the velocity at this y 2 locations and x 4 location. So, if that is related to the velocity at x 1 y 1, but you have to multiple these velocities by something to get this velocity. So, let me elaborate all that.

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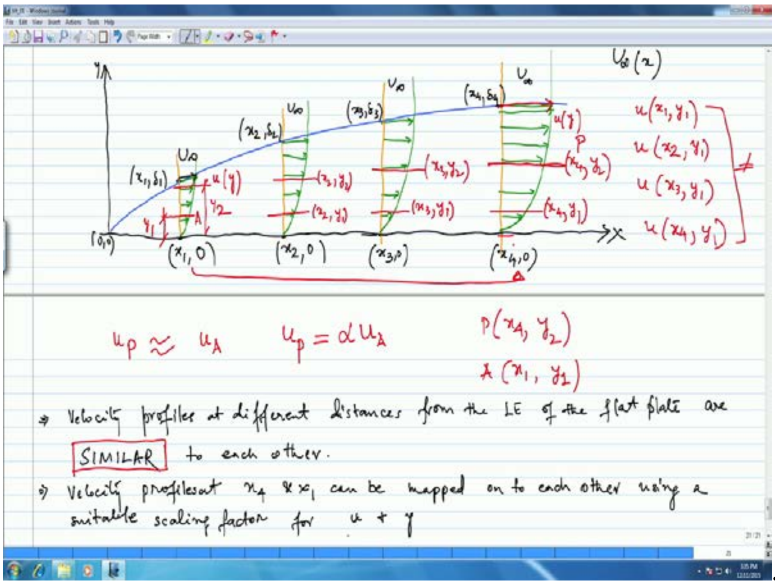
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So, what I am trying to say here is that I take this point if I take this location at p, and I am looking at this location here which is A, right? So, what I am saying is that the velocity of p which is x 4 y 2 is similar to the velocity at A, and velocity of p basically if you multiple by say some scaling factor to the velocity at A, so essentially if I were look at this point you know p. So, p is basically x 4 y 2, and A is x 1 y 1. So, therefore, the velocity at p is similar, is not different, is not much different from UA because it is shifted along the x and it is also shifted along the y, but what I am basically trying to say is that if I take this profile here at located x 1, 0, I can actually map this. I can take this picture and I can map this on to here. So, I will need a scaling factor for that.

So, I what I am saying is that the velocity profile at x 4 you know at the location x 4 is just a scaled version of the velocity profile at x 1. Does that make sense? So, in the behavioral aspect is similar. Behavior as such is similar that it is velocity is 0 at x is equal to y is equal to 0 and at y is equal to the delta. At x 4, it is free stream. You know this is a non-linear change in the velocity and I am going to say this velocity where is with y. So, therefore this is exactly the same picture even at x 1, except that the value, the numbers at say location y 1 y 2 etcetera is different, because if you see that the velocity at the same value of y 1, but at x 4, will be different. So, if you look at x y 1 here which is location A, the velocity the value of u, the small u. This is also the value of u is different. The better value of u at x 4, but basically what we are saying is still the horizontal component of velocity is just you have to multiple. You know multiple the value of u at x 1 by or some factor to get this at x 4.

So, similarly if I take another height which is not the same as you know y 1 or call whatever is the same thing. Therefore, for example if it takes instead of look at that, so this is the free stream, this component at the edge of the boundary layer at x 4, this is u infinity, but if I take this velocity profile at x 1 and import this or impose this on to the location at x 4, I am not going to get that like I am not going to get you know u infinity at delta 1. I am not going to do that. So, all I have to do is stretch this out both in the y direction and x direction, so that I get this.

So, essentially what I am trying to say here is that that velocity profile at different and this is like a statement similar. Write this down that velocity profiles at different distances well from the leading edge of the flat plate. Well, this is a technical term. Let me just use similar to each other. This word is very important here.

So, when I look at the boundary layer, so again what the inference which we got just now from this flat plate is that the velocity profile for example, this is one bullet say and this is the other one. Velocity profile at x 4, yes well vice versa actually you could map x 4 also on to x 1 velocity before x 4 on to x 1. So, this has to be scaled down. This is we scaled up I mean really that, so the velocity put this, the velocity profiles at x 4 and x 1 can be mapped on to each other using a suitable scaling factor, so reversed scaling factor.

So, you know let us use you know scaling factor. So, you have to scale up both the velocity and the y, of course the corresponding y. So, basically for u, for scaling factor for u and y, let us instead of using the scaling you know we let us scale velocities using u infinity and y by delta. That is the most interesting to do I guess.

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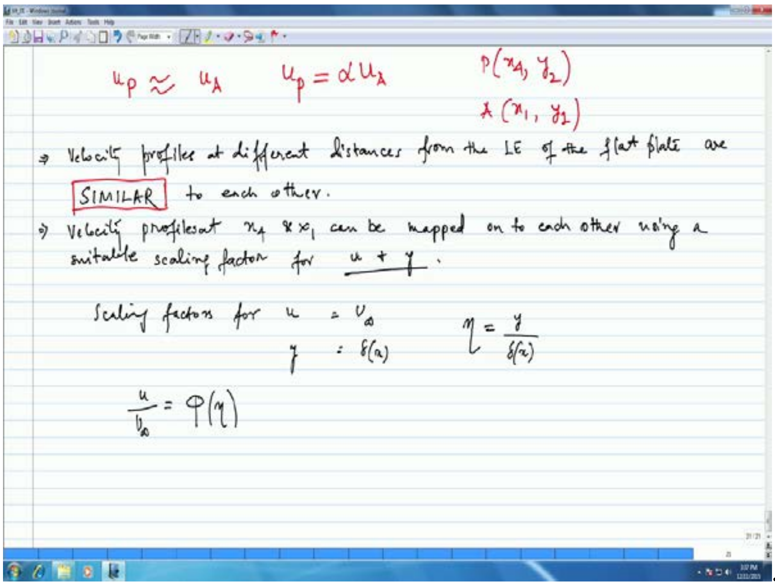

So, let us use you know scaling factors for u. Let that be you know u infinity and for y, let that be delta x. So, when I say that, we will do a little bit of math and see we can come up with something interesting. Basically we are trying to solve the equations now. So, therefore, what we will basically then say is, we will define this because of that we will define this variable eta, which is y by delta x. We will define that value and we are also going to define this. Then, u by u infinity and that is going to be a function of that.

That is the function of eta and this thing is independent of x and this thing actually physically, basically means it is a stream function actually.

What is the stream function? We can just come to that. You must be aware of that is probably, basically we are taking derivatives. You can get the components of velocity back, alright. So, this thing of course as you can say is independent of x. So, now let us sort of look at this. So, we have looked at how the velocity is looking like as we move along x and so on and so forth.

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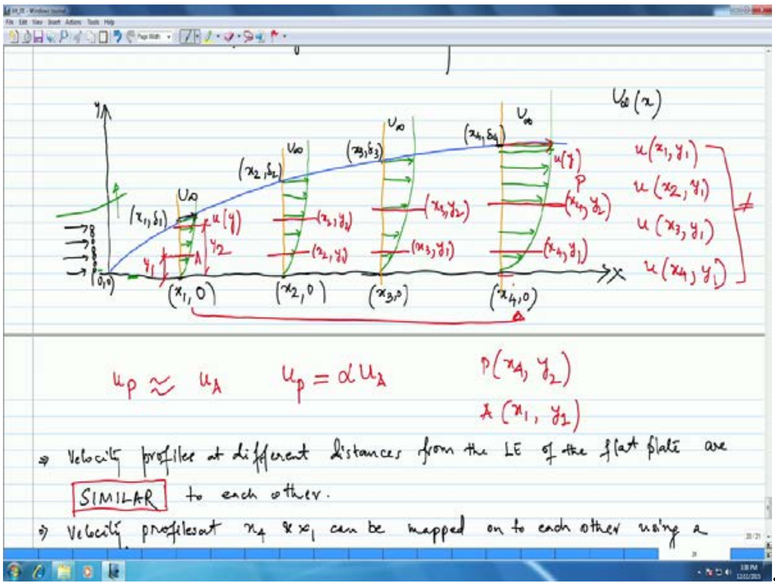

Now, the point is what we basically have said and you are learned so far that now given that the boundary layer exists if I have a fluid, if I will look at this like, so I have set of fluid particle which comes in. So, this is usual velocity profile that we talked here. So, a fluid particle which comes in, now what will happen that the fluid particle at the bottom here is that is going to stick to the surface to the next one here slowly. So, basically these are going to be shifted up.

So, if I would draw a streamline, so it is going to go and then, it is going to go like that. So, streamline is basically telling you that there is a certain velocity component. That means, the only way I can and this is mass, this is fluid coming in. So, therefore I am moving this way. I am moving this way and that is possible because there is a certain exchange momentum. This was an exchange momentum. So, there is a certain momentum, a transport in the wide reaction. So, basically you know this is the cause of this. This is viscosity.

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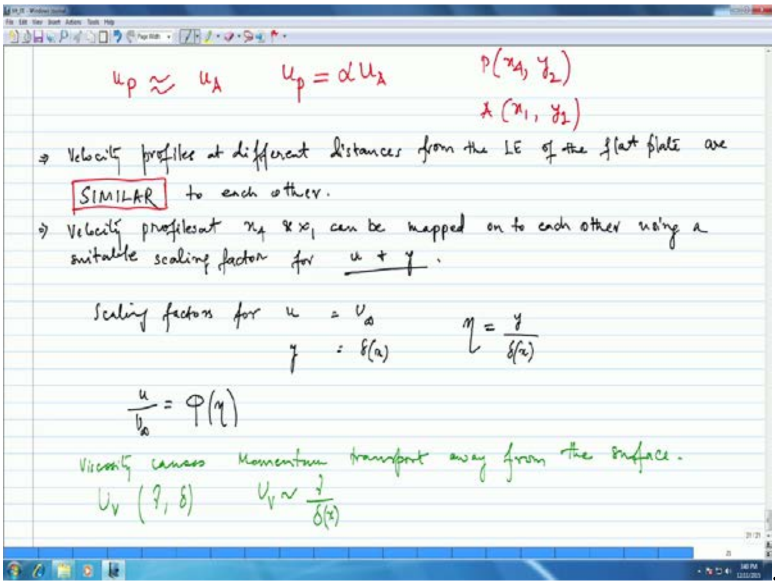

So, therefore the viscosity causes the momentum transport away from the surface and is the cause of viscosity. Now, let this velocity which is responsible for transporting the momentum in the wide direction is, you know let us call that UV, and of course this is function of the viscosity and of course, the amount of boundary layer thickness. So, if I would instead of this depends on this, so if I look at the dimensions, dimensionally scale, this is the velocity. So, unit meters per second if I look at this nu and delta, so I could say that UV could be something like this. This could be a possible function and UV depends on nu and delta and the reason I am writing it that UV is you know is something like this. I am just making it dimensionally correct. And delta of course, this depends on x that changes.

Now, we will come and stop here for this module you know and then, continue this in the next month where we go for the down in getting this solution for thin flat plate. So, see you there.

Thank you for now.