Introduction to Boundary Layers

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Module-01
Lecture-01
Review of Fundamentals of Fluid Mechanics-I

Hi, welcome to the Introduction to Boundary Layers. Like we said, we are going to do a brief overview of few mechanics, some fundamental equations and some concepts and the first two modules and let me start by opposing the question is that, a fluid can flow, a solid cannot, so let us kind of think about that a little bit. Now this thing is a solid, as you see you know this thing is a solid, this whole thing that I have here, this is basically a bunch of coasters and this thing is a solid and this cannot flow. You have a bottle of water, you move the bottle; the water in the bottle moves, there is in a flow exactly. It is not like you open your tap and the water flows, so what is it that, what is up with that of, you know fluid flows whereas solid cannot.

Let me do one thing, let me take out all these coasters from here; let me take this out keep this aside. This is like set of coasters, now these are separate as you can see these are all separate, now hold them together. Now say, apply a force then these things can move at the same time and let me do a thing, what I will do is; I will hold the bottom once, I will try to fix it and I would not let the bottom once move. So, when I apply the force what happens? So, I am not letting the bottom ones move.

What basically happens is that the top ones, the top layers move forward. So, this is nothing, but a flow. So, if you think of a fluid which consist of so many layers, then as it say moves over a surface; I am going to exaggerate this little bit, as say it moves over a surface which could be so many things. It could be a flat plate; it could be moving through a pipe, it could be moving over an airplane wing so on and so forth, so you know your fan blade.

So when that happens, basically you can consider the fluid to consist of layers like that, where one layer slides over the other and this is what causes flow. Now, if you was just look at it from this side and join these points, it might give you, so base join these points in the sense that with respect to the origin here, this has most slightly forward this is most slightly forward so on and so forth. So, if you can of draw this a little bit like that, it is something familiar, you are kind of familiar. Let us sort of talk about this a little more, So, this whole force that I am applying, this force which I am applying, which is making these layers, each layer to slide over the other this is nothing, but basically shear force, so it is the shear stress that we are going to look at.

So, let us see a little bit, how much your force can cause, how much of movement and how do you relate that in terms of a fluid, what is it; is it about a fluid property serve, what does that depend on. Let us sort of go and look at that little bit.

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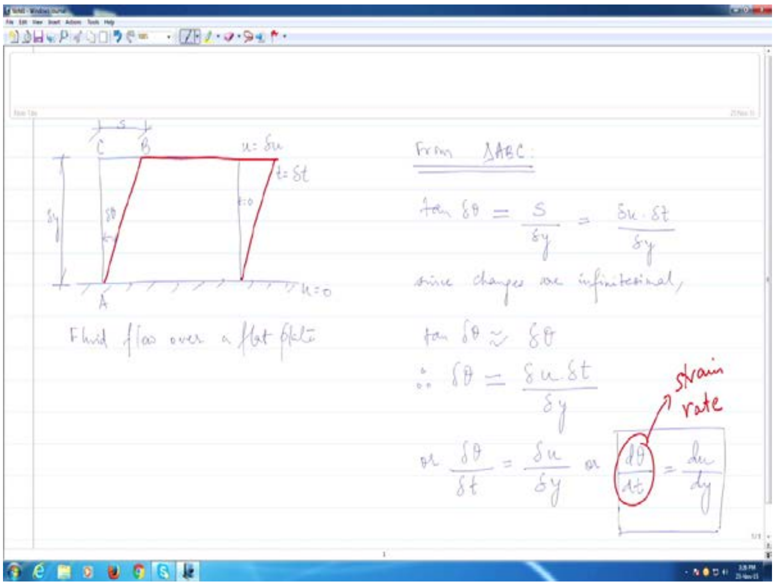

So, say we basically looking at say fluid flowing over a flat plate, so we got a flat plate. So, we got a flat plate like that and yes; this is stationery of course and so we have; let us just consider a fluid element like that, if I consider fluid element like that, so now, what we will do is that this fluid element however now is going to deform a little bit. How is it going to deform now as we just said, the bottom part of this fluid element is going to stick to this, the top part however, is go to move forward. So, you can think of this here again as you know layers, you can think of these as layers.

So, these layers are going to sort of move forward, so let us say that it deforms by something like this and by an equal amount here that; that. So, what we are going to do is let us go ahead and name these. So, let just say that this bit that we did, this is delta, theta; similarly here is a delta, theta. So, this blue thing, the blue is the basically the shape of the fluid element at t is equal to t naught, t is equal to 0 say right. So, this is at t is equal to 0 and this is essentially t is equal to at a certain delta t. Let us call this as A; this is B and this is C, and this distance, so this distance that we have this is s and this sorry that should be straight line.

So, that is delta y and the fact that this moves, it flows with respect to anything, the change and there is certain displacement happening over a time, there is certain velocity. So, let us say that velocity, this velocity is of course is equal to say we could call it delta u .This is essentially a fluid element, simplistic fluid element flowing over a flat plate, should I sort of write that, so this is essentially fluid flow over a flat plate.

Now let us look at the triangle, I am going to write it here. Now, if you see, so from the triangle ABC, what is tan of delta theta, can you see this is A, B, C, so tan of delta theta is this s by delta y, is that fine, s by delta y and this s is basically the entire displacement and like we said the displacement, so the corresponding velocity is delta u and the time during which this is happen is delta t. So, I can also write this as the corresponding velocity, time by delta y, is that right.

Now, of course here the changes are infinitesimal, so since changes are infinitesimal I got the spelling. So, what can we write we can basically say that tan of delta theta, I can write that as delta theta is it not. So, therefore, delta theta is equal to delta u, delta t, delta y or delta theta by delta t is equal to delta u by delta y, now what you can see here. Now, here of course theta and theta is changing only with respective time, theta change is own theta is a function of only time and u is a function of only y because we are not really looking at change of u with respect to the x axis.

Therefore, these partial derivatives I can actually write this as derivatives. So, therefore, this I can write this as d theta, d t is equal to t u, t y, I can write that. Now, that something we get, now what is this d theta d t. So, what you can basically see here that d theta d t is basically giving you a quantitative measure of how much the fluid element is deforming. So, therefore, this is essentially the strain rate, so this thing this, d theta, so this thing is essentially nothing, but strain rate, that is nothing but strain rate.

Now that we have here, so now, the next question is that what exactly decides the strain rate. What is the strain rate depends on, in the sense that I do have a fluid element. So, what helps me that how much will be the deformation or what exactly it is that depend on. So, let us sort of go to the next page.

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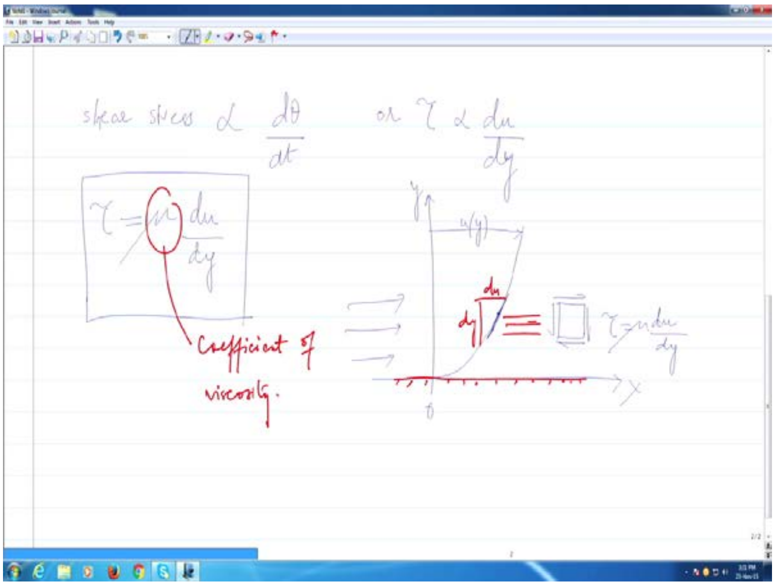

Now, as we kind of, as you must also be aware that shear stress is proportional to the strain rate. In this particular case, in the case of the fluid so d theta by d t is equal to d u by d y. So, therefore, this means that; so, I can write this or shear stress let us denote that by tau is proportional to d u, d y, which is basically the gradient of v horizontal components of the velocity in the wider action, horizontal component in the wider action.

So, this is where basically I think you are familiar with this, so we are going to introduce a coefficient; coefficient of proportionality or basically say the tau is equal to mu, d u, d y; d u, d y. So, this is essentially nothing, but, so this is something that I think you know this. This bit is nothing, but coefficient of viscosity, this is coefficient of viscosity, so simple, alright.

So, if I were for example so therefore, like I was saying you know the layers of the fluid, move with respect to each other. So, for example, say if I were to kind of draw a velocity, you know profile, if I have a velocity profile. So, say I go away from the plates this is say x, so I may have a plates somewhere here and this is essentially the y direction. So let us say I have a plate, may be resting some more like that, so what I am basically looking at is, in some kind of a velocity profile.

So, say the velocity profile is something like that and something like this and here so essentially this bit is nothing, but the velocity and this depends on the y; the y offset basically. So, therefore, now the thing is; if I take a point here and if I take a point here, so I am going to, I am trying to understand the slope of that, if I try to understand the slope of that and I can exaggerate that. So, I can basically say that this is d u and this is d y, so this here. So, this essentially means, the slope of this velocity plot as d u d y.

Now, this essentially, this basically means that, if I have little fluid, if I take consider a fluid element, so this is my plate; I have fluid coming in. So, this is my plate and I have incoming fluid like that, so at a certain point you know this; we just do this origin somewhere and the velocity plot kind of looks like that; velocity looks like that. So, where at some point we took the slope, which basically means that going away from the plate at this point, if I take a little fluid element then it would be having a shear stress you know shear stress which is equal to right.

So, physically that basically means that here if I have a fluid element, then the slope of the velocity curve, mind you the slope of the velocity curve will essentially give you a measure of the shear stress. Shear stress is basically the coefficient of a viscosity multiplied by the slope of the velocity profile. Now, set of let us just do just a simple problem before I go into something else. So, for example, let us go to another page.

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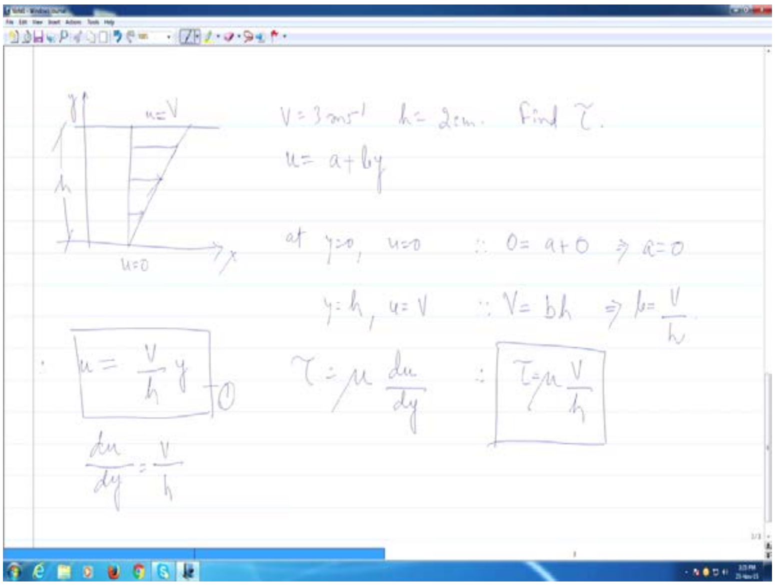

Now, for example, I have x is this all should be straight lines hopefully, but the end of this class I would be more good at doing this. So, say I have this is a very simple sort of plot, so this is a velocity plot again and this is like I said this is a velocity plot. So, this is 0 and this is equal to V; capital V, it is given and this is h, this height is h. Now, this V is given to be 3 meters per second, h is equal to 2 centimeters, the question is find tau.

So, how do we go about sort of doing this? Well, I will go about one particular way, but I think we can do another way as well, let you think about that. Now what I am going to do is find out an expression, I am going do it little bit like a mathematical way. I would like you to do more from your understanding of the definition of shear stress, so let me do that. So, here what I can see is that the velocity is a linear curve; it is basically a linear straight line, so it is equation of a straight line.

So, I am going to say u is equal to a plus b y; where a and b are constants, is basically a straight line, it is a equation of a straight line, then at y is equal to 0, u is also equal to 0, which means that means, 0 is equal to a plus 0, which means that a is equal to 0, then at y is equal to h, u is equal to V. So, this means u is equal to V, a is 0, we have already got that, so this is b into h, this means b is equal to V by h. Therefore, we can write u to be equal V by h into y right u, so this is u is equal to V by this, this fine.

Then like we just did, so tau; tau is nothing but mu; d u, d y. So, from here, from this say let us call this a 1, so from this expression 1, d u, d y is nothing, but V by h. So, therefore, tau is equal to mu; V by h, that is the expression. Now, I kind of did a little bit of math and I got this, but I would request you to kind of think about this from a more conceptual basis, like we just; we kind of, let us, we can go back like I said that if I had to find out these stress is basically equal to new d u, d y and this is a slope of the element here you know at this location and how shear stress is proportional to the strain rate. So, I would request you to go look at this; not sort of do this, a plus b y and not do it more mathematically, but think about that, find out may be the slope or something. So, find out that way, so do this, is the same exercise simple exercise, but do it anyways.

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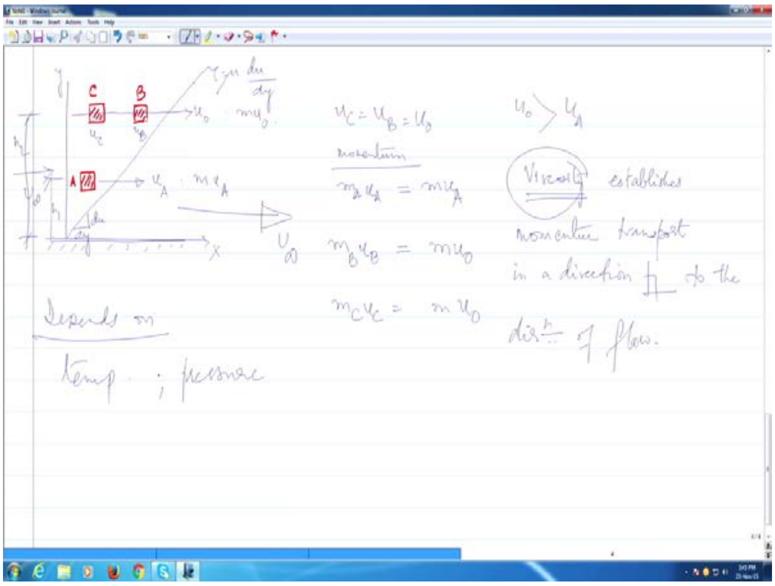

So, now there is something called Newton's law of fiction. Now this viscosity, we will come to the point that what does exactly. So, I guess we have just sentence the coefficient of viscosity is, what you can understand here that fluid flow is essentially is happening because of viscosity, so viscosity plays over important part and friction is something that we use more for solids and friction in fluid is basically a viscosity, so it is a measure right. So, like I said coefficient of viscosity is essentially the sheer stress, which is causing a corresponding strain.

Now, there is something very interesting here which is important to understand because what are the effects of considering viscosity? There are various fluid flow where we will not consider viscosity, the effect of viscosity is not dominant, so we can attempt to ignore that. But if we do then what kind of stuff exactly is you know; what should we know? Or what are the things that effects, so, that is what we kind of trying to discuss here.

So, now, say I am going to look at this that and this. Now again this is basically and I can think of that as the velocity profile and is that alright, fine right. Now let us consider, I am going to consider; a fluid element here and this fluid element is A, I am going to consider two more fluid elements here and this is B and this is C. So, now, this thing right, this thing is moving with a velocity u A, this is moving with the velocity u B, this is going with the velocity u C, essentially these two are moving with the same velocity u naught.

You have free strain velocity oh sorry, this is free strain velocity which comes in here. So, this is free strain velocity and you have a particle, so you have a fluid element A, which moves with a velocity u A. You have two more particles u C and u B, which move; which basically u C is equal to u B right which is, so u C is equal to u B; which is equal to u naught. Therefore, let us also now look at the momentum; the momentum of the three particles now then so this one, momentum of particle A is; n A, u A right and we are going to say that all the three particles or all the three fluid elements have the same mass. So, it is going to be m of u A, when there will be m B, u B and that is equal to m u naught and m C, u C is equal to m u naught.

So, therefore what we see is that, if this is a velocity profile, this thing that you have here is a velocity profile, this inclined line, this is your velocity profile. So, if you have a particle a, fluid element a, which is kind of closer right to the; so, you could probably have a plate or something here, you could have a flat plate some where over here. So, now, you have this particle which is located at some height and you have two more particles which are located somewhere up there and what you see is; now this sort of a velocity profile, this is happening because of viscosity, it is a viscose effect which is making, the bottom of the fluid to stick to it and the other layers are subsequently gliding over each other, sliding over each other.

When that happens, what you see here is that the momentum of this particle is m of u A and moment of these particle is m of u naught. So, clearly as we go from, let us some height here so say this is h 1 and this is h 2. So, as we go from say h 1 to h 2, there is a change in the momentum. So, therefore, and in all probability u naught is larger, u naught is larger than u A. So, if that happens, therefore, you have a higher momentum as we go away from the surface; now from the plate.

So, therefore, this is which is the viscosity which establishes a momentum transport, but this momentum transport is in a direction which is perpendicular to the direction of flow; understand that because the direction of flow is this way, this is your u infinity and this is how it is flowing and the transport of momentum is this way. So, therefore, it is the viscosity let us write that down, it is the viscosity which establishes, this is the viscosity which establishes momentum transport in a direction perpendicular to the direction of flow.

Therefore, viscosity is also called a transport property of the fluid, so it is a transport property of the fluid and it depends on both temperature and pressure and it is a physical property of the fluid and it depends on both temperature and pressure, the temperature dependence is quite dominant. So, having sort of done that, let me see if I should, there is just one more thing I would sort of like to do and so that should take care of it for this module.