Introduction to Boundary Layers
Dr. Rinku Mukherjee
Department of Mechanical Engineering
Indian Institute of Technology, Madras

Module – 04
Lecture – 16
Derivation of Prandtl's Laminar BL equations-IV

Hi. So, welcome back. So, let us go back and look at the equations. So, what I am going to do is, write it back.

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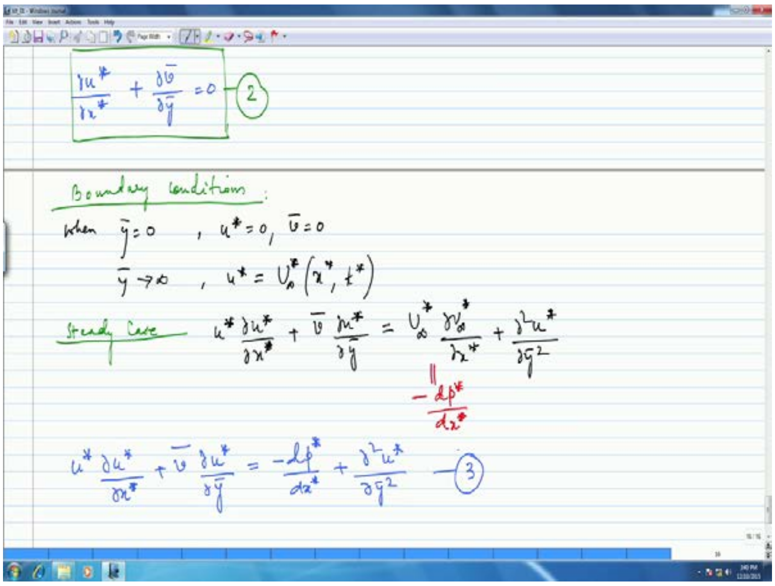

So, let us see. So, what we start. So, if I were to write this art again, basically what are the two equations that we come up with using non-dimensionalization and stretching out the velocity component and the links component, perpendicular to the flow direction is y bar and v bar. So, what we fairly come up with is this. That is the equations are u star del u star del x star v bar is equal to minus d x star plus delta u star del y by square. Let us call this as 3 and we are also going to write down the continuity equation.

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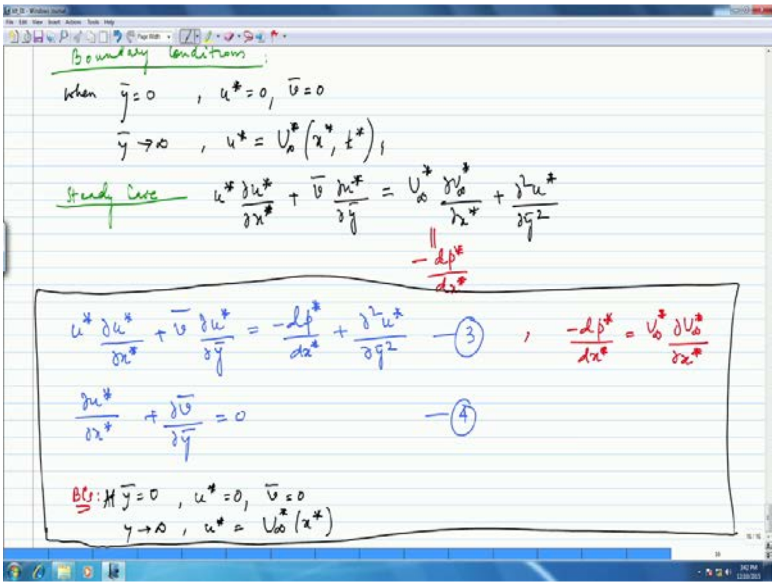

So, that is del u star del x star and this is this. Now, here minus d p star d x, this is important because you see if in these equations this term is basically some if you do not need to calculate, you know this because that is equal to u infinity star del u infinity del x. So, we will do that and then we get this. So, again the boundary condition is something that is you know because using steady case, we will write that out again. So, this is my set of equations like we also write out the boundary conditions, which is same as before except that we would just chase that little bit. So, y bar 0, right at y bar 0. We got this to be 0.

So, basically the velocity 0 and when y is faraway, then u star is u infinity. This is for the steady case. This is for the steady case because what we wrote here was also the t star. So, it took that of here because these are steady case. So, this is what you know come up with. So, this is what we have come up with basically. So, we have come up with this. So, 3 and 4 is what we come with and if you compare this, you know with what we started out. If you compare this, we watch the way we started this out. Let see 1, 2 and 3 if were to go back.

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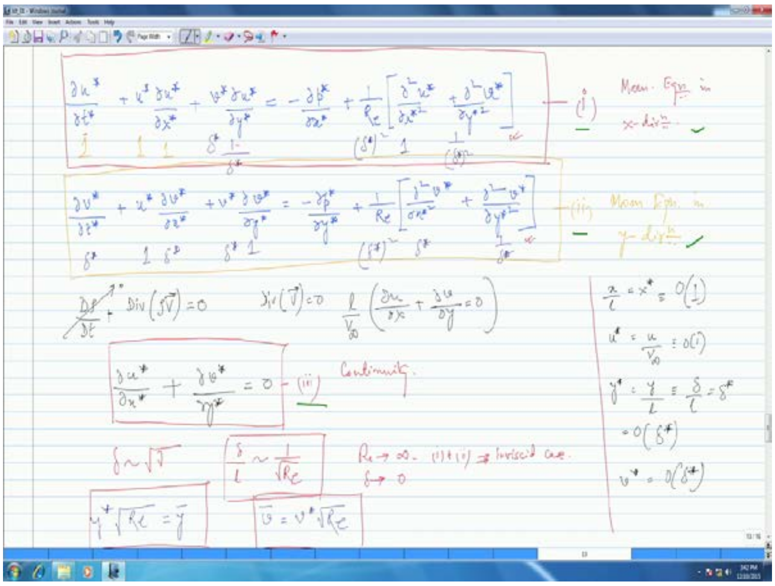

So, essentially you know now before we did the stretching and all of that because this is also non-dimensionalized equation if you see. If you see this 3 and 4, this is also non-dimensionalize equation. So, I would go back to where we were and would be looking at the non-dimensionalize equation here which is 1, 2 and 3. So, if you see this equation, I mean if you look at this particular equation, these equations the first thing that will strike you is that here we got three equations and now, we got two. That is one thing. Definitely this is well, there is change in here in the sense we use the bar, we do not use that here. So, we have done the stretching within the equation that you got now.

The next thing is that we do not have the R e. So, therefore, what I can write I will point that out. So, if I do that, then I will write that triangular boundary layer equations. So, basically I would write this.

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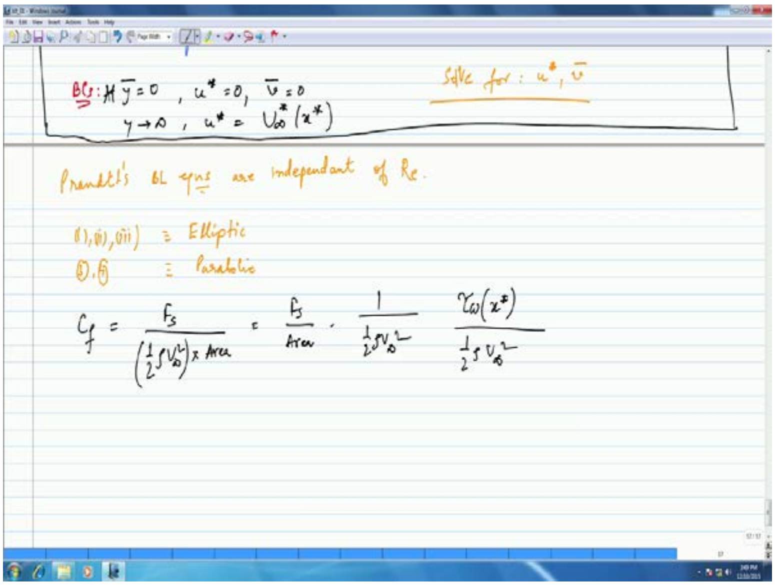

Prandtl's boundary layer equations are independent of R e, and this solution, ok. So, that is very important and we see that we do not have anything you know here. So, therefore basically we need to solve you know just once for the equation. So, that is one and then, well there is one more thing which is numerically important is that the equation that you just saw basically 1, 2 and 3. Now, these equations numerically are elliptic and 3 and 4, that these equations are parabolic. Now, this is what do these mean was I think you have to educate yourself a little bit. If you look at a standard book, it should be able to tell what that means because that is actually little bit beyond the scope of this class. So, I will not be able to you know elaborate on that here, but essentially what I am saying is that the nature, the numerical nature of the equation is different.

The numerical nature of equations you know the non-dimensionalize equations here this 1, 2 and 3, these three equations numerical nature is elliptic, whereas this equation the once that we now see three and four we have two equations here. So, numerical nature is parabolic. Now, also we said that the equations are you know the boundary layer equations. So, 3 and 4, the steady state boundary layer 2 d boundary layer equations, Prandtl's boundary layer equations, now that is its independent of R e as we have written sort that when we sort of you know reverted back to the original function, then of course the velocity is there a function of the x and y only when reverted back. So, as you see therefore once we cannot with that. So, basically we need just one calculation of the unknown. Therefore, for the boundary layer, we started out with the Navier-stokes equations and we have come up with these two equations.

It is very important to understand to this that these equations are basically you know you need to make sure that you work in domain and these are basically valid for the boundary layer. These are equations which give you specifics about the boundary layer. Now, again let us see how you know what kind of information I can get by solving these equations. So, we solve for basically now here what we will do is, solve for u star and v bar. If you see that is what you should get here. Now, again very important parameter for us like we said earlier also is skin friction coefficient. So, let us look at that. So, let us look at skin friction coefficient.

Now, skin friction coefficient I am going to call that CF. So, what is CF? So, what is CF which is basically the force you know the force, the sheer force I can accurately sheer force, write this is one definitions of force by dynamic pressure into the area. Let say area characteristic lines into the characteristic area. Let us go to that way. So, then in that case I can also write this by FS by area into one by half solve. So, infinity square now this FS by area is nothing, but stress. So, in this case is nothing, but stress. Is not it?

Sheer stress at the wall, and in this also of course dependence on x, therefore, this by half rho v infinity square; so, again from definition we know from definition we know that. So, if I would go back let me see if I have to go back to diagram that we had made about sheer stress you know not show that is here. So, we did make diagram saying what drag is. So, this is the diagram.

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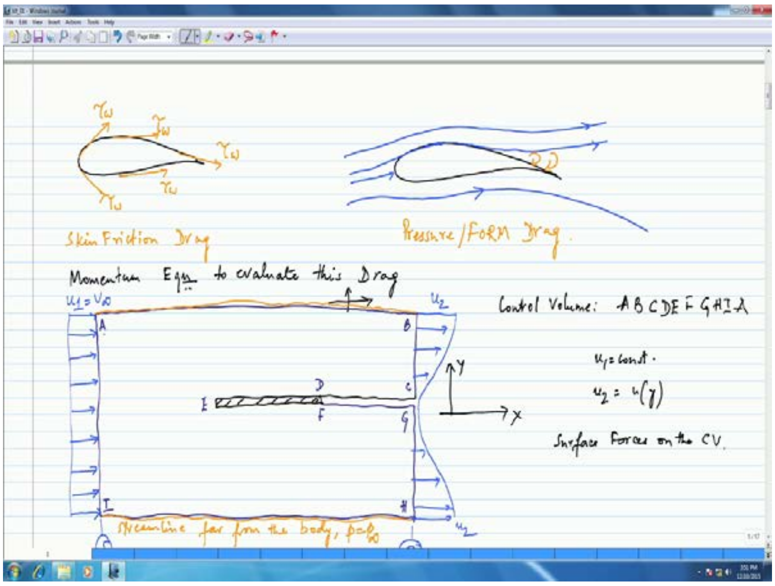

So, this is that I tau all the time talking about. So, the tau is nothing, but you know the pull that the fluid experiences, as it is trying to move over the surface. So, this is the tau wall. See you would always think of like you know pulling the flow fluid like a little. You can take a string and pull it across it that is that the force that you need to you know slide your strain you know a part this surface or along that surface, that is nothing tau. So, this is sheer stress at the wall, when I say as a skin friction coefficient. Now, talking about when I say you know if you slowly get used to it and just a friction efficient coefficient, the movement you say that is skin friction, drag skin friction coefficient, you are basically talking about the drag really at the body at the wall. You know ultimately that is what you are concerned with. Is not it?

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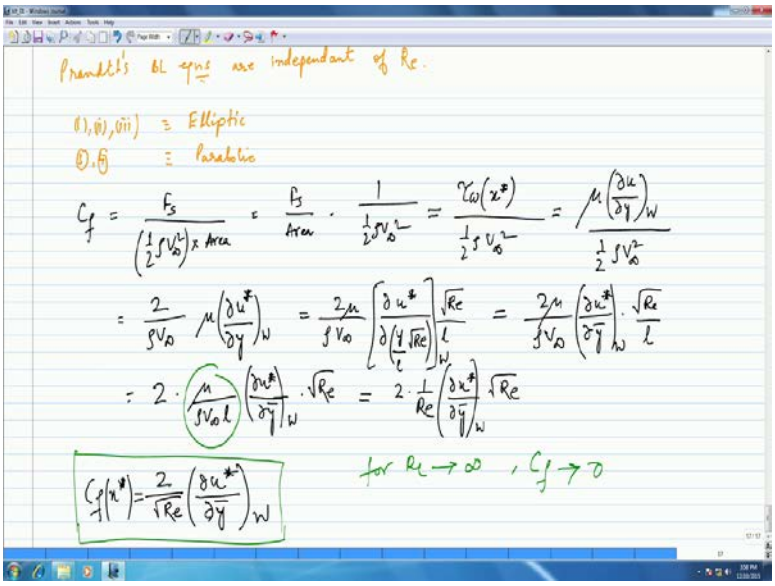

Now, again we know this from definition that this is equal to coefficient of viscosity and del u in this case, del u del y at the wall by half rho v infinity square. So, this is what it is. Now, what we will do here is, again we need the non-dimensionalize form of that because what I am looking for is the parameter that I will I mean when I solve 3 and 4 you know how I can use that solution. So, let me just write this again in the non-dimensionalize form. That will be u star and y bar. So, if I have to do that, then how do I do that? So, let us see.

So, this is 2 by row v infinity and that gives mu del. So, u by v infinity you get that. So, because u star by y I still need the staff at the wall. I still need this at the wall. So, then if I were to do this, so rho v infinity mu del u star and del y by l into R e so that I multiply this by l and Re; if I do that, then what I get is 2 mu row v infinity del u star del y bar R e by l. Now, if you look at this, if I want to look at let me write this answer 2 mu by row v infinity l del u star del y bar into R e. Now, that is essentially equal to, so if you see this term then, row v d by mu. So, this is nothing by 1 by R e. So, what I have here? I again have 2 1 by R e del u star del u bar under root R e, and for somewhere we kind of missed this. This is basically at the wall. So, therefore what I can do is write CF to be equal to 2 by under root R e at the wall. So, now, you see that in order to calculate.

So, this is fine and of course, this you know here as well write this that is CF actually it depends on x star. So, if I do that, now what is basically we see here is that the coefficient of friction.

The coefficient of friction is if you know is basically dependent on Reynolds number, but it is inversely proportional to Reynolds number. So, if you see here that is you know as we increase Reynolds number as we increase with increasing Reynolds number, the coefficient of friction tends to 0.

So, now, the reason I did this here is that once we solve the equations, once we solve the equations here, what we need here is this essentially del u star del y star. So, this is the term that we will need. So, when we solve for 3 and 4, this is the kind of thing that we will need for calculation of the coefficient of friction and again, this is something which is valid at a particular section. So, well I mean that kind of a closes the whole discussion on how we go from the Navier-stoke equation, to the parabolic equations that we developed from there which basically gives me parabolic equations which we developed which are basically the boundary layer equations, and which we developed in order to, yeah. So, this we sorry which we developed in order to you know steady the boundary layers and what we get out of that is, basically our you know skin friction which we talked about. So, more couple things to say here before a kind of close this.

Now, what I am going to do is not sort of you know start anything new. I will start doing this from the next class may be. So, we have done couple of you know problem, the solutions you know using the skin friction. You kind of have an idea to you know use that. One of the reasons I did this you know derivation to derive from the point that basically you need a combination of knowledge of the physics as well as the numerical behavior of their equations to come with something like this. That is what we did. Now, there is a small thing that I will do you know before we kind of because it is something when we you know something that you probably need you know if you when we go into you know further solution and numerical solutions which is you know I gain vectors and I gain values from.

What do you mean by that? So, it is just a little brief overview of that when we sort of do that now. So, I will just take you know couple of minutes to do that. So, let us go and you know do that, but to say basically a lot of these equations can get little numerical exhaustive. So, in order to be able to understand that how we go about that is just we basically very brief overview of you know I gain vectors and I gain values where similar to I think what we did for that, is a brief overview of fluid mechanics now. So, I will write down a matrix. So, I will down what you say a matrix like this.

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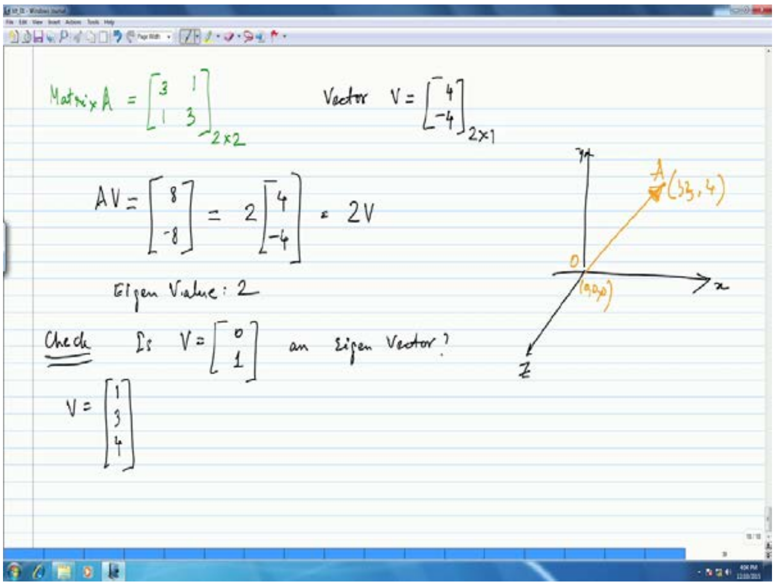

Matrix A is this. So, it is two row and two column matrix and then, we have a vector which is this. It is a two row by one column now when AV. So, AV is equal to what? So, if you do this yourself, you know what and how to do that. So, it is A into B. So, what I will do is take this, multiply the first row with the one column that we have here. So, that is 4 into 3 is 12 minus. So, basically 3 into 4 minus 1 into 4 like plus 1 into 4. So, then this one is 1 into 4 plus minus 3 into 4, otherwise this is essentially equal to 8 and 8. So, this is equal to if you have to write this in terms of vector nothing, so I would write this as nothing, but 4 and 4. Now, this 4 and 4 is nothing, but vector V. So, this is equal to 2 into V 2 V.

Therefore, AV is essentially equal to 2V. So, in this case this 2 is basically I gain volume. So, I gain value is essentially 2 now. So, now what we can do is, you can check this yourself, you can check the vector. I gain values is 2, and I gain vector is this v. So, what you could check? You could check some similar way if this vector here which is 0 and 1 is also I gain vector.

Now, let us look at one more thing. Vector basically will have three elements. So, let us say a vector is something like this. So, now essentially you know the simplest way to have you know represent something like this is nothing, but you have cortices and coordinate system. So, this is say x, y and z. So, the simplest way to represent something like this is that the 4 as simplest you know it is a pointed basically vector. So, you can say this and this is 0.

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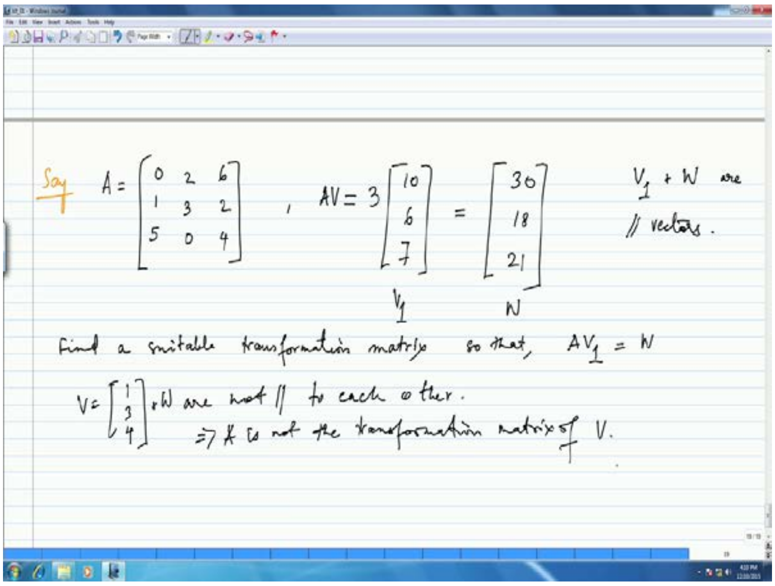

Say we will take A which is 0 1 5 2 3 0 6 2 4, that is A; and using the vector that we did said 1 3 4, this vector that we have taken. Now, do it yourself and see what you get for AV. Do it yourself. What I get here? So, I am going to miss lots of steps here in this. So, basically this and this and then, again I can also write this as this. So, let me call this as you know V 1. Now, I am going to call this as W. I am going to call this as W. Now, the question is, I am going to ask a question here that V 1 and W well are in the sense. So, V 1 and W of course are parallel vectors that at least we know.

So, in the sense that this vector again if I have to represent this, you know if I have to represent this, this way it is length would be 10 6 7, and if I would represent this, it just another vector is that it is just, so anyways see that is three times of this vector. So, this length will become three times of that.

So, I can say that V 1 and W are parallel. So, now the question is find suitable transformation matrix basically, so that A of V 1 is equal to W. So, I need to find out a transformation you know matrix which I can find out. So, basically what I am trying to say here is, that is you know this vector which is 1, 3 and 4 and the W here, so 1, 3 and 4 now this is not you know these are not parallel to each other, these are not parallel to each other, whereas these are.

So, the reason for that is because I can really find something like this. I cannot find a number like 3 you know 2 multiplied by V 1. I can get AW. I cannot do that. So, what do I multiply here and I need to multiply by 13. I will get this. In this particular case I can say that V 1 and W are parallel vectors. So, these are not parallel to each other. So, this one and W are not parallel to each other. Therefore, you know A is not probably, so once V 1 and W are not parallel to each other which means that A is not the transformation matrix of V.

Well, I think we will stop there and pick this sort again from the next module and steady some more of boundary layer properties.

Thank you.