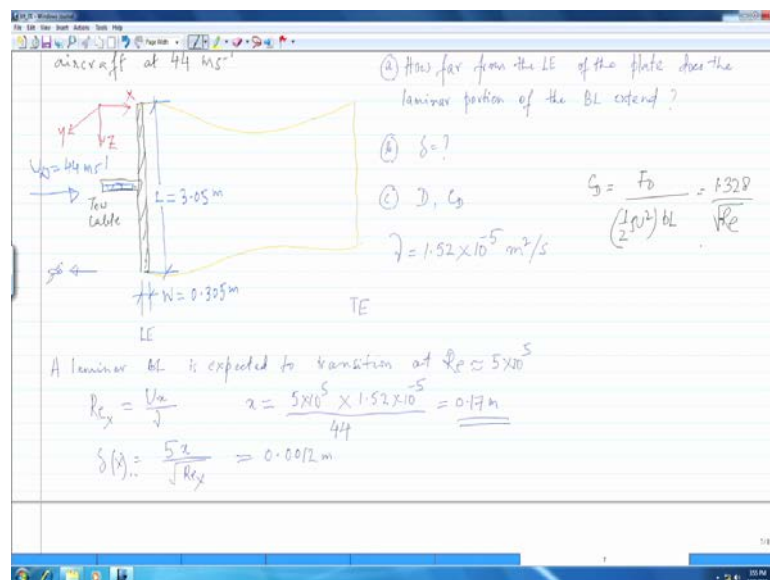


**Introduction To Boundary Layers**  
**Dr. Rinku Mukherjee**  
**Department of Mechanical Engineering**  
**Indian Institute of Technology, Madras**

**Module - 04**  
**Lecture - 12**  
**Skin Friction Coefficient-II**

Hi. Welcome back. Let us go and calculate the drag force, coefficient of drag from what you know the problem that we started out in the last module.

(Refer Slide Time: 00:28)



Now, we know these are formulas that we have been using now for some time.  $C_D$ ,  $C_D$  is what?  $F_D$ , the force by dynamic pressure half  $\rho V$  squared area. So, say the width is this and the length is that. So, that is the area and this is also equal to  $1.328$  by  $Re$ . So, as you can see what we are going to do is basically use this relationship to come up with one expression to find out the drag. So, this drag we are talking about is the drag force.

(Refer Slide Time: 01:25)

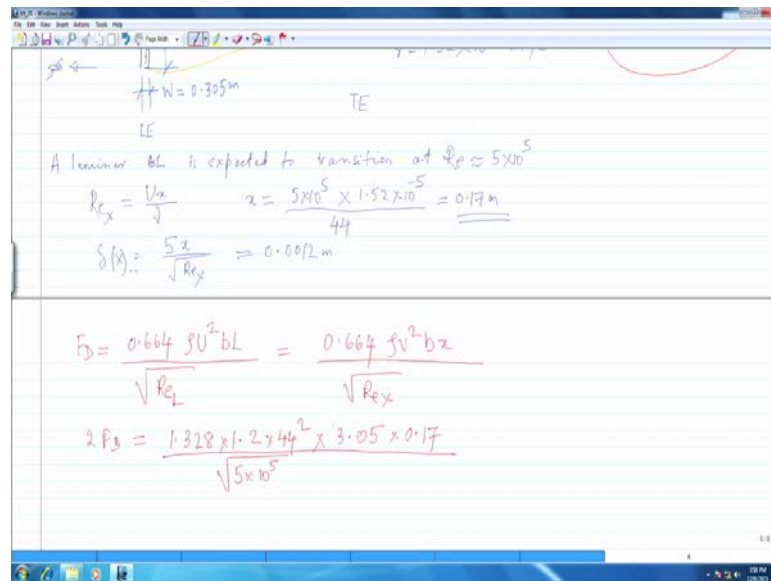


Diagram showing a flat plate of width  $W = 0.305\text{ m}$  with leading edge (LE) and trailing edge (TE) labels.

A laminar BL is expected to transition at  $Re \approx 5 \times 10^5$

$$Re_x = \frac{U_\infty}{\nu} \quad x = \frac{5 \times 10^5 \times 1.52 \times 10^{-5}}{44} = 0.17\text{ m}$$

$$\delta(x) = \frac{5x}{\sqrt{Re_x}} = 0.002\text{ m}$$


---


$$F_D = \frac{0.664 \rho U_\infty^2 b L}{\sqrt{Re_L}} = \frac{0.664 \rho U_\infty^2 b x}{\sqrt{Re_x}}$$

$$2 F_D = \frac{1.328 \times 1.2 \times 44^2 \times 3.05 \times 0.17}{\sqrt{5 \times 10^5}}$$

So, if I do that what I get for F D is this is 0 point 664 divided by 2 into rho U square bL by under root R e L. So, if I do that what I get is rho v square b x. This thing, and drag on both sides so that is 2 F D. So, then well I am going to write this also, what I get is 1 point 328. So, the density is essentially 1 point 2. Free stream velocity is 44. Then what is the breadth? The breadth is 3 point 5 and x is 0 point 17 meter and this is very important to understand. Let me write this down instead of, fully attention to that by R e which is this. So, what are we instead of doing here, you know the math is simple, but as long as you understand what we are doing here. So, now 3 point 05 is basically these lengths and 0 point 17 meter is the length till which the laminar boundary layer extends.

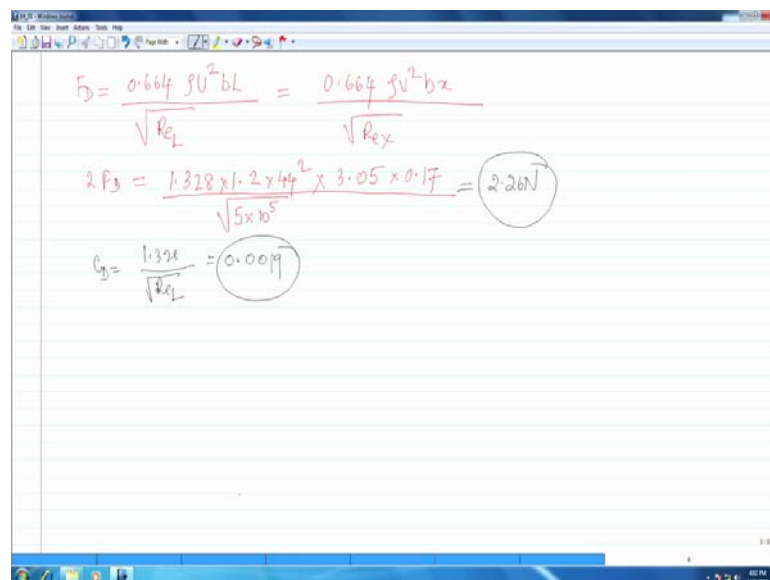
So, when we are talking about b into L, well it does not matter. You can take this as L and this is b. I know that is the definition, but in the sense in this case the boundary layer is essentially developing in this fashion. You know, the boundary layer is developing this way. So, the boundary layer profile is the boundary layer say velocity profile which is something like this.

So, therefore in this particular case, we are looking at 3 point 05 you know, so this is the total length over which the boundary layer is developing and the x, so the x is 0 point 17 meters. So, that is essentially along this. So along these lengths, we have a laminar

boundary layer which is developing for 0 point 17 meters starting from this edge. If I would exaggerate you know if I would look at that, so basically this is my plate, and my boundary layer is developing with laminar boundary layer for about say this is 0 point 17 meters something like this and the height of the boundary layer here is 0 point 0012. This is 0 point 0012 meters and this width. So, this 0 point 17 meter is still is laminar boundary layer. You have boundary after that is well, but it is turbulent. So, this entire width is 0 point 305 meters, it is like almost double of that.

So, therefore in this particular case when we are talking about breadth, the breadth is basically this length and that length is the complete length of the plate and when we say x, so this is the x over which I have the boundary layer development. So, therefore, that is the area I am looking at and the Reynolds number again. Reynolds number, we take the Reynolds number which is the kind of the limiting case almost, when the boundary layer will transition from laminar to turbulent. So, therefore, that is the Reynolds number that we have taken here. So, if I calculate that, that comes out to be 2.26 Newton. This is drag on both sides.

(Refer Slide Time: 06:14)



The image shows handwritten calculations on a digital notepad. The first equation is  $F_D = \frac{0.664 \rho U^2 b L}{\sqrt{Re_L}} = \frac{0.664 \rho U^2 b x}{\sqrt{Re_x}}$ . The second equation is  $2 F_D = \frac{1.328 \times 1.2 \times 4^2 \times 3.05 \times 0.17}{\sqrt{5 \times 10^5}} = 2.26 \text{ N}$ . The third equation is  $C_D = \frac{1.328}{\sqrt{Re_L}} = 0.0019$ . The results 2.26 N and 0.0019 are circled.

And, so that C D is basically 1 point 328 R e L and that comes out to be 0 point 0019. So, that is the total drag. So, this is the total drag and this is the C D. Now, the total drag

is what you say, we calculate this only if till you know laminar case. Now, the total drag is way larger because lowest turbulent now that something that we have not considered here. So, if you calculate the drag, I mean on the real banner, it will come out to be much larger. So, you should not be surprised because what we have done here is basically consider the boundary layer till the laminar portion of it. We have not considered that turbulent part.

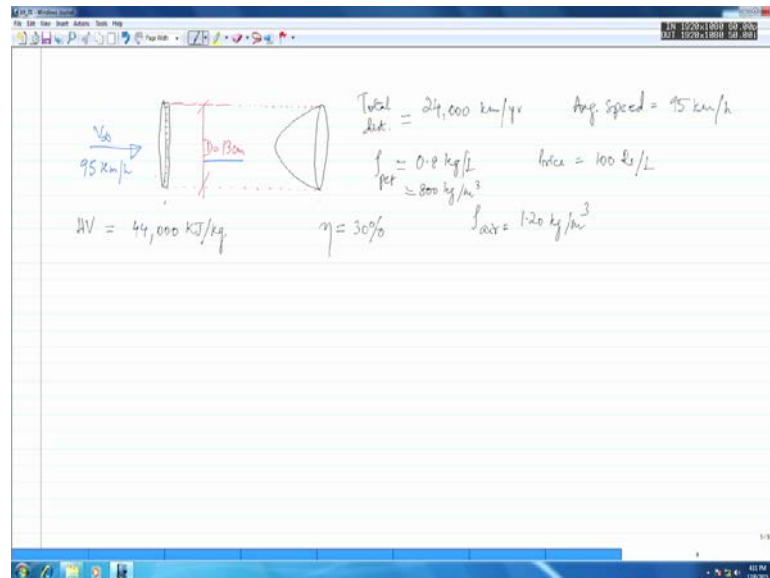
So, that is one as per looking at how we look at the drag. This is another important way, interesting problem that we are going to look at now. This is basically trying to give you an idea or trying to understand that, where can we use this kind of say; why is it does not even affect us you know. So, now cars have been undergoing a lot of, you see so many different types of cars with so many designs, some are really weird and one does not know I mean one does not always analyze. So, what they are doing sometimes you do, some cars even look very nice and sometimes you know I always wonder why somebody would make designs like this you know, is this even design? So, now the point is that you know design of a car I mean this is just even a smallest of things you can change the drag. So, that is why it becomes very important and one of the things is the rear view mirrors.

Now, you could you know have very flat mirrors. You know the mirrors (Refer Time: 09:00) in your house. So, it could be just a very flat mirror. You know this is a mirror. So, if I am the driver, so I am looking at the mirror like that you know. So, the air drive in the car is going to hit you know, this is like a flat plate going to hit it just like that. On the other hand, you could make this little more streamlined and you know that is more non-technical term which pretty much everybody uses streamlined body. So, anyway you take instead of a flat plate, you take a little bit of a hemisphere. Take a say football cut into half, so one and half is the hemisphere. So, you take that and then you have a mirror here. What is the difference? Does it give you fuel efficiency; because of the drag or coefficients of drag let say is an interesting problem, right?

I am not going to write the whole thing. So, in f x to reduce the drag coefficient and thus improves fuel efficiency of cars. The design of the side view mirrors has changed a lot.

So, now, determine the amount of fuel and money saved per year as a result of replacing a 13 centimeter dia flat mirror by one with the hemispherical back.

(Refer Slide Time: 10:27)



So, basically this is the problem. So, this is a flat, this is basically a disc. So, mirror and in this basically driver will look from this side. So, the mirror is on this side. So, if I say A side and B side, the mirror is actually stuck to this on the B side. So, the driver actually looks at here. This is it, and instead of this I could also have the hemisphere like I said. Basically, what I am trying to say is. And this is actually supposed to be a straight line. For some reason I am not able to draw it. That is fine. So, if I do that, then this is equal to 13 centimeters and then here free stream which is coming in and this is actually the free stream is 95 kilometers per hour. So, determine the amount of fuel and money saved per year as a result of replacing a 13 centimeter dia, a 13 centimeter dia flat mirror which is this; flat mirror by the volume of the hemispherical back.

Some more information is given. The car is driven 24,000 kilometers a year. It is 24,000 kilometers per year. The car is driven 24,000 kilometers a year at an average speed of 95 kilometers per hour and this is a total distance. Average speed is 95 kilometers per hour. Now, the density implies of you know gasoline, petrol or whatever, so density of the petrol or let me write here, the density of the petrol is 0 point 8 kg per litre and the price

is 100 rupees per litre, respectively. Now, heating value let us call that HV of the petrol is 44,000 kilo joules per kg, and the overall efficiency of the car in the make of the car using is 30 percent.

So, essentially it means that you know if you burn say 1 kg of you know the fuel, you will get 30 percent of the 44,000 kilo joules of energy. What you will get? Now, drive your car, that is what it means and of course, we know the gamma of air which is 1 point 2 kg per meter cube and this if I convert, so this basically gamma of this. So, this basically becomes 800 kg per meter cube. So, we have this information. Now, how do we go about? So, the question is even the fuel and money saved per year, you know if I replace the flat mirror with the hemispherical back mirror, simple.

So, now another interesting thing is that several experiments have been done to calculate the coefficient of drag, experimentally taking various shapes, you know volts are a source of shapes. So, they are available to us; that is available from literature as if you know consult an advice standard it should be able to get that. So, from that now C D is given to be C D for a flat disc like this is given to be 1.1 and for this is given to be 0.4. That is C D, OK.

(Refer Slide Time: 17:02)

The image shows a handwritten solution to a physics problem, likely from a presentation slide. It includes diagrams of a flat mirror and a hemispherical back mirror, and calculations for drag force, work done, and fuel efficiency.

**Diagram 1: Flat Mirror**  
 A flat mirror is shown with a velocity vector  $V_0 = 95 \text{ km/h}$  pointing towards it. The drag coefficient is given as  $C_D = 1.1$ . The diameter is  $D = 0.3 \text{ m}$ . The drag force is calculated as  $F_D = C_D \left( \frac{1}{2} \rho V^2 \right) A$ .

**Diagram 2: Hemispherical Back Mirror**  
 A hemispherical back mirror is shown with a velocity vector  $V_0 = 95 \text{ km/h}$  pointing towards it. The drag coefficient is given as  $C_D = 0.4$ . The diameter is  $D = 0.3 \text{ m}$ . The drag force is calculated as  $F_D = C_D \left( \frac{1}{2} \rho V^2 \right) A$ .

**Calculations:**

- Total dist. = 24,000 km/yr
- Avg. speed = 95 km/h
- $f_{\text{pet}} = 0.8 \text{ kg/l}$
- Price = 100 Rs/l
- $f_{\text{air}} = 1.2 \text{ kg/m}^3$
- $HV = 44,000 \text{ kJ/kg}$
- $\eta = 30\%$
- Area  $A = \frac{\pi D^2}{4}$
- Drag force  $F_D = 1.1 \times \left[ \frac{1}{2} \times 1.2 \times \left( \frac{95}{3.6} \right)^2 \right] \times \left( \frac{\pi \times 0.3^2}{4} \right) = 6.1 \text{ N}$
- Work done to overcome  $F_D$ :  $W = F_D \times L = 6.1 \times 24,000 = 1,464,000 \text{ kJ/yr}$
- Fuel efficiency  $E_{\text{fuel}} = \frac{W}{\eta} = 4,880,000 \text{ kJ/yr}$

Now, when drag force, we know if I have to calculate the drag force. So, that is  $F_D$  that is  $C_D$  into half  $\rho V^2$  into area, where this area here is basically the frontal area for both geometries. What is frontal area mean? So, essentially you know if one were to look like this, you know if one were to basically the top fuel you know. So, if you look from the top, it does not matter whether you are seeing you know hemispherical back or whatever, we will see you know circle. That is what you will see. So, therefore for both the geometries which is given  $A$  is equal to  $\pi D^2$  by 4 this is same for both.

So, now, let us go ahead and find various things for the flat geometry now. So, what I am going to do? So, this is for the disc mirror or say flat disc mirror. So, if I do that, this  $F_D$ , is equal to somewhere use this  $C_D$  is 1.1 and then, half  $\rho$  is 1 point 2 Velocity. Somewhere you convert that to meters per second. So, to do that I divide that by 3 point 6 half  $\rho v^2$  into areas which is  $\pi 13$  centimeters which is  $\pi d^2$  by 4. So, that gives me basically 6 point 1 Newtons. So, that is as simple as you can see. So, essentially for to calculate the drag, total drag all I need is the area, the dynamic pressure and the  $C_D$ . Now, like I said the  $C_D$  is available in literature and there has been you know done over a period of time using our experiments, we can use that. So, for given a flat disc, just that the area is given, the dia given to 13 centimeter. So, I can actually use that you know here. So, I can calculate the area which is this dynamic pressure and the  $C_D$  here. So, what I get is 6.1 Newtons.

Now, our objective here however is to find out the amount of fuel and money saved if I use this, if I use the hemisphere instead of the flat plate; So now work done to overcome this drag force. So, basically work done to overcome  $F_D$ . So, let us call that is  $W$ . So, that will be force into displacement. So, that is basically force into displacement which is equal to 6.1. So, it basically runs for 24,000 kilometers. Therefore, I can write that as what I get is this, kilo joules per year. So, it runs a total of 24,000 kilometers per year. So, that is given in the problem, so that I get  $W$ . So, this is a total amount of work which it does.

So, what is the energy required for that? So, energy input is basically  $W$ , by the efficiency. So, because you know that the efficiency is only 30 percent, you need to give more. So, 30 percent of the total energy will be the actual work output. So, the output is

given to you some kind of calculate the input, so  $W$  by  $\eta$ ; this is 0 point 3 basically. Then, that comes out to be 488,000 kilo joules per year. So, now, we should be in a position to calculate the fuel. So, I know the energy input. So, then I should be able to calculate the amount of fuel.

(Refer Slide Time: 22:20)

Work done to overcome  $F_D$ :

$$W = F_D \times L = 6.1 \times 24,000 = 1,46,400 \text{ kJ/yr}$$

$$E_{in} = \frac{W}{\eta} = 4,88,000 \text{ kJ/yr}$$


---

Amount of fuel:  $\frac{m_{fuel}}{\rho_{fuel}} = \frac{E_{in}/HV}{\rho_{fuel}} = \frac{4,88,000/44,000}{0.8} = 13.9 \text{ L/yr}$

Cost:  $13.9 \times 100 = 1390 \text{ Rs/yr}$

So, the amount of fuel is the mass of fuel by the density of fuel. So, this is the energy input by the heating value and the rho of the fuel. So, that is equal to 4 and rho is 0.8 and what I get is 13 point 9 litres per year. So, what I get is 13 point 9 litres per year, so that is the total amount of fuel. So, hence the cost that I incur is that is also given, so 13 point 9 litres into 100. So, because we said that price is 100 rupees per litre, so 1390 rupees per year. So, that is basically the cost and the amount of fuel.

If I use the flat disc, now the question is that what if I use. Now, the thing is you could instead go ahead and do the same thing for the hemisphere and find out. Do the same thing and then, we can answer the question that how much do I save. I just do a little more. So, basically drag force, and the total work done to overcome in is directly proportional to the coefficient of drag. So, you know from seen even this picture here, you can see that the drag coefficient of the flat disc is 1.1 and it is 0.44, the hemisphere. So, I am going to just use that stuff. So, basically we can say that the percentage



reduction you know in fuel consumption is related to the percentage reduction in the coefficient of drag, so I am kind of use that.

(Refer Slide Time: 25:21)

Handwritten calculations on a digital whiteboard:

$$\text{Amount of fuel} : \frac{m_{\text{fuel}}}{\dot{m}_{\text{fuel}}} = \frac{E_{\text{in}}/AV}{\dot{m}_{\text{fuel}}} = \frac{4,00,000/44,000}{0.8} = 13.9 \text{ L/hr}$$

$$\text{Cost} : 13.9 \times 100 = 1390 \text{ Rs/hr}$$

$$\text{Reduction Ratio} = \frac{(C_D)_{\text{disc}} - (C_D)_{\text{hem}}}{(C_D)_{\text{disc}}} = \frac{1.1 - 0.4}{1.1} = 0.636$$

$$\text{Fuel Reduction} = 0.636 \times 13.9 = 8.84 \text{ L/hr}$$

$$\text{Cost} = 0.636 \times 1390 = 884.04 \text{ Rs/hr}$$

So, I am going to use a calculator. The reduction ratio in the C D and when I do that basically means C D of the disc, C D of the disc minus C D of the hemisphere by C D of the disc. So, that is basically 1.1 minus 0.4 by 1.1 and that comes out to be this. So, therefore fuel reduction. So, fuel reduction is 0.636 into what the amount of fuel which is 13.9 and that comes out to be 8.8 litres per year. Hence, of course cost reduction due to that is 0.636 into 1390 which comes out to be 884.04 rupees per year. So, that is interesting as if I do to like that basically. So, what I did is basically I calculated you know the ratio. So, I think what is important for you to understand that fuel and money input of energy etcetera is all connected to the coefficient of drag. That is very important and we have a literature available to us experimentally calculated several values of C D which is available for us to use.

So, this is in a practical sense or you know where we can use this. So, all I did here is cal. So, I used that information, very infer information. So, in all I have to calculate the percentage reduction. I want to calculate the reduction in the C D, rational C D. So, based on the C D I calculate the reduction ratio. I am trying to emphasize that. So, then

all I do is just you can see is less than 1. So, when I go from the disc to the hem, the reduction ratio is this. If I do that, then all I do is then multiply the reduction ratio with the amount of fuel in the cost and then, what I get is this.

So, therefore, I have an advantage that I use, I end up using less fuel and hence, you know less money plus again of that by using a hemispherical back mirror. So, that is another you know nothing to drive from the point that, how we are kind of using the concept of drag and how do we go ahead and calculate it you know and what sort of formulas are available to us.

So, I am literally also we can do this and also, you know we also looked at if we can use a momentum you know theory to come up with something like this.

(Refer Slide Time: 28:57)

The image shows handwritten mathematical derivations on a whiteboard. The top section defines the drag coefficient  $C_D$  as the ratio of drag force  $D$  to the dynamic pressure  $\frac{1}{2} \rho V_\infty^2 A$ . The drag force  $D$  is given by the integral  $D = \int_{A_H} u_2 (u_1 - u_2) dy$ . Below this, the velocity profile is defined as  $u_1 = V_\infty$  and  $u_2 = u(y)$ . The drag force is then expressed as  $D = \int_{A_H} u (V_\infty - u) dy$ . A boxed equation shows the velocity profile  $u = V_\infty \left( \frac{y}{\delta} \right)^n$ . The bottom section shows the calculation of the drag coefficient  $C_D = 2 \int_0^{\delta/c} \frac{u}{V_\infty} \left( 1 - \frac{u}{V_\infty} \right) A \left( \frac{y}{c} \right) dy = \frac{1.328}{\sqrt{Re_c}}$ , with the boundary layer thickness  $\delta/c = \frac{5}{\sqrt{Re_c}}$ .

You know the drag and we come up with something like that. If we have a proper you know expression for the velocity profile, we should be able to get the drag analytically. On the other hand, we are also using formulas like 1.328 by you know the Reynolds number and this is dependent on the Reynolds number at these points. I would say that you know the drag; I mean you have Reynolds numbers less than 1 when we have flows.

So, those are basically creeping flows and it is of very low Reynolds number and there the universal forces are almost negligible.

So, the  $C_D$  there is also inversely proportional to the Reynolds number. So, it is like  $24$  by  $Re$  and then of course, you have slightly in the laminar range who is slightly Reynolds numbers till before like we said in around 5 million and that is in the laminar range. After that of course you have turbulence range and the  $C_D$  in is the drag is much larger when you have turbulent flow.

So, I think we will stop here and go onto other things from the next module. So, this is basically the drag and the drag is something that we cannot estimate over you know visualize without considering viscosity and hence, the boundary layer. So, this is not something that we can do with in-viscid theory. So, we will stop here and take it up next time.

Thank you.