Foundation of Computational Fluid Dynamics
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Lecture – 39

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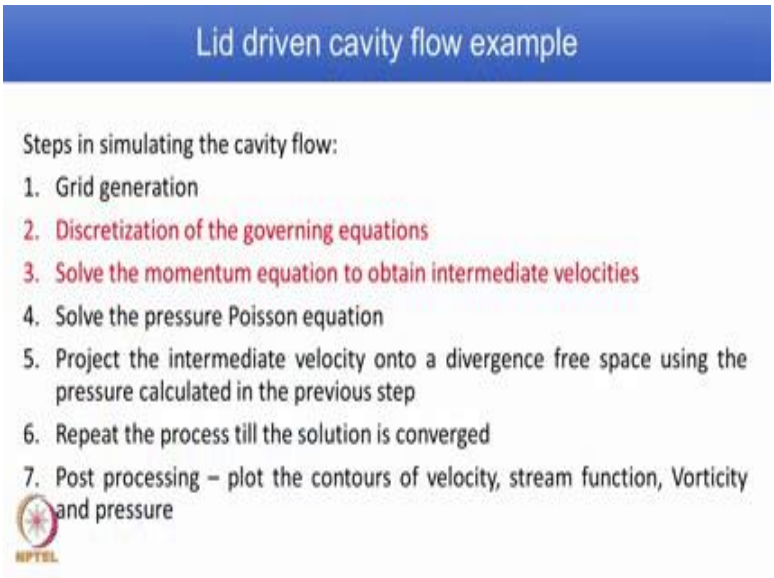

Greetings and welcome again to this course on CFD. In this module, we are going to talk about discretization of convection term and diffusion term in detail. Last class, we have listed algorithm steps to do the test case problem of flow in a lid driven gravity, grid generation, discretization of the governing equation, solving the momentum equation to obtain intermediate velocities, solved the pressure Poisson equation, project the intermediate velocity on a divergence free space, using the pressure obtain from the pressure Poisson equation, repeat the process till the convergence is obtained. Once you get solution then we do post processing. So, in this particular module, we are going to talk about two steps two and three that is discretization of the governing equations and solving the momentum equation to obtain intermediate velocities. As we did in module one, in this module also, we will show corresponding code and explain each step.

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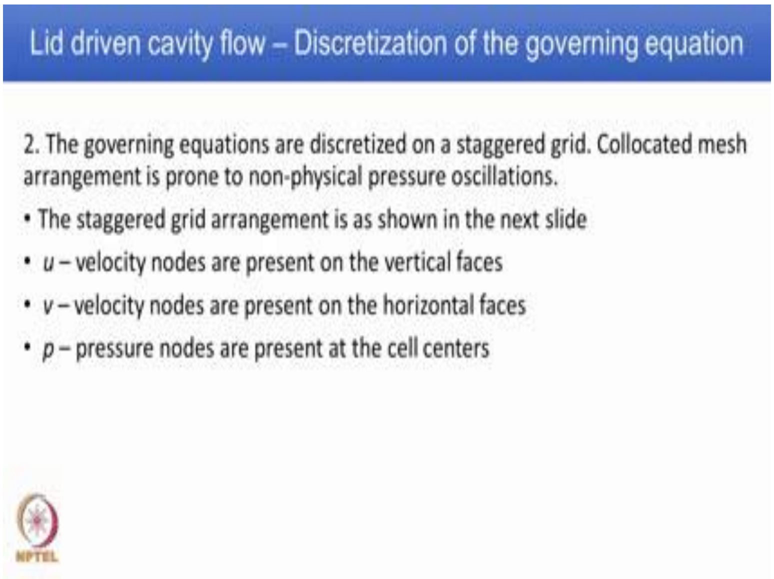

The governing equations are discretized on a staggered grid we mentioned in the last module, we have three options staggered grid, collocated grid and semi staggered grid. Collocated grid results in oscillation, hence we go for staggered grid. And we are using finite difference method to solve the equation, the staggered grid arrangement is shown in the next slide. U-velocity nodes are present on the vertical faces and v-velocity nodes are present on the horizontal faces, and pressure is stored at the centre of the cell.

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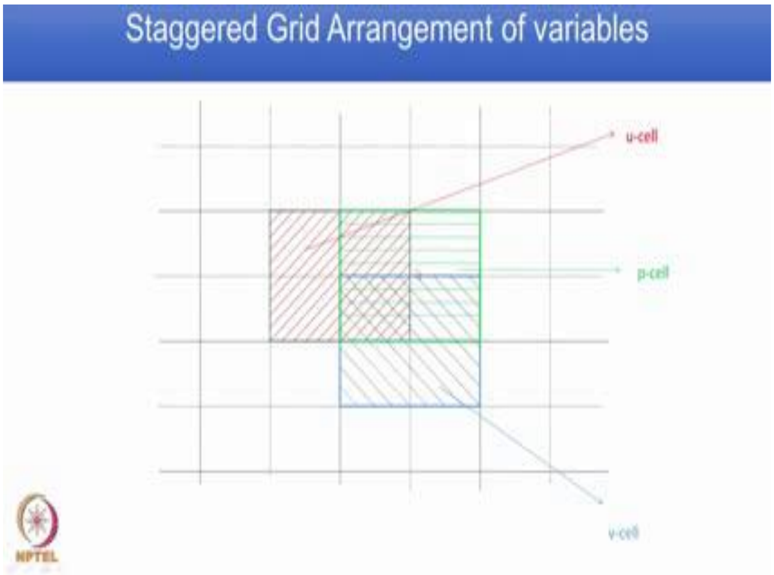

And this is a schematic to explain what is staggered grid. So, we have vertical thick lines as I am showing here, then you also have a dashed vertical lines as I am showing here. Then we have horizontal thick lines, I am showing now. And in between two horizontal thick line we have a dash horizontal lines as I am showing here. And this is staggered arrangement for final volume procedure. So, u velocity is stored as shown here, this is a u velocity, this is a finite volume cell for solving u momentum equation, and this is a finite volume mesh for solving v momentum equation. And pressure or any another scalar in stored in this finite volume. So, as you can observe u, v and pressure or stored at different location that is why the name staggered grid.

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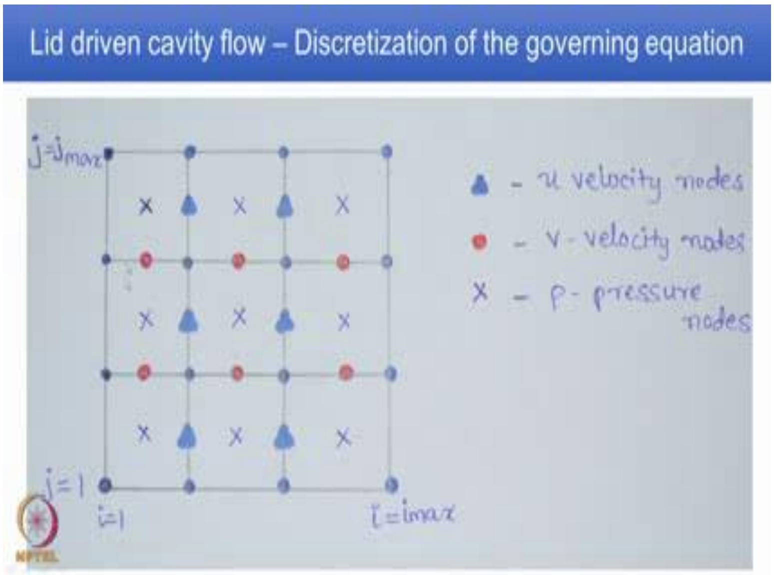

For the problem that we have considered you are following finite difference procedure, hence for the finite different procedure and for the problem the staggered grid arrangement is shown here. We have i that is grid lines x direction running from 1 to i equal to i max, we define four grid in x direction, so 1, 2, 3, 4 vertical lines; similarly four horizontal lines for grid in y direction, so we have 1, 2, 3, 4 and that is running from j equal to 1 to j equal to j max, because it is staggered, we have u velocity which is shown here by a triangle and v velocity which is show by circle and pressure at the centre of the cell which is shown by into mark. So, you can observe in finite difference procedure for staggered grid, velocities, how the velocities are stored, u velocities are stored on the vertical face, v velocity are stored on horizontal face, and pressure is stored at the centre of the cell. We have defined 4 by 4 mesh arrangement and that results in nine cells.

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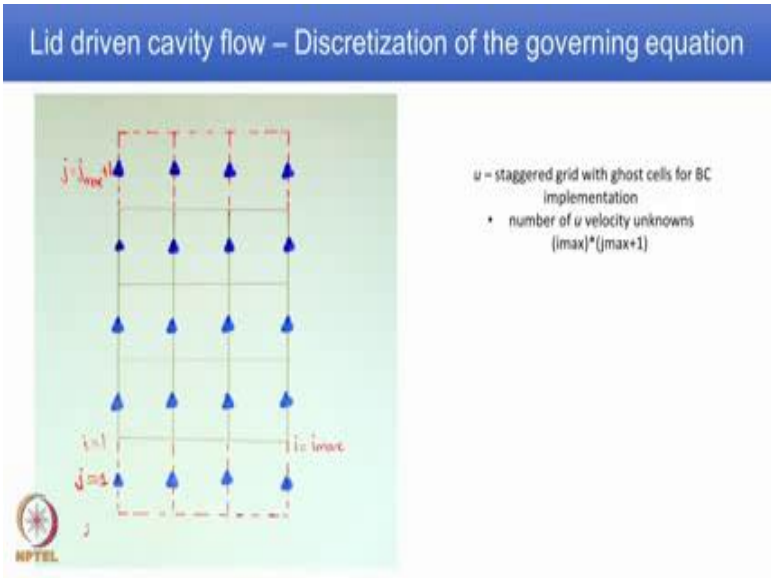

Now, we have define boundary conditions in two different forms you can have a Dirichlet type of boundary condition, where use primary variables and you can have a Neumann type of boundary condition where we use pressure. Now to apply boundary condition, we need to have what is known as a ghost cell that is cell beyond the actual computational domain. So, what is shown here is for y direction the thick black line is actually is the actual grid arrangement, so 3 by 3 then we have a ghost node define extending the domain. So, we have a red colour that is a ghost line. So, j is actually j max plus 1 because j max is actually the last horizontal black colour line.

Now, we define one more ghost node in y direction in on the both sides. So, one on the upper side, j is equal to j max plus 1; another one on the lower side that it is beyond this j is equal to j one this shown by the red colour. This is required to apply the boundary condition. So, u-staggered grid with the ghost cell for boundary condition implementation. So, number of u velocity unknown becomes i max, because we do not have a any extra line in the i direction when you solve u momentum equation, we have only extra in j-direction, so that is j max plus 1 is considered. Number of u velocity unknown becomes i max into j max plus 1.

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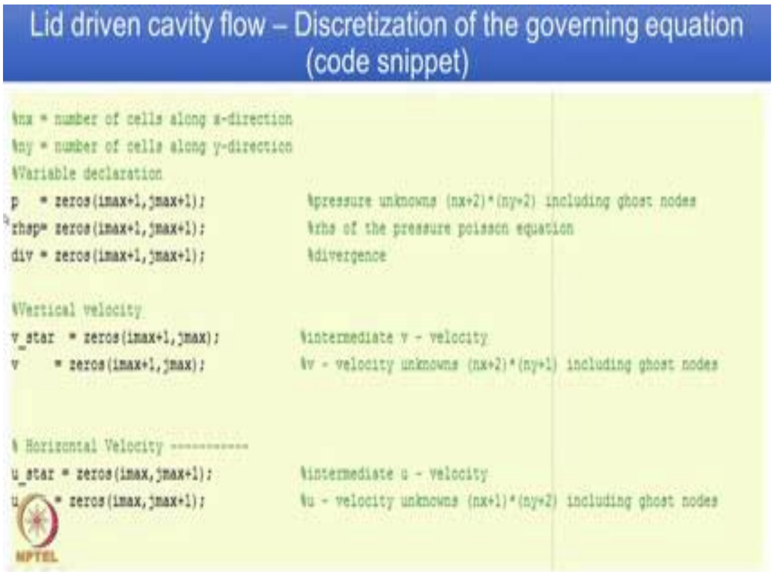

Similar arrangement for v cell, we will look into that when we do the v-momentum equation. And this is the code snippet which actually does what we have just now explained n x is the number of cells in x direction; n y number of cells along y direction, and we have a variable declaration, p is for pressure. Now we have defined array size for the p zeros i max plus 1 and j max plus 1. So, pressure unknowns are n x plus 2 into n y plus 2 including ghost nodes, because when you come to pressure, it is appearing both the equation. So, we have a ghost node appearing x direction as well as in y direction. Then this is r h s p is the pressure Poisson equation, you have a source term on right hand side, and that what is define here as r h s. And there again initialised with zeros of i max plus 1 and j max plus 1. Then we have to calculate divergence and that is also stored vertical velocities are actually v. So, v star is a temporary velocity which is the first step in the projection method. So, v star are define zeros i max plus 1 to j max and v is actual velocity again define zeros i max plus 1 and j max.

So, v is a velocity unknown this is with n x plus 2 multiply by n y plus 1 because when you solve v momentum equation you required ghost nodes in other direction, so that why it becomes n x plus 2, and n y plus 1 is number of nodes available in the y directions itself. Similarly for u velocity which is otherwise horizontal velocity u star and u they are also define array with array i max and j max plus 1 and i max j max plus 1. So, you can observe here for v star it is i max plus 1 j max; for u star it is i max and j max plus 1. This is for intermediate velocity and u velocity. And when you solve u velocity then we have a ghost node defined in other direction. So, it becomes n x plus 1 star n y plus 2, it is a number of minimum required array size.

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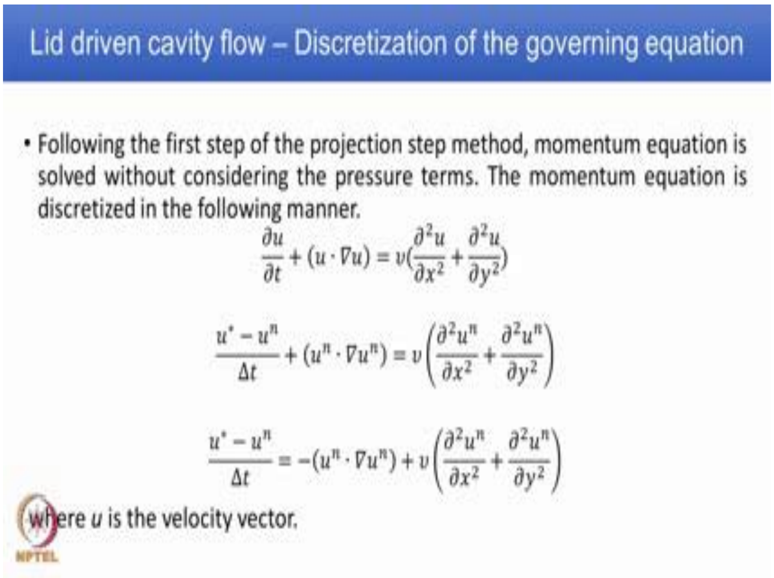

We follow the projection step method. So, momentum equation is solved without considering the pressure term momentum equation is discretized in the following manner. We will explain first with the help of u momentum equation, the procedure is same for v momentum equation. So, u momentum equation after neglecting pressure term is written here we need to discretized and the discretized equation is as shown here. So, it is explicit we already mentioned. So, we have the all the known quantity with superscription only unknown is star quantity. So, u star minus u n by delta t and all the known's are taken to the other side as shown here, where u is the velocity vector and of course it is for v also its included.

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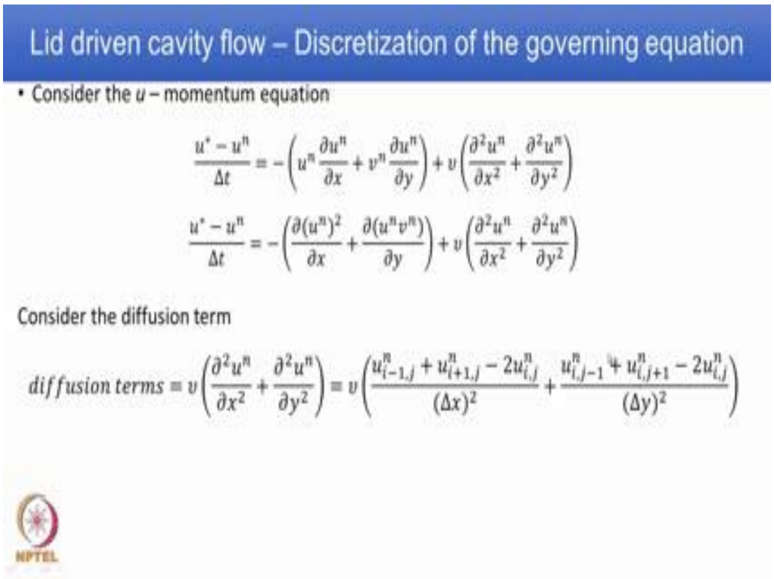

Consider the u momentum equation, the above discretized equation is written specifically now for u momentum equation as shown here. Now there is one small difference between the first equation and second equation for convection term. This u n can be taken inside the partial derivative and that is written here as dou by dou x of u n square. You can actually perform this operation that is dou by dou x of u n square you can write it as 2 u n dou u n by u x, and other term; and one term actually goes to zero because of the continuity. So, it is possible to write the convection term in different form and one form is as shown here. This is for convenient because u is define at one location. So, anyway you can take into the part inside the partial derivative, and any additional terms which is coming because of this partial derivative inclusion will go to zero once you satisfy the continuity, and v n again is taken to the other side as shown here.

So, if you do the partial derivative for both these term together, you will have one extra term from the first term as dou u by dou x. Similarly one extra term from the second term as dou v dou y, if you sum them up, it is actually the continuity which is actually zero. Consider the first diffusion term that is a second term here. So, it is written as shown nu dou square un by dou x square plus dou square u n by dou y square and mu is maintain. Now we follow second order central differences scheme for the derivative, the second derivative that is what is shown here. So, for the first term in the bracket dou square u n by dou x square the discretized equation is shown here, because we mentioned it is explicit, we have all superscript n. So, u evaluated at i minus 1 j n plus u i plus 1 j n minus 2 time u at i comma j n by delta x squared. Similarly for the second derivative in y direction and that is what is shown here.

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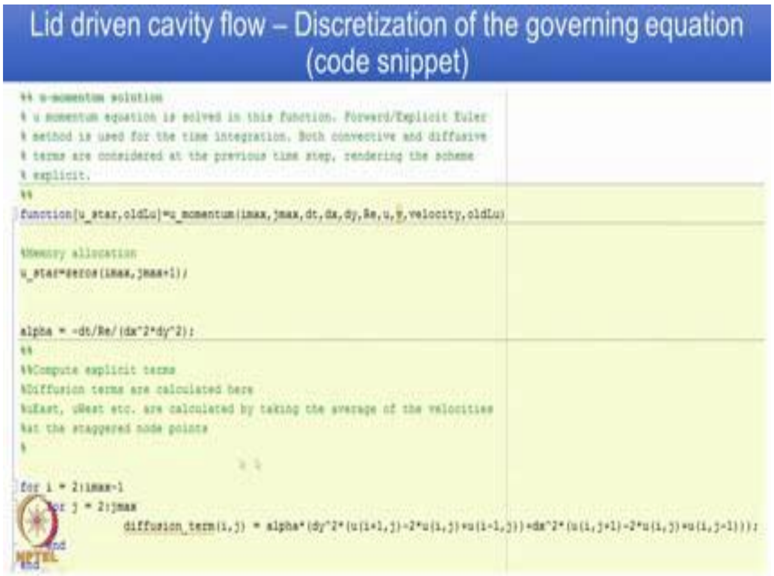

Corresponding code, so we have memory allocation u star alpha is minus d t by Re delta x squared multiplying delta y square. Compute the explicit term, diffusion terms are calculated here and we follow staggered grid. So, we have the east, west, north, south terminology to be used here. So, u east and u west are calculated by taking the average of the velocities at staggered node points, because we want u at staggered location. So, for i equal to 2 to i max minus 1 and for j equal to 2 to j max diffusion term i comma j equal to alpha star d y squared multiplying u i plus 1 comma j minus 2 times u i comma j plus u i minus 1 comma j. So, this first three terms in this particular bracket or for the first derivative dou squared u by dou x square second order central differences scheme for dou square u by dou x square because we have alpha define has minus d t by Re delta x square and delta y square, we do not have a delta y square, we are actually multiplying here alpha, so that needs to be accounted properly, hence we have delta y squared multiplying the entire factor.

Similarly, when you do the second derivative for y direction that is dou square u by dou y square, we have only delta y square, we do not have a x square where as alpha is genetically define considering both delta x squared and delta y square. So, extra delta x square needs to be accounted properly, hence we have delta x squared multiplying the finite difference of dou square u by dou y square and i shown here, so u i comma j plus 1 minus two times u i comma j plus u i j minus 1. So, this is actually central differencing one is the point of interest and one on the right side one on left side.

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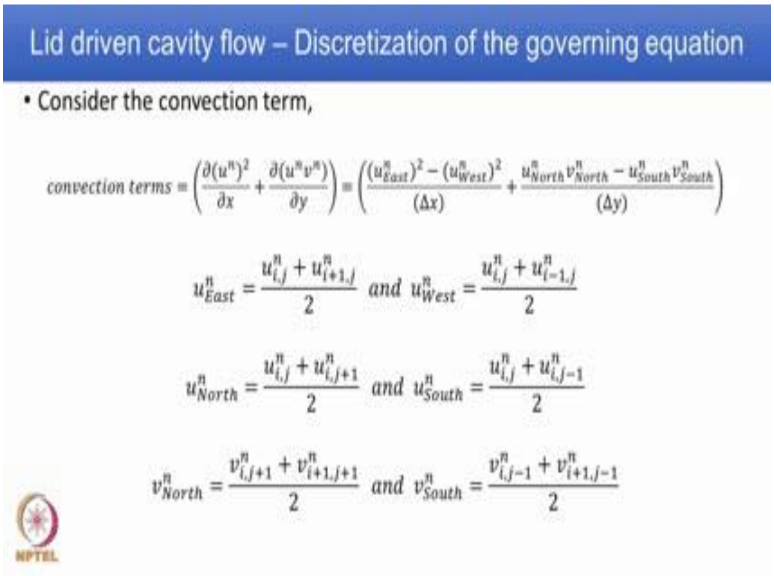

Next convection term we have u dou u by dou x plus v dou u by dou y and we can derive convection term considering the u and v inside the partial derivative. So, the first term is actually dou by dou x u n squared plus dou by dou y u n v n. So, if you actually perform the partial derivative on the terms inside the bracket, you will get two additional terms and those two terms together we will go to zero because of the continuity equation. Now we are following staggered grid arrangement on finite difference procedure. So, we have a north east west south terminology the coming into the picture. So, we have first term in the discretized form is shown here u east n square minus u west n squared by delta x plus for the second term discretized form u north at nth level. V north at nth level minus u south n th level and v south nth level by delta y.

Now what you have to be careful is we have to evaluate u at east west north and south and there done separately here now we have to recall what we did in finite volume treatment on convection term it is a convection terms are non-linear that is u multiplying dou u dou x we need to evaluate u at phases and we need to get dou u by dou x at phases and that is what is followed here we explain three different procedures that is pure upwinding central differencing type of approximation, linear approximation, QUICK approximation, hybrid type etcetera and what is shown here is a linear approximation or central difference in type approximation. So, u at east is u i comma j plus u i plus 1 comma j divided by 2; similarly for u at west, and u at north, and u at south, and v also we follow the same and that is from j direction. So, v at i j plus 1 and v at i plus 1 j plus 1 divided by two similarly v at south v at i j minus 1 v at i plus 1 j minus 1 by 2. We explain all this in detail, when we did finite volume formulation for convection term.

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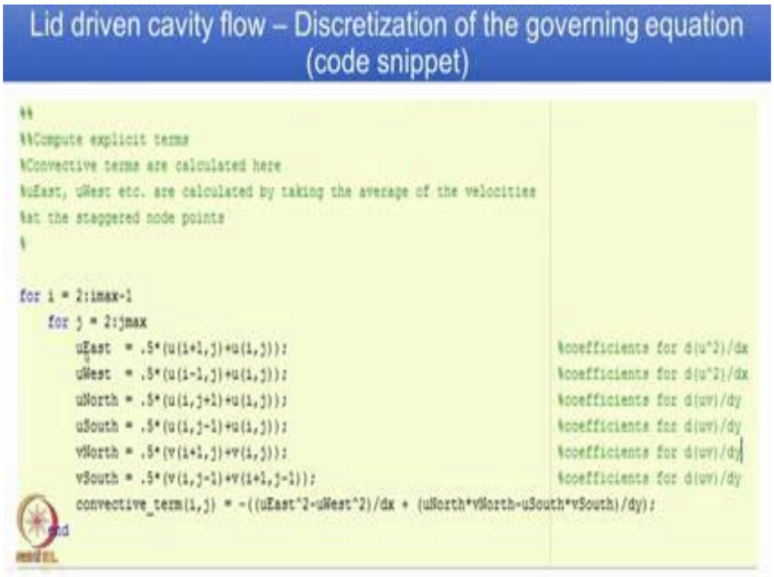

Corresponding code snippet is given here. So, u for i equal to 2 to i max minus 1; for j equal to 2 to j max u at east, u at west, u at north, u at south, v at north, v at south all are calculated. Once you calculate, then you write a convection term for i comma j as explain the discretized equation and it is shown here, dx and dy are already defined, hence they are directly used. Now we have dx and dy for simplicity sake uniform and equal. If it is not uniform then we have to take corresponding weightage in dx and dy will also be as a function of array.

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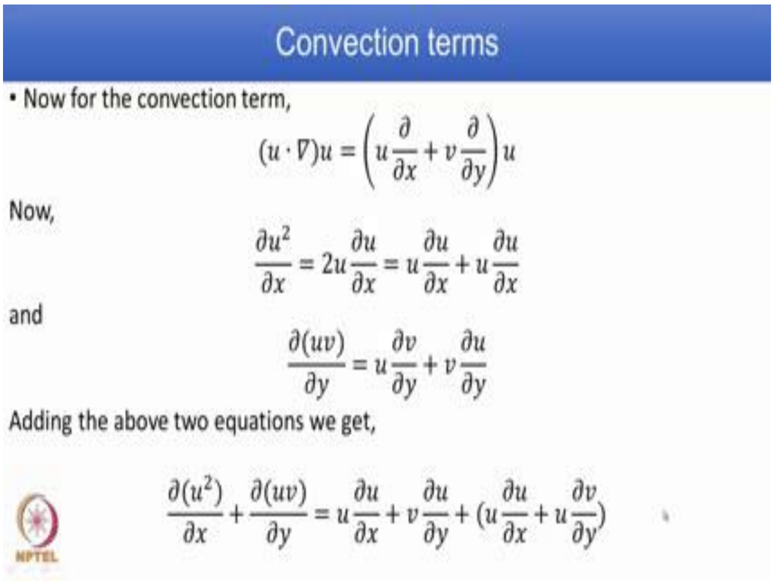

In the last slide, I mentioned about rewriting convection term, I did not explain at that time why are how it can rewritten and there is no change in the actual equation. This slide as well as the next slide, I going to explain in detail, how the convection term can be written. So, u dot dell into u is written as u into dou by dou x plus v into dou by dou y of u dou u square by dou x equal to two into u dou u by dou x equal to u dou u by dou x plus u dou u by dou x. So, two times u into dou u by dou x it is expanded as 1 u into dou u by dou x plus u into dou u by dou x and then dou by dou y of u v as u into dou v by dou y plus v into dou u dou y. If you put these two together then we get dou by dou x of u squared plus dou by dou y of u v equal to u into dou u by dou x plus v into dou u by dou y plus u into dou u by dou x plus u into dou v by dou y. So, we have 1 u into dou u by dou x and 1 v into dou u by dou y grouped separately as shown here.

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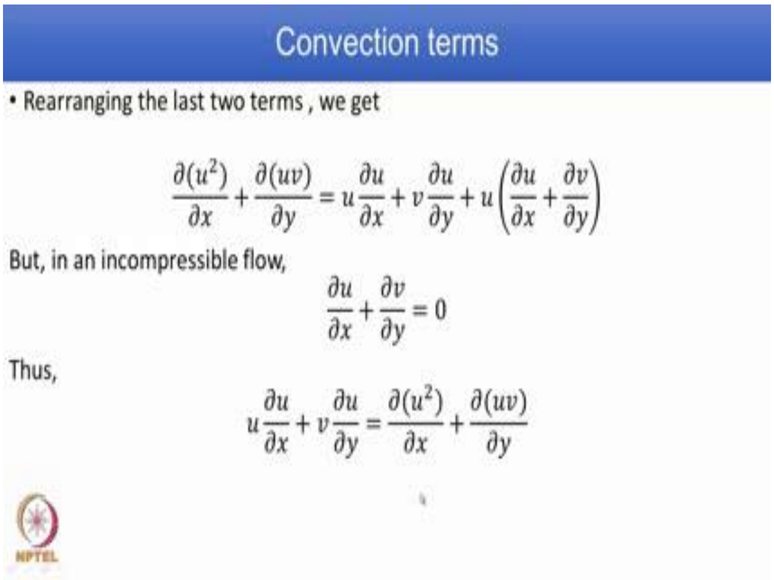

We rewrite that equation again as shown here, but in an incompressible flow, we know the continuity equation for two-dimensional flow as shown here, dou u by dou x plus dou v by dou y equal to 0. Hence the rewritten form of the convection term that is dou by dou x of u square plus dou by dou y of u v is same as u into dou u by dou x plus v into dou u by dou y, hence there is no additional term introduced, because of this rewriting.

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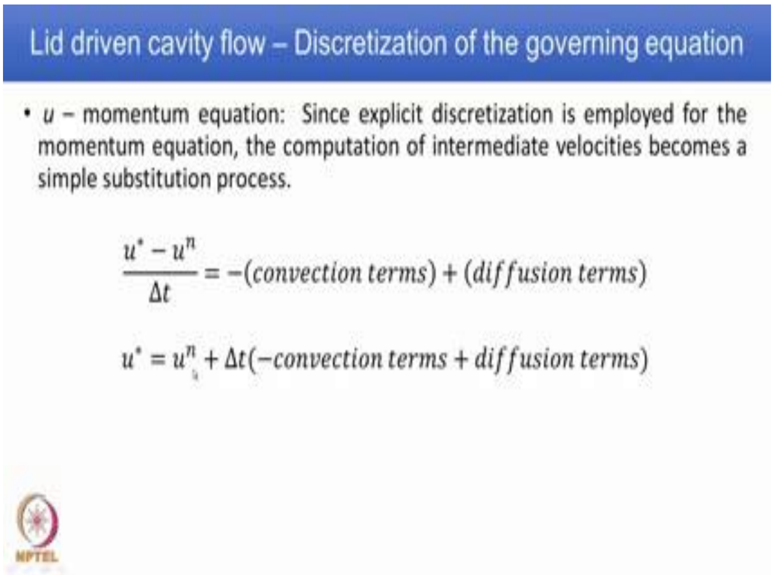

So, u momentum equation, we follow explicit discretization the computation of intermediate velocities become a simple substitutions because we do not need to store, it is directly related to velocity component in the form of discretized equation. So, u star minus u n by delta t equal to minus convection terms plus diffusion term without pressure term. So, the left hand side, convection term is brought to the right hand side, and diffusion term on the right hand side remain same, we follow explicit formulation hence it is written as simple as shown here. So, u star equal to u n plus delta t into this terms.

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So, corresponding code is shown here for i equal to 2 to i max minus 1 and for j equal to 2 to j max. We calculate intermediate velocity, we use u star i comma j as u i comma j plus delta t; in the code we use dt multiplying convection term and diffusion term end. Once you solve equation, you need to ensure implementation of boundary condition and that is applying boundary condition u star on the left wall, u star 1 comma 2 to j max is equal to 0, because we are following Dirichlet type of boundary condition using the primary variable we mentioned u equal to v equal to o on all the three walls. So, we have a left wall, right wall bottom wall and the top wall we have velocity specified for u. So, u star 1 to i max j max plus 1 that is in the top line grid line is equal to 2 into velocity minus u star 1 i max and j max.

In this module, we have seen in particular convention term discretization, diffusion term discretization, and we learned what is known as to ghost node, ghost node cell in x direction and y direction when you solve respectively u momentum equation and v momentum equation for the implementation of boundary condition. We also showed corresponding codes and including implementation of boundary condition. In the next module, we will proceed with the some other explanation and proceed with all the next stage in the algorithm.

Thank you.