Introduction to Boundary Layers
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Module – 03
Lecture – 11
Skin Friction Coefficient-I

Hi. So, welcome to the next module. So, we are going to continue to talk about drag a little bit. So, we did you know derive some formulae in the previous module in terms how to evaluate drag in terms of velocity. Now let us see what this exactly mean. So, if you (Refer Time: 00:48) so this is the value that we were, this is the relationship which we developed last time.

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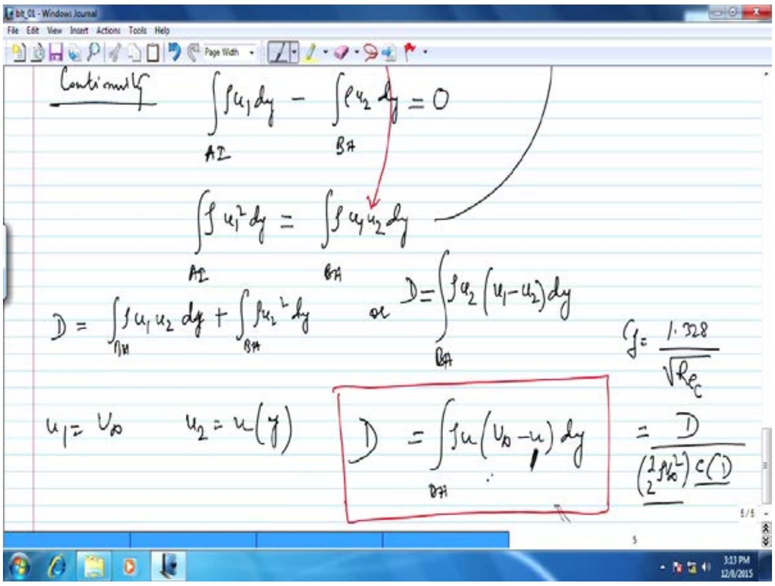

So, we said basically if you are able to now given expression for this velocity then one should be able to estimate the drag quite well. So let us see, let us sort of take a formula and see take a little bit of problem and see if there is been possible. For example, let us take an expression for this u right, this velocity this is the free stream velocity and see if that kind of even works.

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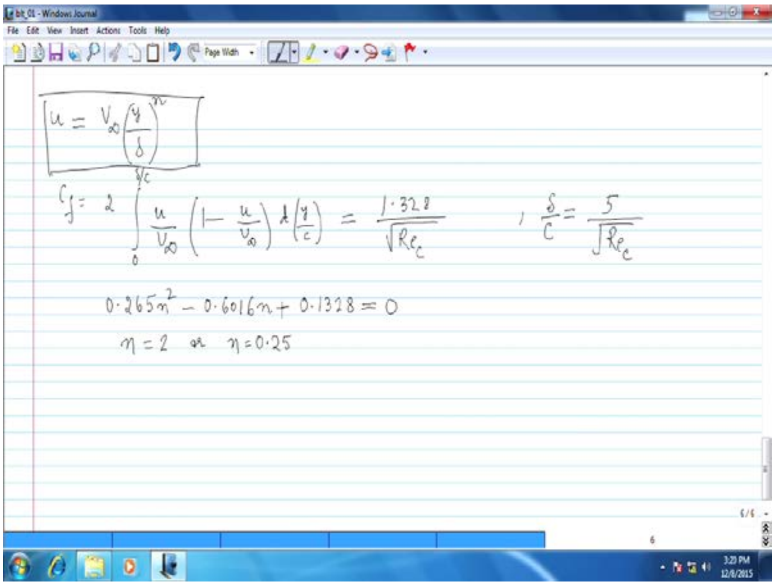

Now, say we have a definition of u, which is something like this. So, u is, so this n is basically a power. In the sense that the velocity is obeying some kind of a power law as you can see in terms of and you can see that when y is equal to delta; delta is usual boundary layer thickness. So when y is equal to delta, so u is equal to v infinity so that kind of follows that. And we are looking at laminar flow. So, if this is a kind of thing that we now looking at, this is a kind of velocity we looking at. Then the coefficient of skin friction, so like we did come up here. So, we are using this same definition which we have used earlier which is this, this bit. So, we are using the same bit, so it is 1.328 by R e based on code. So, this here is basically equal to drag by the dynamic pressure into the area. So, in this case unit one is the unit depth into the plane of paper and c is the code line, that gives you basically the area, so that is c f.

Now for the drag, we have this expression basically this is the expression which we are going to use. So, which is what is I am doing and I am going to kind of miss a couple of steps and I leave you to sort of work through that. And what I get is something like this; the expression that I get is something like this. So, c f is 2, so it goes basically from zero to delta by c 1 minus u by u infinity d, basically we are non-dimensionalizing the lengths by c by the code length you can see. So, you got delta by c and y y c. So, if I get this and this whole thing is equal to 1.328 by under root of R e of code. This is equal to that and also we know that if you already note pointing that out delta by c is something that we have developed earlier. So, 5 by under root R e of c. We are going to use these definition as well. So, if I do that then this what I get from here is basically this equation 265 n square minus 0.6016 n plus 0.1328 is equal to 0. So, this is an equation which one needs to solve.

And if I do that I get two values right it is a quadratic equation. So, I get two values of n which is, n is equal to 2 or n is equal to 0.25 actually (Refer Time: 06:17). So, n is equal 2 or n is equal to 0.25. So basically, now I can draw the velocity profile, I can draw the velocity profile for two cases; one is 2 and one is a 0.25. So, if I use that here in this expression here, in this definition of the power law that we were using you should be able to plot this.

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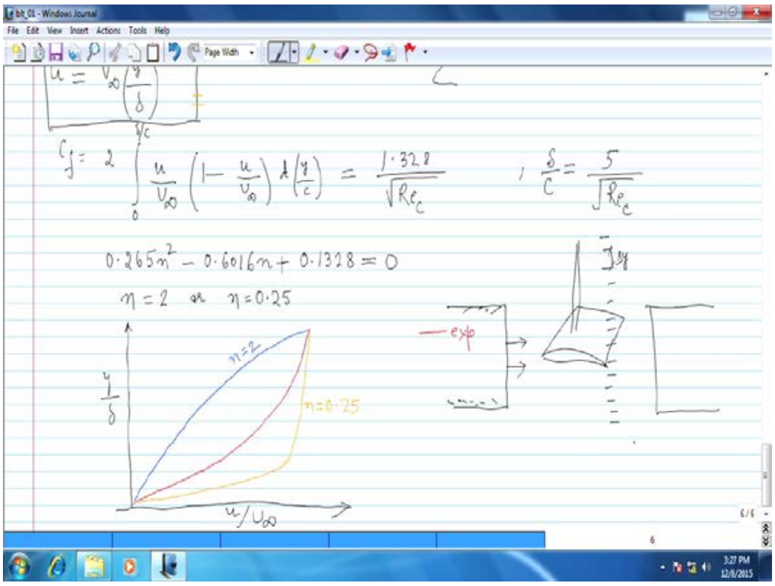

Let us see I mean this is a drawing which I will do not to (Refer Time: 07:04). So, you will have to bear with me. So, say if I were to do that, this is my these lines will need to be straight and this is not coming out to be straight. So, this is say y by delta and this is u by u infinity. If this is so, now this is not 2 scale. So, this is what this is basically from experiment let me write that here, so this is what we get from experiment. And what we get from here is something like this, something like that. And this one right this one is for n is equal to 2, this value; and say the other one is something like that and that n is equal to 0.25.

So, basically what one can say from this plot is that this actually does not work too well. This power law that we came up with to give into took take any value of u that is not holding up too well you know given this. Because this quite far apart from the experimental value of that. So, one will have to think about a better way of doing this. So that is one thing to show and that is also another way to let sort of derive from the point that when you come up with the theory how do you go and cross check whether you know that is actually even working or not.

Now the interesting thing is that we like when I sorted this I said you know you could sort of small experiment to do that. In fact, some of our students of you know actually done that, and what is interesting is that we did you know the talk about of wind tunnel and having a model there. Now it will be very interesting that you know. So therefore, what we see is that if I have an analytical expression for this velocity here then I should be able to get an estimate drag. So, I tried this it is seem to work very well again something else might, you have to give it trying yourself.

But there is a another way. For example, if you are you know in say the wind tunnel. So, this is say your wind tunnel. So, you have a test section the kind of a wind tunnel that we have so it is basically something like this. So, we have an open test section. If I were to do that, so something like this we will have and there is a fan here and so and so forth. So that is the one which blowers and this is a distraction terminal. Say if I hang you know a wind model or you know truly model depends on what you going to use. So, say if I do that, say if I draw a wind model like that. So that is basically in the test section and this is going to be hang from the top, it is going to be mounted like that. There is a mechanism on top to do that and you can change the angle of the drag. As you can see here actually now, so actually this is probably the wrong way. So, what I mean is let us erase this bit here ok.

So, basically I mean the flow is going to coming from this way. It is going to coming and form what you can see here that there is seems to be little bit of an angle of the drag does not matter. So the point here, is that one could actually you know take a little bit of a traverse and you could measure velocities at different locations like that as you go on top and bottom of this wing, you could actually take locations and just measure this, measure it. And then that value when you input into your equation here so that should basically then you write this in the form of summation. I will let you do that go ahead and write that. So, then you go step by step in small steps. So, you get this let this be said delta y. So, you go step by step and you calculate the velocities.

Now the question is do you directly get the velocity here or you get somewhere else, I am not telling you that. So, try and think about that. This is interesting way of doing this experiment, which is students can do. And you can actually set this up and get a feel for the drag across the body. And it could be any body it does not have to be you know wing or anything and it could be anything. So that is one of the things that I wanted to talk about. Also what I think I kind of did not mention or forget to mention, so what are about to mention. So, what we were talking about the previous module. Basically, what I if I were to sum this up, so drag is a phenomenal which is basically contributed by pressure as well as shear forces. And most of the times it is you know difficult to differentiate the two pressure drag, form drag and is something these is resulting out of the skin friction. So, usually you end up calculating the total drag. So, what you going to calculate here is total drag. So, it is difficult to kind of differentiate the two.

Now, as we also talked about the (Refer Time: 14:33) paradox. So, we used in inviscid theory, we use in inviscid theory or let us say the inviscid theory is sufficient to calculate effectively calculate the lift. And the important thing is that the shear stresses, when you integrate that in the lift direction do not have a significant amount, they do not have a significant component. So therefore, the lift is primarily because of the pressure distribution, which we can evaluate you know fairly well with inviscid theory. Have that theory fails if we you know try to calculate the component of that in the direction of the drag and hence the paradox. Then you have to take refuge in the viscous theory and hence I come for a boundary layer to come for the drag, all right.

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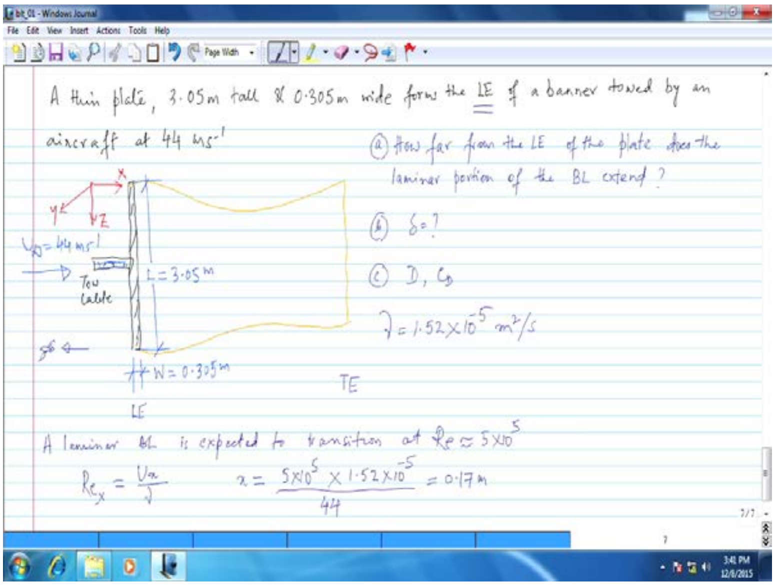

So, now let us look at another example, and see if we can, this is also another way kind of slowly it should give you this whole idea, it should give us some idea as to how we will use this knowledge or where what the cut type of problems we are trying to solve know that kind of trying to understand this physical phenomenon. Now say this is an example. Now let say we going to put it like this. A thin plate 3.05 meters tall and 0.305 meters wide forms the leading edge of a banner towed by an aircraft at 44 meters per second. So this is the tow cable and this is the thin plate, which is 3.5 meters wide and 0.305 meters wide, 3.05 meters tall. And it is got a banner. So, basically I can say that is got to banner something like that. You seen these one right so this banners keep flying and so on. So, you got a banner and it is got a banner like this and essentially so this height this is l, this is 3.05 meters tall, and this is the width. So, this is the width which is 0.305meters.

This is what we have, and this parts is the tow cable, this is the tow cable. And you have a free stream at 44 meters per second which comes in right what we are going to draw here, we can do that later but let us just do that for now. So, I am going to draw an axis system, so this is x, this is z and this is y. So, this is y and this is x. This is basically that picture that we are looking at. Now the question is, how far from the leading edge of the plate does the laminar portion of the boundary layer extent? that is first question. The second is, what is the boundary layer thickness at the downstream end of the laminar boundary layer? So, what is the you know I am not going to write the whole thing. So, what is the thickness at the downstream end of the laminar boundary layer, so I am just going to write that what is delta. And number c is, find the drag force on the plate contributed by the boundary layer and the corresponding drag coefficients. So, basically find the drag and the c d. So, find the drag force on the plate which is contributed by the boundary layer and the corresponding drag coefficient which is given is kinematic viscosity which is 1.52 (Refer Time: 21:49) so that is basically what we looking at.

So, that is basically the question. So, you know there is this a plate here to which a banner is attached and these thing is being towed by an airplane just like that. So, what we basically therefore, so when we said if this plate forms the leading edge of the banner. So, this is the leading edge of the banner, this is trailing edge of the banner and since the airplane is actually towing it in this fashion. So, the airplane is moving this way, the airplane is moving this way so that you know we can say that the incoming velocity is basically from 44 meters per second and it is in this direction that is the free stream. So, if I do that, so now how do we sort of go about this.

Now a laminar boundary layer you know is expected to transition at around 0.5 million Reynolds numbers. Because now the question is like I have repeatedly said earlier also that we going to use some very ready to use formulae calculate lot of things delta so and so forth, but it is very important that you know that what regime a flow on u m. In the sense that, is that laminar or turbulent and use formulae accordingly. Because that really determines the drag is very different if you consider turbulent flow. Or for the for that matter you know all the boundary layer parameters will be very different if you consider turbulent flow instead of nominal flow. So, it is very important that we understand that.

So, although this is part of the question in this problem that how far from the leading edge of the plate does the laminar portion of the boundary layer extend, but that is something that one must cross check or you know ask every time you do a problem like that. In this particular case, a laminar boundary layer is expected to transition we kind of know this right at a Reynolds number of this right. Therefore, now Reynolds number I can writes, say some length u x by mu right. In that case here, so we what we known here is we know this, so in this formula what we know is the velocity free stream mu is given and R e x something that we know right. So, we can find out the corresponding x from there. So, here x is therefore, equal too so 5 into 10 to the power 5 that is Reynolds number into nu which is 1.52 into 10 to the power minus 5 by the free stream which is 44. And what we get is 0.17 meters. So, how far from the leading edge does the laminar portion of the boundary layer extend? Well, till about 0.17 meters right which is kind of less than half of the total width, if you want to see this.

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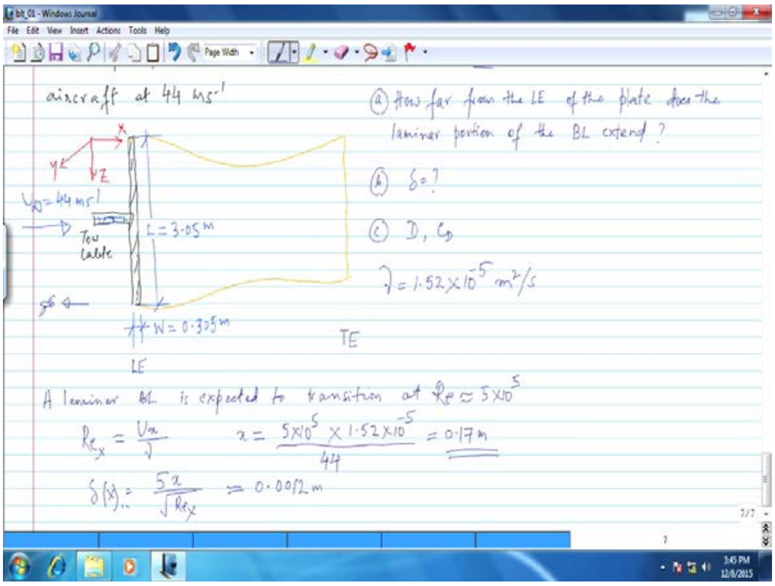

And the next question is, what is delta? What is the delta at the downstream end of the laminar boundary layers, so at the edge of the laminar boundary layer. And the way we will do that we know that right. So, delta the formula for delta right x is basically that is this right. So, delta x, where we get this x from because you remember the formula is basically delta by x right depends on that so if I do that then what I get is 0.0012 meters. So essentially, we have a laminar boundary layer extending up to 0.17 meters away from the leading edge of the plate, and the maximum height of the boundary layer is 0.0012 meters.

Now if this is so what is the third question, so what we need to now do is calculate the drag and the c d. So, let us continue to do that in the next module. The only I would like to pull your attention to here is a this a little different right from the plate that we were considering so far the flow was kind of always coming, you will have a plate like that and the flow would somewhere sort of come like this and then the boundary layer will developed something like that. But right now that is not what it is, so what we see here is not that. So, you got a plate like that. And there is a free stream, which comes like this sort of say perpendicular to the plate. What we have said it so far is that the direction of the velocities is kind of parallel to the plate is not it, and that is what we say we keep a plate parallel to the direction of flow. But that is not what we doing here, it is actually perpendicular to it. So, please sort of you know note that.

And what we are therefore, now saying that a boundary layer is kind of developing this way you know over the plate. There is very little plates to develop with you know the width of the plate is like 0.305. Then there is this part you know which is kind of flexible. So therefore, it is important to find out that how far the laminar portion of the boundary layer extends and what we see is that lines within the plate width which is 0.305. So actually, we are looking at this is very small little you know distance and in that the maximum the delta x, where we take the x to be 0.17, which is the edge of the kind of laminar flow and that turns out to be 0.0012. So, about what, about 1.2 millimeters right. So, let us come back and then you know calculate the drag and the series. So, I will close this here.

Thanks.