Foundation of Computational Fluid Dynamics
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Lecture – 05

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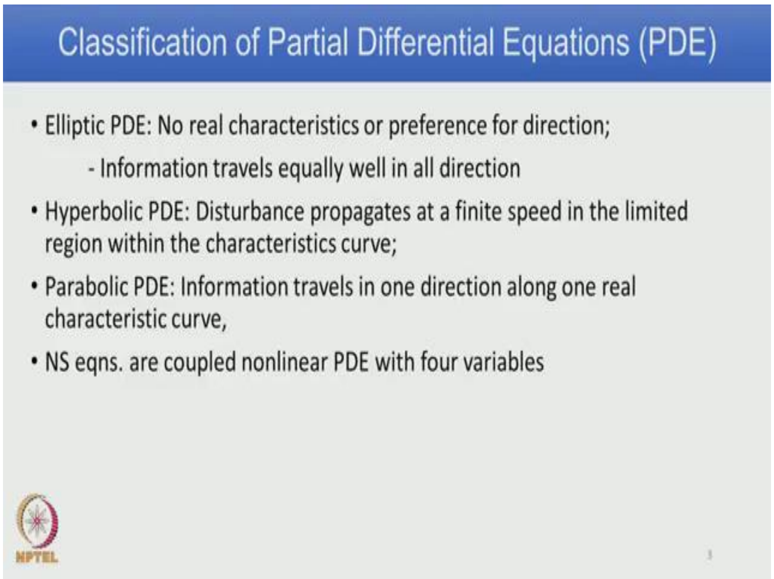

Greetings to you all. We now move onto module five of this course. And this will end week one syllabus. We actually did basic review, vorticity transport equation; then towards end of last class, we started doing something about classification of PDE. Review of last few slides, as I mentioned elliptic PDE may have no preference for direction for information travel or solution propagation. Hyperbolic PDE disturbance propagates only at a finite speed in limited region. And today we are going to see hyperbolic PDE. We also went through parabolic PDE by taking example of diffusion equation. And we also explained we are able to classify Navier-Stokes equations and any one particular category. It is a coupled non-linear PDE with four variables.

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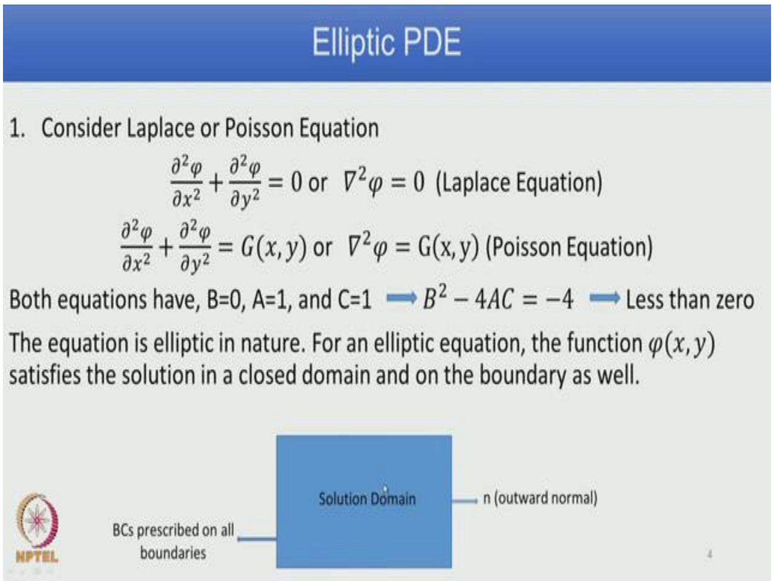

And we had seen this elliptic PDE by taking example of Laplace equation and Poisson equation. And then we observed, the solution propagate in all direction and this is the solution domain.

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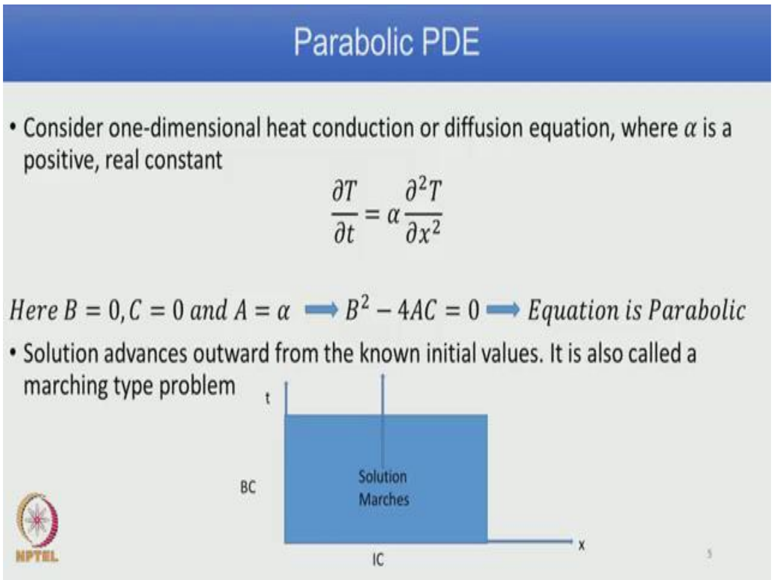

And we did parabolic PDE by taking example of one-dimensional diffusion equation. And notice that the solution advances outward as shown here, from the initial value, this IC is the initial value and solution propagate in one direction outward and such as solution or problem is called Marching type problem or Marching type solution.

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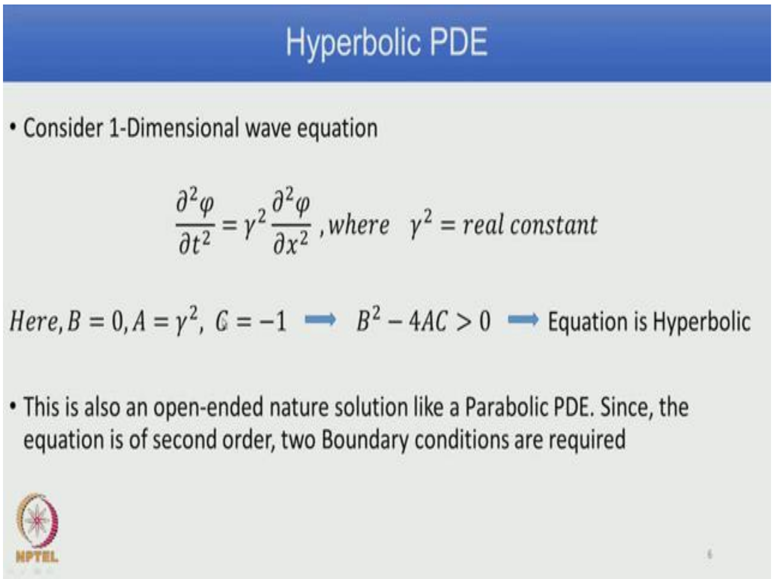

The third category, specifically called hyperbolic PDE; we consider one-dimensional wave equation, and this is the second order dou square phi by dou t square gamma square dou square phi by dou x square, where gamma square is some real constant. And we find out by fitting this with the generic equation, we find the discriminant value, B square minus four A C and we observed that is greater zero, and we classify for such situation equation is hyperbolic. As we have seen in parabolic PDE, this is also open-ended nature of solution; that means, it progresses and along the two characteristics curves. So, two boundary conditions are required, because it is the second order equation.

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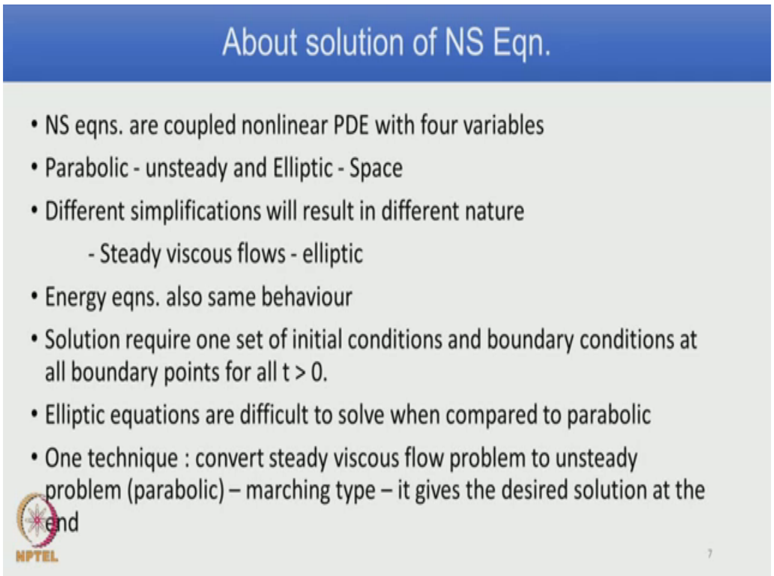

Now, we know that Navier-Stokes equation in full form does not fall under any one category; it is a coupled non-linear PDE with four variables. It is parabolic, because it has unsteady term; it is elliptic in space, because diffusion term. On the other hand, you can simplify the Navier-Stokes equation for different situation, hence it will results in different nature. For example, if you consider steady viscous flow problem then you do not have a unsteady term, you only have diffusion term then the equation falls in the category of elliptic nature. We can also classify energy equation and simplify energy equation for different situation to get different nature of equation. So, solution require one set of initial conditions and boundary conditions at all boundary points for time greater than zero. Elliptic equations are generally very difficult to solve when compared to parabolic.

So, usually what we do, we convert steady for example, we have then here steady viscous flow equation; it is a elliptic equation, it is difficult to solve, so we convert steady viscous flow or a steady problem elliptic nature into unsteady problem by adding time derivative term. Now, you march a solution until you get a desired solution that is until you reach steady state condition. So, we pseudo treat the governing equation, change the form from elliptic to parabolic. The advantage is we have one initial condition, progresses in time. It is only a marching in one direction, solution, convergence obtaining is easy and finally, you get a steady state solution, so this is one numerical trick that is followed to deal with elliptic nature of equation.

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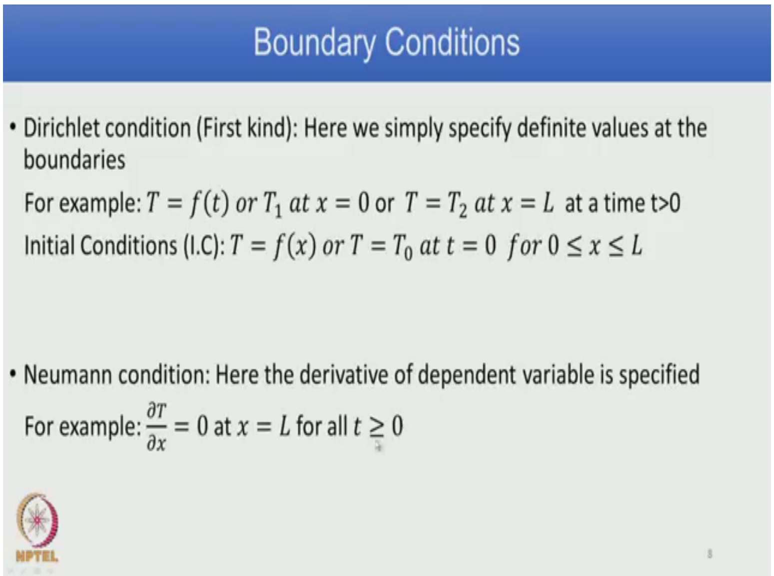

Now, we define nature of equation before that we got all governing equation. The next step is to know something about boundary conditions. There are four types. The first one we see Dirichlet boundary condition, here we simply specify a definite value for the variable. So, for example, T if you are solving temperature equation, t is the variable, so you can specify t as a function of time, or T 1 specifically a value T 1at x is equal to zero on one set of the boundary. And T is equal to 2 T 2 at another boundary at x is equal to L for all time greater than zero. So, specifying a particular value or a function then it is called Dirichlet boundary condition.

Initial conditions, so you can also specify for time is equal to zero, T as a function of x for all x going from zero to L. This mean the domain computational domain extend in x-direction from zero value to L, end of the domain is L. So, along that x-direction specific value of temperature T is equal to zero is given for time is equal to zero. This is initial condition. And for time greater than zero at x is equal to L, we have one value; at x is equal to zero, we have another value. So, you specifically give that value such a type of boundary condition is called Dirichlet boundary condition. Next is a Neumann boundary condition, here we specify the derivative of the dependent variable. So, for example, you say dou t by dou x equal to zero at x is equal to L. So, in the previous case, we said T is equal to T two a specific value, here we are specifying the derivative, dou T by dou x equal to zero at x equal to L for the same time greater than or equal to zero.

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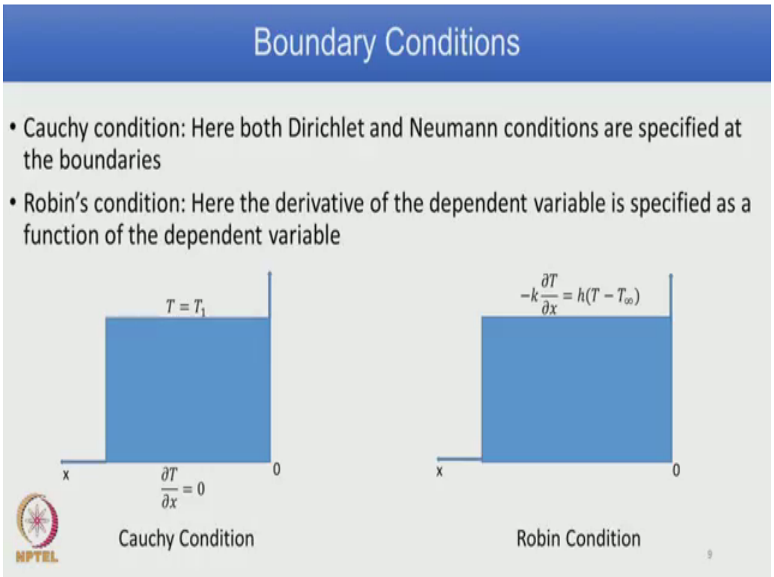

Next, it is a Cauchy boundary condition. We have a mix of Dirichlet as well as Neumann type. So, for example, in this problem figure, along this boundary zero to x, we specify a derivative condition dou T by dou x equal to zero on one side; on the other side, we specify a Dirichlet boundary condition specific value at T is equal to T 1. We have also another type, what is known as a Robin's condition, where the derivative of the dependent variables itself is specified as a boundary condition. So, in this case, for example, dou T by dou x is given as T minus T infinity with some coefficient, so this is what is known as a Robin's boundary condition. So, whenever you are going for a solution, you need to identify what kind of boundary condition you want to impose, whether it is a derivative boundary condition or a specific value of the boundary condition.

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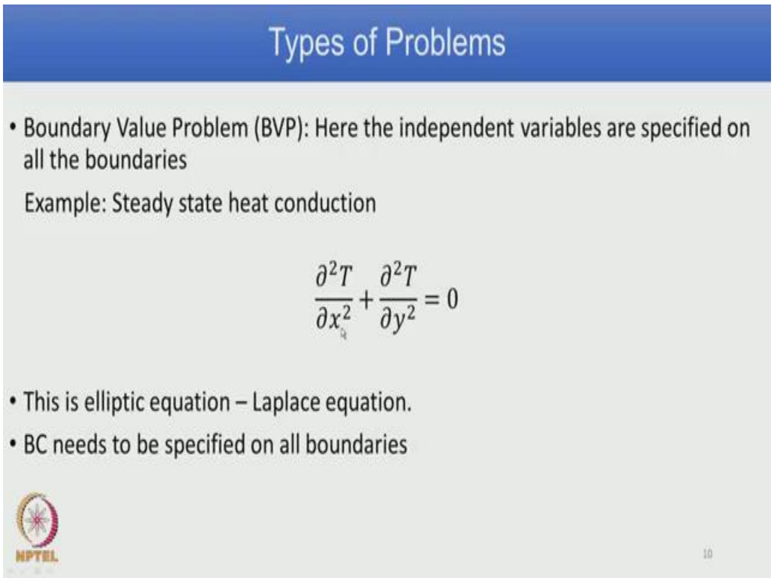

Next, we have also have to know what is the type of problems. So, there are two classifications; one is boundary value problem, here independent variables are specified on all boundaries. For example, steady state heat conduction equation is given here, dou square T by dou x square dou square T by dou y square equal to zero. And we know from previous class, this is the Laplace equation, and this equation falls under elliptic nature of solution category. Now, boundary condition is specified on all the boundaries. So, in this, variable is T – temperature and domain going from x as well as in y. So, you specify boundary condition in all x and y.

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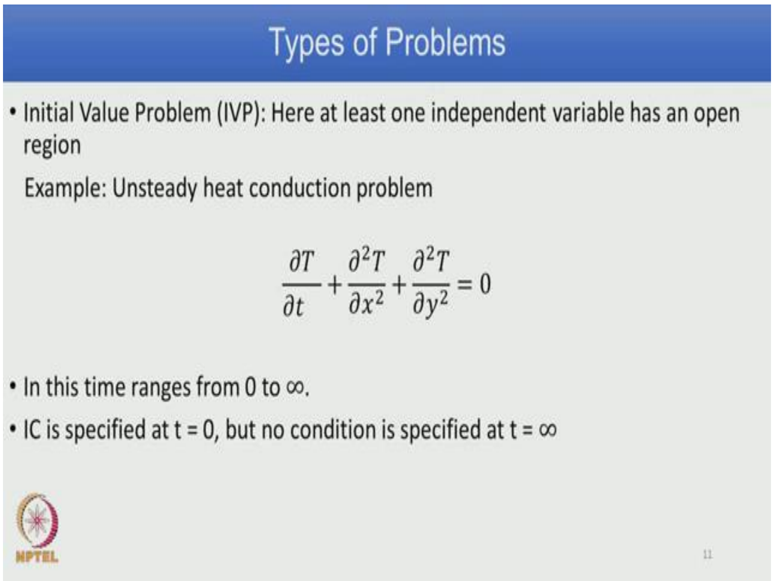

Next category is initial value problem, here at least one independent variable has an open region. Example - unsteady heat conduction problem, same equation that we had seen before; if you add time derivative, it becomes unsteady heat conduction problem. And we know it is a parabolic in nature, and the time starts from zero to infinity range. And we specify temperature value, temperature is a variable here, is specify a temperature value at time is equal to zero. And there is no value specified at time is equal to infinity, so that is why it is called initial value problem. The one variable, the variable has a open ended solution.

By now, you have learned some aspects of CFD, and you will learn more aspects in future classes. After this, let say you are ready to practice CFD. You have multiple options here; either you write your own code or you have inherited code from somebody or from open source. And another option to use any commercial software; here again in commercial software, all the features are available, but some micro programming maybe required from problem to problem. Whatever the case, all the numerical strategies that you are choosing, for example, grid, order of accuracy, convergence criteria, matrix inversion procedure, time steps and so on, all these have to be proved. Now, this stage of CFD is what is known as a validation case. For this validation, there are many test problems available.

These problems are simple in flow or flow over simple geometry, and they have features representing features in complex flow. And these problems are tested and many results are available in open literature either numerically or experimentally. So, one has to take these standard bench mark cases, and first to prove all the numerical strategies that you have chosen for this problem, compare the results obtained by your own strategies against results available or reported in the open literature that stage is what is known as validation stage. Now, there are many test problems available; in this particular lecture, I am taking only few examples, some of them represent internal flow problem, some of them represent external flow problem. Problems are flow in a cavity, flow through backward phasing step, flow passed a cube mounted in a channel, and boundary layer problem. We will go into details of these flows from now.

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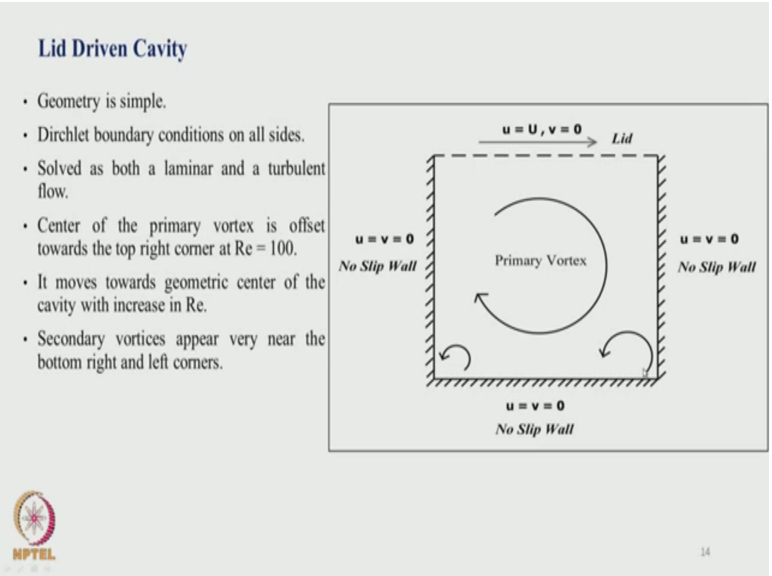

So, here you will see few examples. The first one, I am showing here is lid driven cavity. So, it is a cavity the hash mark that is shown on the side to represent it is a wall; and the top, there is a lid and that is driven with a velocity u equal to capital U and v equal to zero. And a boundary condition imposed for this problem on all the sides, because you have already mentioned it is a wall, on all the sides we put the velocity equal to zero that is u equal to v equal to zero and such a boundary condition is called no slip. And we also know if we specify a value for the variable, it is called Dirichlet boundary condition. So, why are we interested in this problem, this problem is very simple in nature, because it is rectangular, but there are many features that are present inside the problem appears in many other complex flow as well. So, there is a primary vortex and there are vortex in the corner depending on the velocity condition, the vortex will grow in size, and you get velocity distribution etcetera.

Depending on the velocity and depending on the size then you can define the Reynolds number and it becomes either laminar flow or turbulent flow. The centre of the primary vortex is offset, at top right corner for Reynolds number hundred. Then it moves towards geometric centre of the cavity with increase in Reynolds number. So, the problem though it is simple by going on changing the Reynolds number, it becomes slightly complex and the way the vortex behaves also differ. Secondary vortices appear very near both right and left bottom corners.

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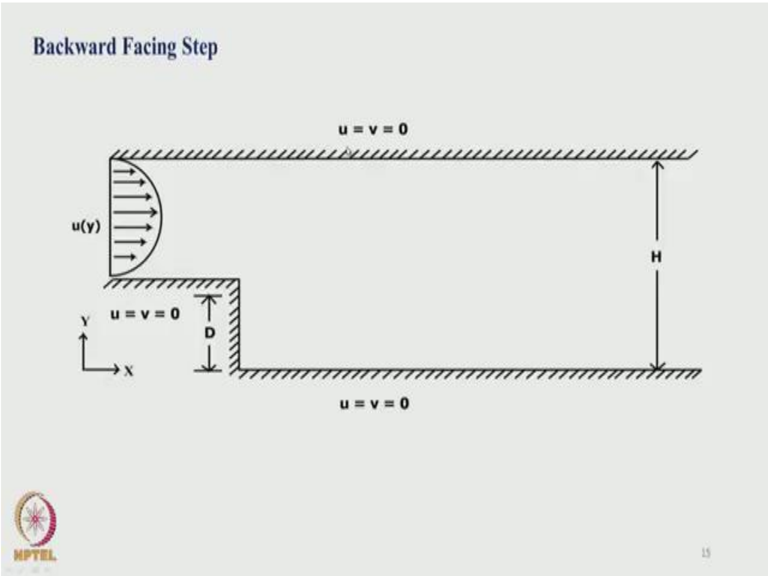

Next problem – test case problem is usually the backward facing step. Here simplified, this is the inlet with velocity given as a parabolic inlet; and the top, there is wall that is why you see hash mark; and in the bottom, there is a wall up to some distance then there is a step and then these are step is also a wall, and then there is another wall in the bottom, so because the step is facing the flow backward this is called backward facing steps. The step has a height given by D, in the channel original channel width has a height H. Now, because with this wall, and the top and everywhere is specify a boundary condition u is equal to v is equal to zero.

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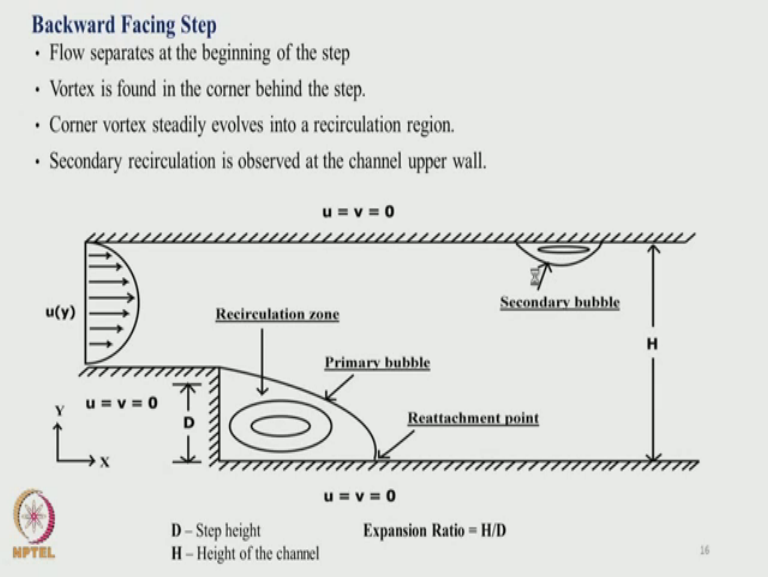

And if you look at the flow pattern, so flow separates at this corner and then it comes and reattaches. There is a recirculation zone, and reattachment point. The way where it reattaches, what is the recirculation zone length depends on the Reynolds number and the flow condition. And similarly, you also get to observe, these also secondary bubble on the top wall, and the size of the bubble where the location the bubble depends on the condition of the Reynolds number or any other flow conditions. There is also a another characteristics what is known as a expansion ratio, which is nothing, but the channel height to the step height and that decides also some flow parameter. Corner vortex steadily evolves into a recirculation zone, depending on the Reynolds number. And secondary recirculation observed on the channel upper wall.

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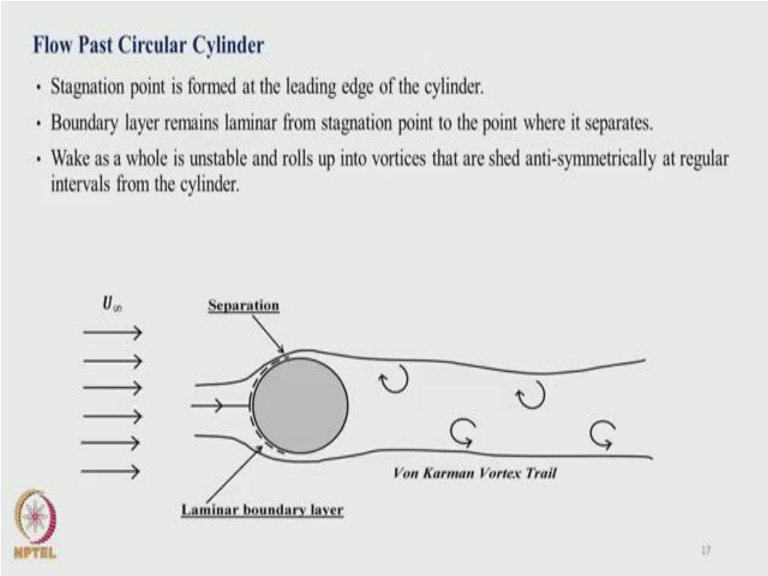

So, we have seen example for two external flows. The next example is flow passed cylinder circular in cross-section, square in cross-section; this falls under category external flow. So, here flow pass a circular cylinder depending on the incoming left hand side is the incoming velocity, what is shown here the uniform velocity of u infinity. And there is a separation point, and in this problem separation point for this geometry is not fixed, it keeps moving oscillating on the surface, and where it oscillate, it depends on what is the condition here, inlet condition and whether the flow is laminar or turbulent. Now, behind the geometry, there is a one common famous one common vortex shedding, and you can have a situation where it is laminar boundary layer transition and turbulent boundary layer or it can be laminar boundary layer, laminar separation laminar wake, or you can have a turbulent wake. Wake is nothing, but the flow region behind the geometry that is what is marked here, and it is unstable and there is a shedding anti-symmetrically at regular intervals from the cylinder.

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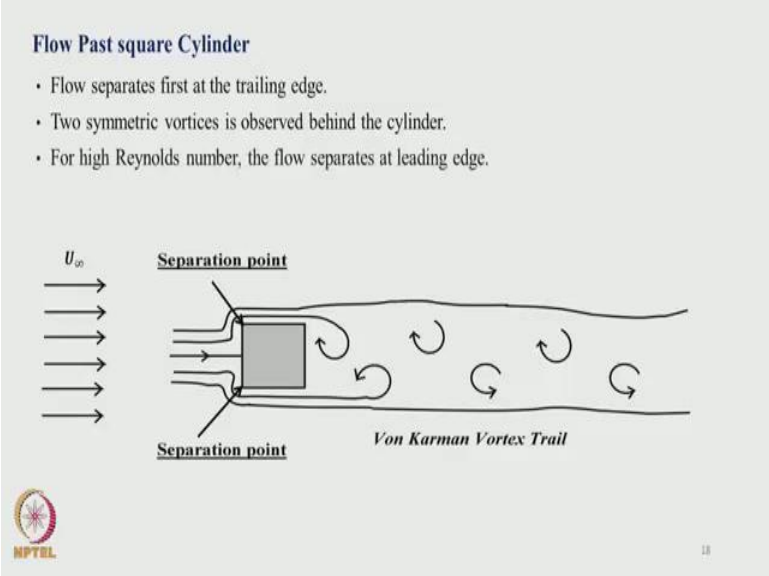

Similar problem, but the geometry is square. The big difference between circular cylinder and square cylinder is separation points are fixed that is at corner both top as well as bottom. Now, that decides the dynamics the wake very differently. The flow separate from this corner, the separation angle, ((Refer Time: 20:36)) rolls into wake or vortex roll up vortex shedding in the wake. So, for high Reynolds number, the flow separates at leading edge.

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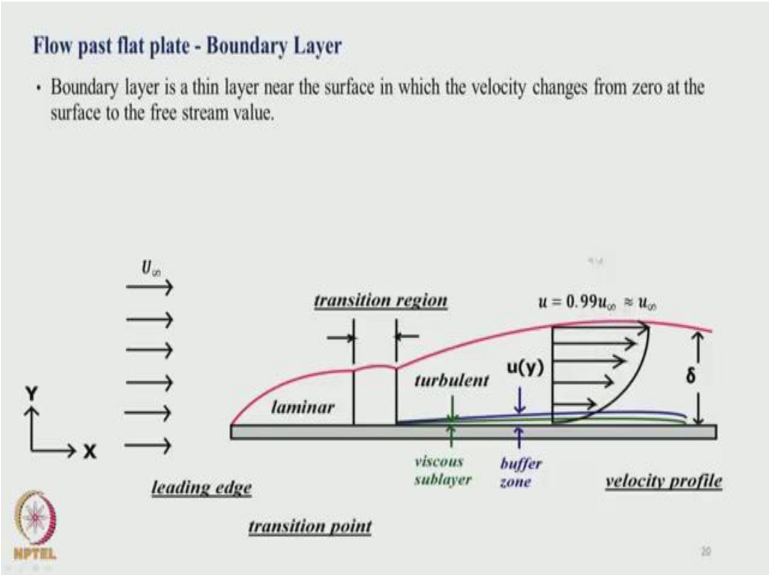

Then third problem is the flow past flat plate and that forms a boundary layer. In this problem, that is shown here incoming flow is the uniform velocity and what is shown here is the plate representing flat plate, and so the flow starts from the leading edge. Initially it is laminar then there is a transition then there is a turbulent. And later we will see the definition of boundary layer thickness, so whenever the velocity reaches ninety-nine percent of the free stream velocity then you call that is a boundary layer thickness. And the turbulent boundary layer again has sub classification, viscous sub layer, buffer layer and so on. So, this forms another test case.

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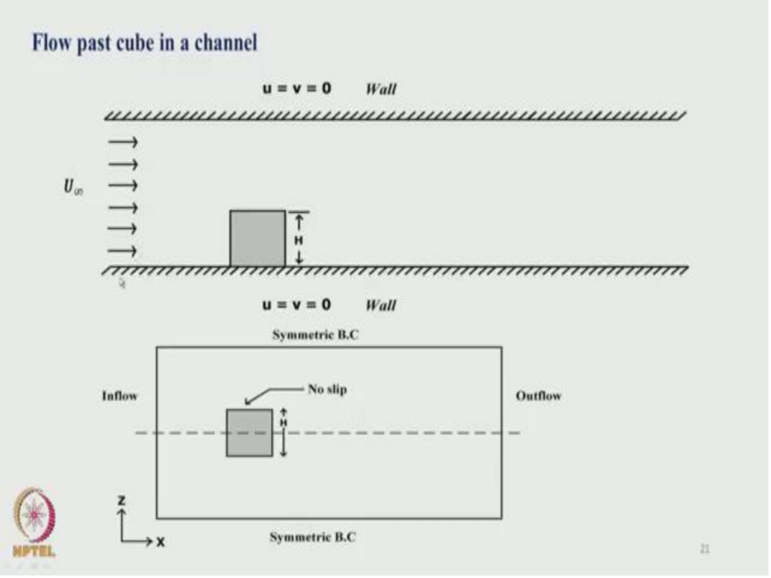

The next problem is flow past cube mounted in a channel. And you see two views of the same problem, here is the channel, so upper is a wall and lower is a wall. And there is a cube that is fixed to the bottom of the wall. And if you see from the top, and you see that it is mounted in the centre in z-direction. And U infinity is the velocity – incoming velocity or it can also be a here it is shown as uniform velocity, it can also be a parabolic velocity profile.

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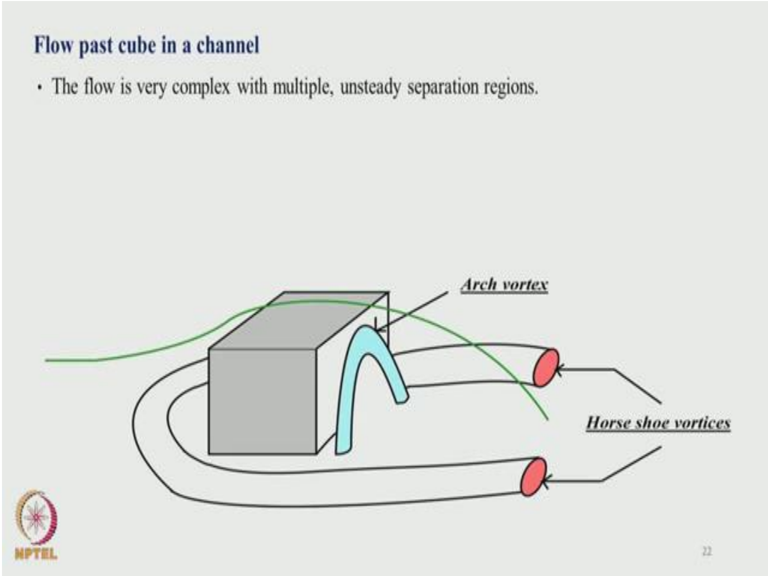

Now, why it is so complex, the geometry is very simple, so the incoming flow we will feel the pressure or the presence of the geometry and there is a separation from the wall, and forms what is known as a horse shoe vortex, and then behind the geometry it forms what is known as arch vortex. So, prediction of this is critical. Now, this geometry has many application, so depending on the height to the site, it can be represent models of high rise building or ti can be thought of as chips on a electronic board. So, all these test problems all are very simple, but the flow feature represent flow feature noticed in many complex situation. So, whenever you write it code or whenever you setup a numerical scheme in a commercial software, you are suppose to validate by taking bench mark cases and many literatures are available to get results and to conform the result that you obtain again result reported in standard literature. So, with this, we come to end of week one, in week one, we have seen all the important topics. And in week two, we will actually start CFD techniques.

Thank you.