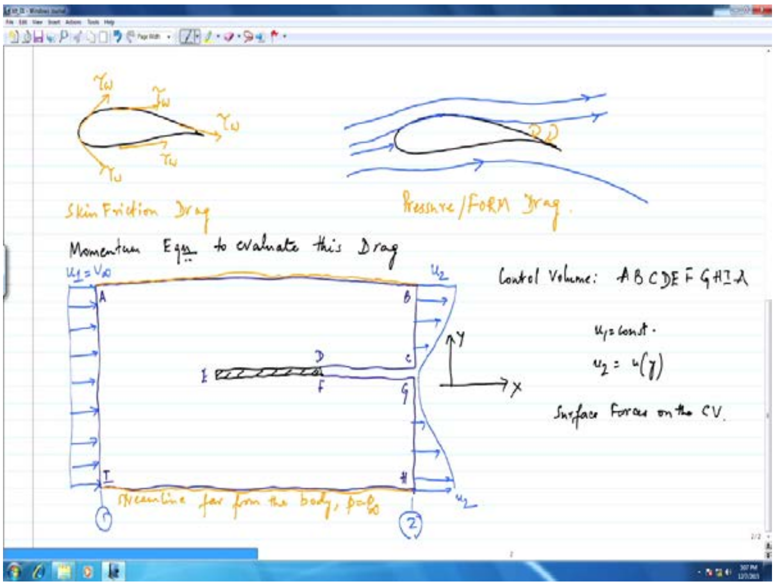
Introduction to Boundary Layers

Dr. Rinku Mukherjee
Department of Applied Mechanics
Indian Institute of Technology, Madras

Module – 02
Lecture – 10
Concept of friction drag

Hi, welcome back. So, let us continue using the momentum equation. Now, here so, let me draw an axis. So, this is what we were doing.

(Refer Slide Time: 00:24)



You have this flat plate which is DEF, and we have taken a control volume around this which starts from A and it runs just inside a streamline which is very far away from the flat plate. Essentially this streamline is looking at free stream. So, we draw a boundary just inside that streamline which is AB. We come down and we run far enough on the off side of the flat plate like on the downstream side. We go far enough and we come down, we make this cut here at, so that we can run back to the flat plate, run over the surface and wrap ourselves around the flat plate, come on F, come here back at G which is exactly below C. We become the cut and then, we come down again at H and again run along just inside another streamline which is very far away from the body and we go back and close the volume, so basically a control volume which is A, B, C, D, E, F, G, H, I, A.

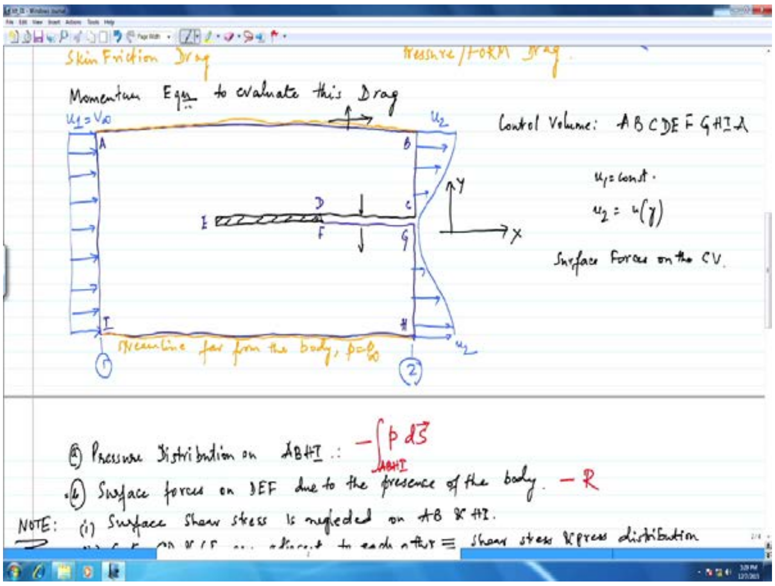
So, we go from as you see from A, we go straight down here to B, we come down to C, and we go along D, and then we walk along the surface like that, touch the surface completely and then, go back here at G, come down, go inside this free stream surface and then go back to A. So, essentially that is my control volume and the access system that I am looking at is something like this. So, let say this is X and this is Y of course. So, as we know from experience so far that we have taken basically two sections 1 and 2 and U 1 is constant.

So, basically what you saying U 1 is constant while U 2 is a function of Y. So, I think this is something that way kind of we know used now. So, now what is interesting to staying is the way we would like to calculate a drag is basically look at surface forces on the control value. So, in order to, because that is our purpose, we want to evaluate drag. So, what we are going to look at is surface forces on the control volume. So, in order to do that let us instead of go and derive, number one it will come from two things here. So, that basically will come from pressure distribution on AB. What does that mean?

So, I say pressure distribution on AB HI. So, AB is essentially the top boundary of the control volume which is inside, the far end streamline and HI, is the same on the other side. So, we will be looking at the pressure distribution on AB HI. Why are you not looking at anything else? Well, we will come to that and the other things; there will be surface forces, on DEF due to the presence of that body, in this case the flat plate. Now, please try to understand here that when I am talking about DEF, I am not talking about the flat plate. I am talking about the control volume. The DEF part of the control volume actually wraps around the flat plate. So, we have taken the control volume in such a way that AB is just inside or just in sync with a streamline which is far away from the flat plate, on the top side and so is HI on the bottom side and DEF is that part of the same control volume, which is running like a thin skin on top of the wrapping, basically the flat plate, which is DEF.

Now, what do we need to note here? So, what we note here is let us instead of this here, what I will note number one, the surface shear stress is neglected on AB and HI. The surface shear stress is neglected on AB and HI. Now, why is that? That is because these two boundaries of the cut, these two possible control volumes are far away from the flat plate and it is basically of the free stream, and the viscosity effects are negligible, that is far end.

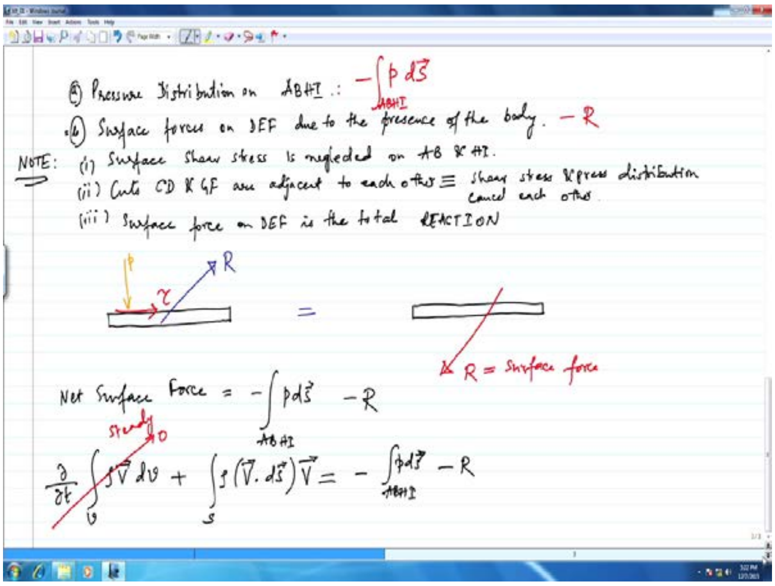
(Refer Slide Time: 07:31)



The second thing is that, the cut that we made in the control volume BCD and GF, so these cuts CD, sorry. So, CD and GF are adjacent to each other. What does that mean? It is basically side by side of the same lines you know to each other, such that the shear stress distribution as well as the pressure distribution I, is equal and opposite and hence, they will cancel each other. So, this basically means that the shear stress and pressure distribution cancel each other. Let us just say cancel each other because you can just instead of think about if I want to just quickly do this here. So, I could have a pressure, acting like that in a pressure acting like that.

So, these two, 1 and 2 that is acting on CD, and this at 3 and 4, these are acting on F and G, FG. So, these will basically cancel each other out. So, therefore they do not cut each other which essentially mean sheer stress pressure and distribution cancel each other out.

(Refer Slide Time: 09:44)



Now, the other thing is that the surface force, now basically what we looking at is this surface force. This surface force on DEF, is basically, is a total reaction which is equal and opposite to the; now on these cuts, however, why this sheer stress pressure distribution, cancel each other because they are equal and opposite.

Now, if there is going to cover some kind of surface force on DEF. So, on DEF there is going to be a force which is going to force by that, then when I say surface forces and DEF is basically a reaction, is a total reaction due to the presence of the body. Now, because of the presence of the body, there are surface forces. So, here when I write B here, when I say surface forces DEF, so due to the presence of the body which basically means that this is like a complete reaction to the presence of the body.

So, what is the presence? What is the body doing by is presence. Well, it is basically causing sheer surface forces, due to sheer stress and pressure distribution. So, what I mean? So, if I would write that, the surface force on DEF is the total reaction. This is actually a reaction, total reaction equal and opposite to the shear stress and pressure distribution could by the flow over some number. All of that is a total reaction to the shear stress and pressure distribution caused due to the flow of fluid over that surface of to the body.

So, instead let me just sort of write this down a little bit, so that will make sense. For example, what I mean is that I have this surface. This is my surface, this is my flat plate. Now, on this, I am just drawing at one point just still get this clear and something like this,. So, this is the shear stress and this is the pressure. This is being caused due to the flow of fluid. Now, if I sum all this up, you can see this. So, I have pressure and effect. So, if I put the effect of both pressure and the shear stress, so I am going to just say that is going to be a reaction. Let us call that as R per unit length. So the depth is into the plain of a paper. So, per unit length, so, therefore with my surface force, what is my surface force then?

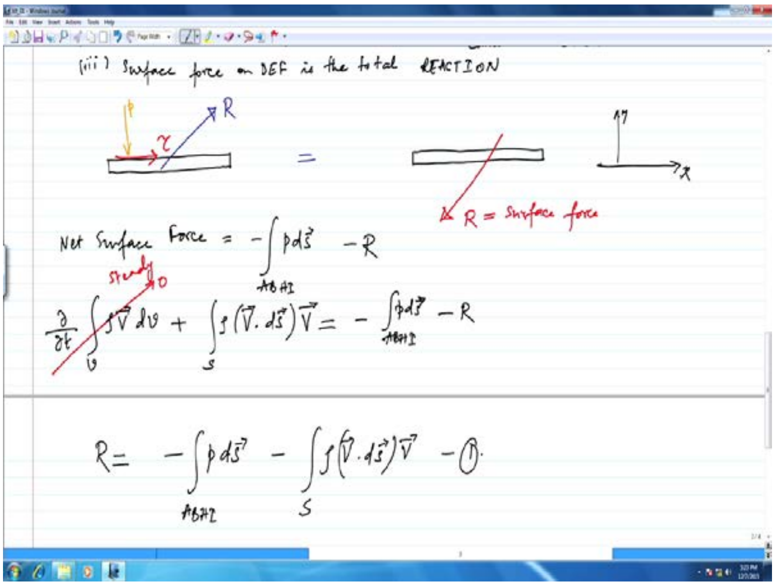
Then my surface force is essentially, this is the force which is being caused and my surface force or DEF is reacting to this. So, therefore that basically means is that there is an equal and opposite. So, this is also exactly equal and opposite to this R. So, this is basically the surface force on the DEF. So, all way, the reason I am kind of trying to say on this a little bit is because this surface force consists essentially of pressure forces and shear stress. So, when I do an integration of the pressure and shear stress among the surface, I get a reaction and surface force is basically, magnitude of that is equal to that. So, that is why this.

So, therefore now that we have talked, so when I say pressure force here, pressure distribution on AB HI, so AB HI, let us go back here. So, let me say that I am going to do integration. So, P and this is AB HI and I am going to add a minus sign what is that mean. So, P is basically the distribution of pressure on AB. DS is a small element along the part of the control volume which means I can take directive factor. Why do I have a negative sign because the pressure. So, this is DS is basically the vector pointing out of the surface. It is the vector direction of here. It is like the area normal and area in this case where unit you know depth, so DS into 1. So, DS is basically u unit link. So, DS is the small elementary link somewhere along AB or HI and the pressure is also directed opposite trail.

So, therefore, you have the negative sign. So, this is my pressure distribution and the surface force by this time, surface force essentially I can write this as minus R because this is acting opposite to this R. So, what I am going to therefore write now is that net surface force, is equal to; you just did this AB HI minus R. So, this R is essentially over DEF, so AB HI and DEF. So, now, what we going to of course we know that great of change of momentum is basically equal to force, we know that. So, these are the net surface forces. Now, if I write from the momentum equation as I would request you to just look up the entire momentum equation, in case you have forgotten. So, the momentum equation I would write that as delta t is a volume integral rho v. This is DV is volume, plus this is over the surface V dot this and that is equal to the total surface forces, which is minus R.

Now, of course we are going to look at a steady state case. So, this is going to get this 0. Why? It is because this is our case is essentially steady. So, therefore, I can write, let me write that R is equal to minus rho V dot DS end of the surface and let me call that is equation 1.

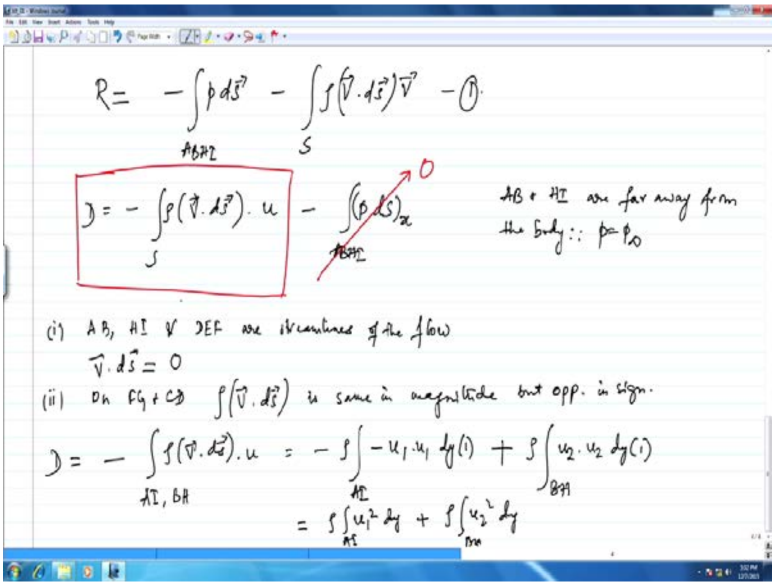
(Refer Slide Time: 18:46)



So, now we are basically going to do what the way we been doing so far, going at each. Now, this equation is a vector equation as you can see. Now, our main object here was to calculate drag, so this might as you can see, this is the total surface force. So, what we have been able to get this expression 1 using the momentum equation, and also understanding that this surface force is basically resulting at the reaction to the pressure and shear forces.

Now, this surface forces when taken component in the X direction, so we have drawn, this is my X and this is my Y. We have taken this is an access system. So, when I take the component of this reaction along the X that should be what is going to cause my drag. I forgot to mention something in terms of the lift. I will go back and mention it. Let me get over this. So, if I take components along the X direction, so that gives drag. So, if I just add what I will get is minus V dot DS into U, use in the x direction minus and all this means is that this second integral.

(Refer Slide Time: 20:46)



Here all these means is that you take the component of the pressure in the x direction, you integrate that and take the component of the pressure force in the x direction, Again now, AB and HI are far away from the body which we already mentioned. So, essentially what we have there is free stream pressure. So, if I just add now this if you need to convince yourself, if you just consult a standard book, you should get that. So, basically this as long as P is constant, this integral is going to reduce to 0 is going to integrate 0 as long as P is a constant and in this case it is.

So, therefore what we get here is this also tends out to be 0 because of this reason that they are far away from the body and therefore, the pressure that is essentially fixed impression. So, what we are left out with essentially is just this. So, this is essentially an expression form of drag. So, now what we are going to basically look at is break this down over the several boundaries that we have. Now, to in order to do that if you look at this, let me write that down here. So, each of these points, now AB HI and DEF which is wrapping around the body are streamlines of the flow; are basically along streamlines of the flow. So, what happens to V dot DS is basically that.

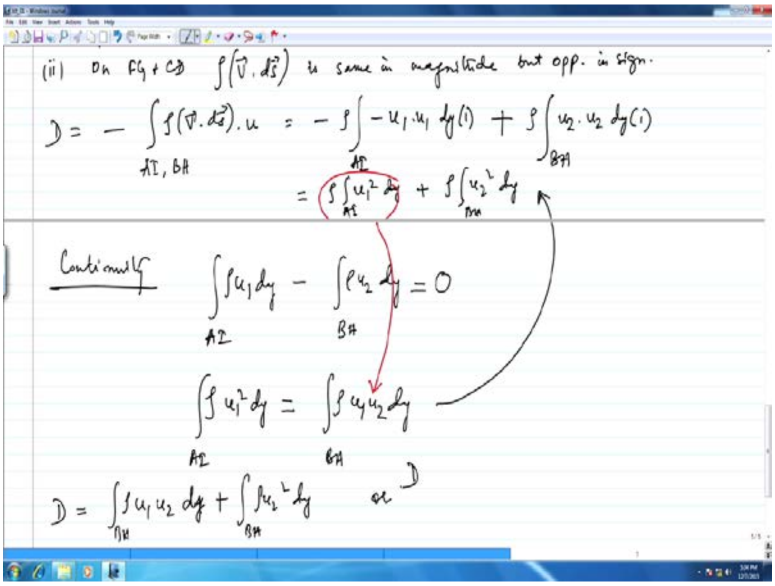
So, now, if you have the boundary, so if you look at this if you have the boundary, so DS is going to say directed somewhere like that, and your velocity is directed something like this. So, if we do a dot product, what you get? So, what we get is basically 0. So, DS is perpendicular to them. So, you can explain yourself, basically this is V DS cos of 90 which is 0. Secondly, again CD and FG, therefore from this integral from this integral here V dot DS is 0 for AB HI and DEF.

Now, again for D for the two cuts basically now for these two cuts, so CD and FG. Now, what you should see is that the mass flux is equal and opposite. So, you can whatever comes in here, is going to go out there. So, essentially the direction is opposite but the magnitude should be same. The magnitude is essentially same. So, therefore now because CD and FG is same, so what happens? Therefore, on FG and CD if you look at this integral rho v dot ds, so this is same in magnitude but opposite in sign.

Therefore, the kind of this is essentially nothing, but the mass flux. So, therefore this also cancels out. So, therefore, we have left out is basically this; so, on the boundaries AI and BH we are going to do this, V dot DS into u. So, we are going to do this only on these two boundaries. So, basically that is what we are left out with. So, AI and BH is essentially if you see here, if we go back, so what we left out with this AI which is the inlet and BH is the exit. So, if I do that, then what comes up with is here minus rho. So, V dot DS is minus u1 then, this also essentially is u1 and ds is nothing, but dy into 1. So, similarly this is for AI, this is the inlet and for this is A, what we get is rho and we get u2 into u2 and again dy into 1. So, basically what I am going to get here is essentially that.

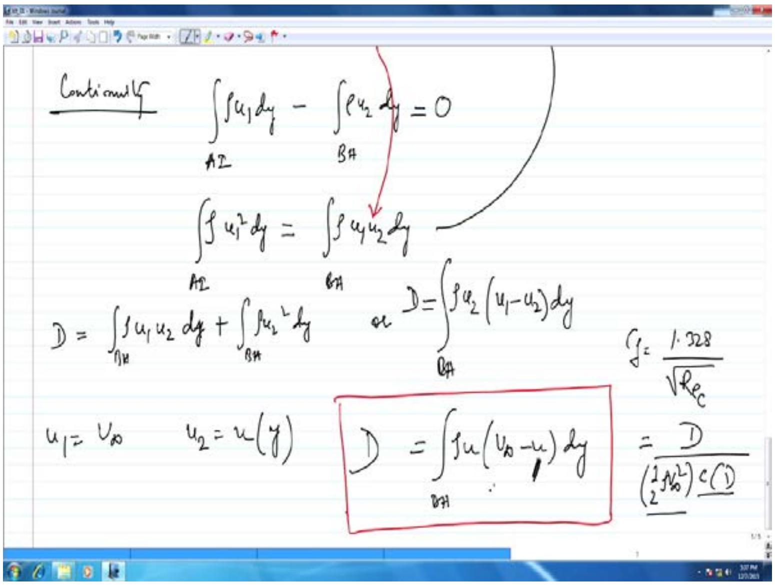
So, this will come down to rho u1 square dy. This is over AI, plus rho u2 square dy and this is over BH.

(Refer Slide Time: 29:14)



Now, what we get is, if we do the same thing from continuity if I look at continuity, what I get is rho u1 dy minus rho u2 dy. This is on the inlet and this is at the exit is equal to 0. Now, if I multiply by u1 throughout, if I do that, then what I get is rho u1 square dy AI is equal to limited is rho u1 u2 dy BH. So, this is what we get from continuity and we use this expression. So, we use this expression back to back into this. If I use this expression back there, then what I get is drag is basically going to replace this by this part. So, essentially what I am doing here is this plot is something that I am going to replace by this. So, if I do that, what I essentially get is D and the reason I was able to multiply by u1 because u1 is constant, we sort of proved that before, D is equal to this drag. Then, I can write that as essentially rho u1 u2 dy over BH at plus rho u2 square dy over BH, say or D is equal to rho u2 u1 minus u2 dy over BH.

(Refer Slide Time: 31:26)



This is again over BH. This is again equal to; if you see u1 is basically the free stream and let me just write the u2 as u. So, then this becomes rho u infinity minus u dy. So, the density is of course known. So, u1 is basically this which is the free stream, and u2 is u which is a function of y. So, this is an expression for D and what is interesting about this drag that now you can actually write this drag, if you look at this,. If you look at this, then you can write this drag in terms of velocity. There is no connection between this and the boundary layer things that we learnt so far.

Now, the only relationship let me just say is that when we wrote this skin friction drag, for lambda boundary layer is 1.328 by under root Re code, so this thing is skin friction drag is nothing, but this drag we do not mention this. So, half rho v square, this is u infinity square into c into 1. So, this is dynamic pressure and this is the area, which is see here is essentially D that we have just come out with here. So, this is lambda 4 flat plates. So, what you can see here is that now I can actually find out the drag and hence, the coefficient friction if I have some idea about the flow, this velocity profile at the exit, so we could do let of small little problem before we start the next module.

So, I will just stop here in kind of do that in the next module. So, essentially what we have done here and I talked a little bit briefly about the usefulness of this. So, this is basically the same, we got an expression for the drag in terms of the free stream and the velocity at the exit, which is something that is easy to measure. At least there is something I can measure and this is same drag I use to calculate the skin friction coefficient. So, I will stop here and I will continue this again from next class.

Thank you.