Foundation of Computational Fluid Dynamics
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Lecture – 31

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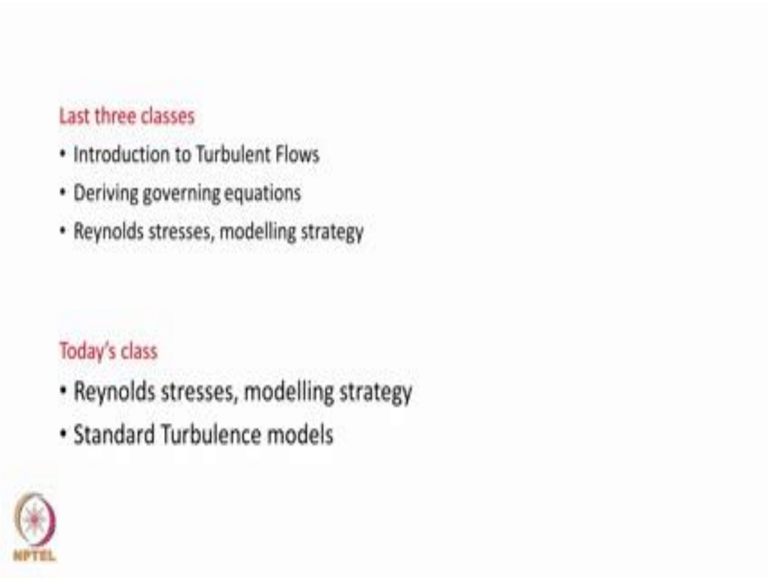

Greetings and welcome again to this course on CFD. Last three classes, what we had sen introduction to turbulence flows, we try to derive governing equation for turbulence flows, try to understand the term Reynolds stresses. Reynolds stresses has two main component one is normal stresses another one is shear stress; totally there are six unknown stresses. Then we also learned few other new terms like turbulent kinetic energy, dissipation rate, specific dissipation rate, eddy viscosity concept. In today's class, we will continued that try to understand something about how to model or what are the models available to close the system of equation; particularly we will focus on what is known as the standard turbulence model.

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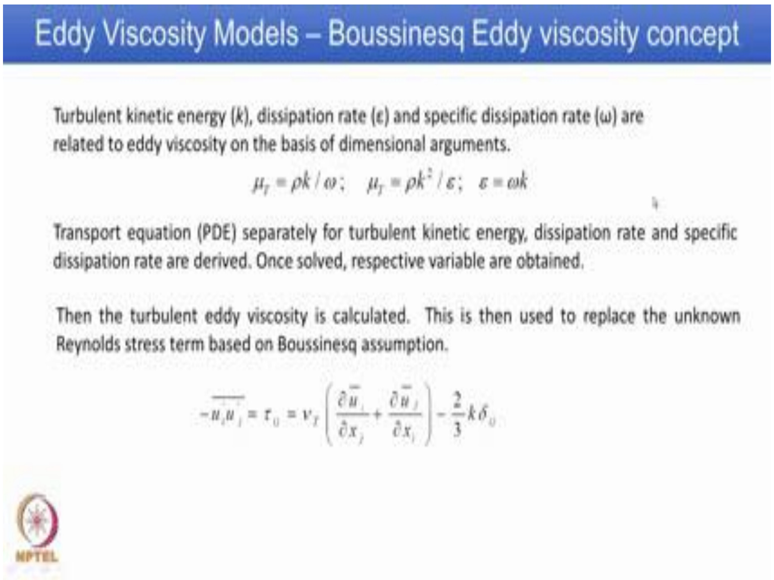

Just to recall what we did in last few classes, we defined a term called turbulent kinetic energy use a symbol k, dissipation rate epsilon, specific dissipation rate omega and they are all related to eddy viscosity on the basics of dimensional argument as shown here mu t as eddy viscosity, t is a subscript used to denote it is for turbulent flow equal to rho k over omega or mu t equal to rho k square over epsilon and k epsilon on omega are related to each other as epsilon is equal to omega into k. And you can check with the units on each side whether they are matching. We can setup transport equation which are again basically partial differential equation separately for each of this quantity turbulent kinetic energy, turbulent dissipation rate, specific dissipation rate.

If you are setup that equation and you are solve that equation then respect variables will be obtain after solving those equations. Then turbulent eddy viscosity is calculated using this relation and it is used to close or to model Reynolds stresses based on what is known as the Boussinesq assumption. The Boussinesq assumption is shown in the form of expression here. We learned that u i prime u j prime bar is a Reynolds stresses, u i prime is fluctuation is one direction, and u j prime fluctuation in another direction is equal to tau i j or in some sense you can use r i j equal to nu t multiplying dou u i by dou x j, which is a velocity gradient and dou u j by dou x j which is also velocity gradient. And this is based on mean velocity minus 2 by 3 k into delta i j; delta i j is a Kronecker delta we learned. When is i equal to j it will have value 1, and i not equal to j, it will have value zero.

Now if you look at this equation nu t is related to either k and omega or k n epsilon as shown here, and the product in the velocity strain is actually based on mean velocity. So, we solve Navier-Stokes equation, average Navier-Stokes equation, we will obtain u bar we solve k equation or epsilon or omega equation, we get respective quantity then that is replace that is the turbulent stress is replaced using this relationship. One additional information as I mentioned last class, we need to have out of the three minimum two quantities to be known. What are those three quantities, velocity scale, length scale and time scale.

If any two is known, then third can be related, so these equation that is equation for turbulent kinetic energy, dissipation rate and specific dissipation rate, they are suppose to give information when it is solve information about velocity scale, length scale and time scale. For example, k equation when it is solved, it will give information about velocity scale. If you take square root of k, which is a basically meter per second that is actually velocity. So when you solve k equation, it will give velocity scale; when you solve epsilon equation, it will give length scale; and when you solve omega equation, it will give the time scale that is how this epsilon is equal to omega k relationship is obtained. So, any two should be obtained to close a system.

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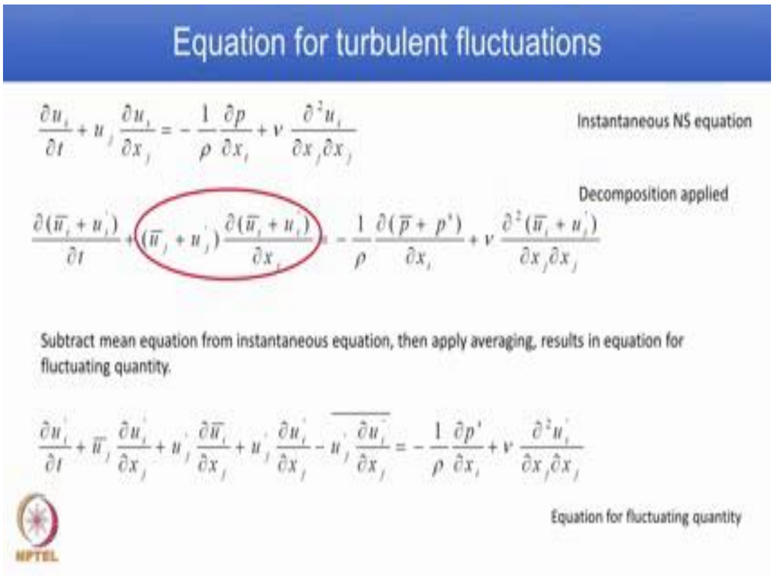

We will go back and look at the equation once again. We again insist on knowing the index notation. So, Navier-Stokes equation is shown here, first term on the left hand side is a time derivative term, second term is convection term equal to pressure gradient term plus viscous diffusion term. So, i will take a value of 1, 2, 3; for every value of i, you get one equation. For example, if you substitute i is equal to 1, then you will get u momentum equation or x momentum equation. If you substitute i is equal to 2, you will get second momentum equation; and i is equal to 3, and you will get third momentum equation. We also mention if the index is repeating then it is a summation. So, second term index j is repeating, and for every value of j, you will get one unique quantity and each one of them are summed up finally. And we also learned that if the index is repeating that index is called dummy index if the index is not repeating, it is called free index. So, the free index here is i the dummy index actually j.

Now we apply the decomposition; decomposition is u, for example, can be decomposed into u bar plus u prime, where u is a instantaneous quantity, u bar is a mean and u prime is a fluctuation. So, you substitute the decomposition for each variable in this term. So, first term for example, dou by dou t and u i is written as u i bar plus u i prime. Similarly u j and u i and pressure and another terms. So, each term is written with the decomposition for the respective variables. Now the first equation as I have been telling before is a Navier-Stokes equation, but in the case of turbulent flow context it is interpreted as instantaneous velocity component used in the Navier-Stokes equation. So, this is after decomposition applied.

And in this equation, it is easy to write after doing the time averaging for each term, in the case of convection term it appears the special treatment. Now if you subtract this mean equation from the instantaneous equation, then you apply averaging then it will result what is known as the equation for the fluctuating quantity. Now how this is arrived what it be say we said u instantaneous quantity is decomposed and return in the form of u bar plus u prime mean and fluctuation. So, if you want fluctuation, you subtract mean from the instantaneous. So, u minus u bar will give you u prime. The same is used for the entire equation as such, so the first equation is a instantaneous Navier-Stokes equation and the second equation is a Navier-Stokes equation with decomposition applied, and you are subtract one from the other then you get equation for fluctuation quantity and that is written as you shown here.

Now if you look at this equation again very closely, you have i and you have j, j is a index that is repeating which is called dummy index; and i is a unique index that is the free index.

So, this equation is written for one fluctuating quantity for every value of i, i is equal to 1, you get one fluctuating; i is equal to 2, you get another fluctuating quantity. So, you can write this equation with other with index change, but still the basic information is that is further fluctuating quantity.

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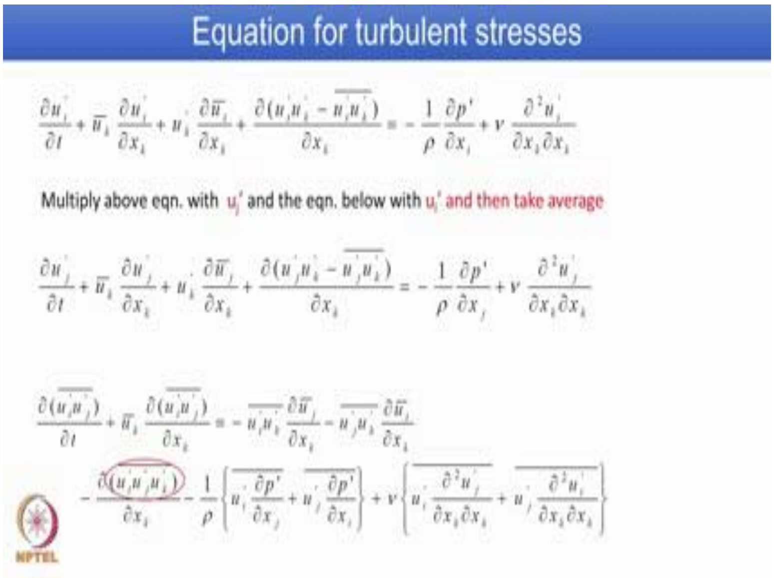

So, we write that equation with the index change for j which is a dummy index and i is the free index is maintain now we can rewrite the equation with some other index because of the meaning the index is it is value 1, 2, 3 except that the repeat index it be maintain. So, in this case for example, this is the original equation we obtain and rewrite this equation again with i replace with j and k is retained that is a repeat index write you can observe that for all the term i is replace with j and k is retained that is repeat in the index is maintain. It means it is a equation for fluctuating quantity, but the other quantity u j the first equation is equation for fluctuating quantity u i prime the second equation is a equation for fluctuating quantity for u j prime.

Now what do we do, we multiply the first equation by u j, second equation by u i, if sum them up then take average. So, multiply the first equation by u j prime and the second equation by u i prime then take average why did we have to do Reynolds stresses, and the Reynolds stresses is actually u i prime u j prime we have u i prime here first term for example, in the first equation u i prime. And if i multiply the equation by u j prime then I get equation for u i prime u j prime; similarly the second equation u j prime is there. I want u i prime u j prime, so I multiply that equation by u i prime. So, you do this operation that is first equation is multiplied by u j prime second equation is multiplied by u i prime then take average that will result in a equation as shown here all the mathematics is done and then final expression is shown here.

So, in these equation for example, the variable is u i prime u j prime bar u i prime u j prime bar is actually Reynolds stresses in other word we are able to obtain the transport equation or p d e equation or governing equation for Reynolds stresses. So, it is possible to obtain equation for fluctuation as shown here and equation for Reynolds stresses as shown here if can recall equation for mean momentum equation that we had we had a new term u i prime u j prime appearing and we define that the Reynolds stresses term and we want to close the Reynolds stresses term and we want for that we started deriving equation. And you observe here, it will result in additional unknowns rather than closing these systems. This equation will result in additional term that is used in u i prime u j prime multiplying with another term u k prime for each variables we get one term totally it will result in twenty seven terms, hence system is never close and the many other new unknowns.

For example, look at this term pressure term here it is u i prime dou p prime by dou x j. So, this u i prime itself we do not know and there is p prime is a fluctuation derivative of the p prime multiplying by u i prime we have no idea how to replace this term, that is why it is difficult in the case of turbulent modelling.

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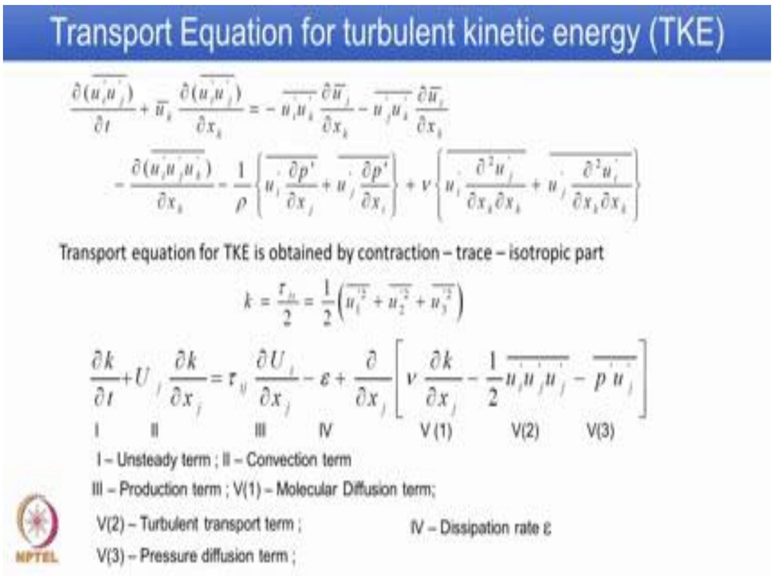

Nevertheless, deriving this equation is very helpful we are going to see that. So, the same equation is written once again here. Now what we have do there is a procedure is called trace isotropic another word contraction or trace in the isotropic part. Let us see the isotropic part will learn that the Reynolds stresses term u i prime u j prime bar is actually a matrix second order tenser along the diagonal of the matrix you have the normal stresses of the diagonal you have the sure stresses. For example, u 1 prime square u 2 prime square u 3 prime, these all are normal stresses and half diagonal u one prime u two prime bar u 1 prime u 3 prime bar this is a sure stresses. So, when you say rays contraction for a second order tenser then you only consider isotropic part isotropic part or along the diagonal. So, in these equation, if you substitute same value for the index i as well as j, so i is equal to 1, j is equal to 1, i is equal to 2, j is equal to 2, i is equal to 3, j is equal to 3, then i will get 3 equation corresponding to three normal stresses.

Now some term together that you give you the turbulent kinetic energy k is in the form of expression it is shown here tau i i. So, we wrote tau i j for Reynolds stresses. Now we have same value for the index i as well as j. So, it is written tau i i by 2 and what is tau i i is a normal stresses. So, u 1 prime square bar plus u 2 prime square bar plus u 3 prime square bar now this is what is known as the turbulent kinetic energy or t k e. So, if I replace if I reduce this equation using the trace or isotropic part or contraction then I will get a equation for turbulent kinetic energy that is the idea that is what happen it is shown here this is a transport equation for turbulent kinetic energy.

Again we try to understand each term first term is a time derivative term second term is a convection term these two term are in left side on the right side we have first term what is known as the production term fourth term is a dissipation term and there are three terms in the fifth term first term in the fifth term it is actually diffusion, but the diffusion is by molecular viscosity mu second term is also a diffusion, but it is by u prime this case u j prime. So, it is called turbulent diffusion and third term is also diffusion, but that is by pressure. So, you have first term in the fifth term as a first molecular diffusion or viscous diffusion second term is turbulent diffusion and third term is pressure diffusion. Now in this equation, if you look at we have replays second term and third term, and try to combine along with the first term using the defined quantity for k epsilon or using the Bosque negative concept and that is the modelling here done on this.

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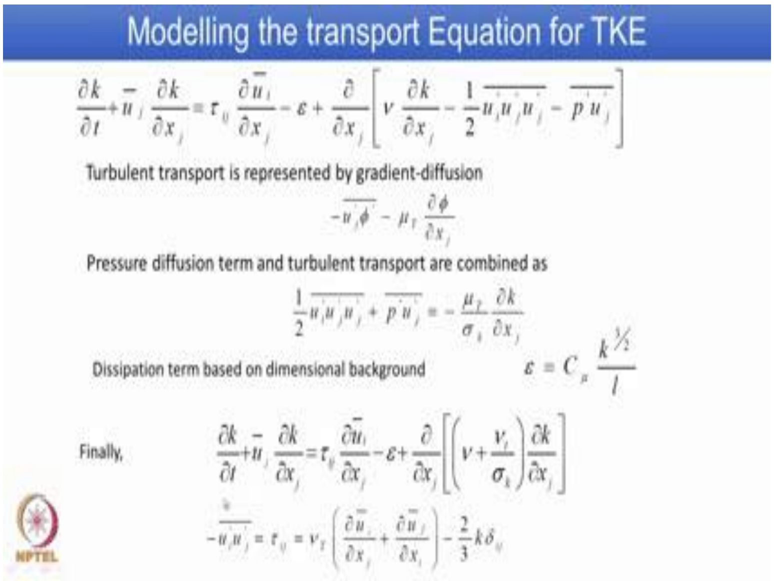

Now we will look at the details. So, turbulent transport first that is turbulent transport is the second term and the fifth term that is the term I am showing here. So, u i prime u j prime is a Reynolds stresses and that is transported using the third quantity j prime that is the way interpreted I repeat again u i prime u j prime is a variable of this equation. For example, for Reynolds stresses and the agent that is respond for transporting the term is u j prime that is why it is refer has a turbulent diffusion term in very generic form, we write that is a five that Reynolds system you can interrupted has five as a u j prime is our quantity. Now this if you say diffusion happens because the gradient that is a general concept.

So, in this case what is the term that is deriving diffuse that is a 5. So, five can get diffuse only if there is gradient in the five. So, mu t is a turbulent viscosity and dou phi by dou x j is a diffusion gradient that is available diffusion to happen now the pressure diffusion term that is p prime u j prime and turbulent diffusion term they are put together and they are written as mu t by sigma k dou k by dou x j into the minus sign. Now this dou k by dou x j is again, it is a gradient kinetic energy because this is the equation for kinetic energy. If you solve the variable, you will come to know this only k, hence we use gradient of k as a known these two unknown or replace with the quantity that is known that quantity that will becomes to known when you solve this equation with k itself. Hence we write dou k by dou x j. Now dissipation term epsilon is a model based on dimensional argument as seem you k to the power of three by two by l. So, finally, we get the model turbulent kinetic energy equation as shown here and we have boosting viscosity concept to close to replace the Reynolds stresses as shown here.

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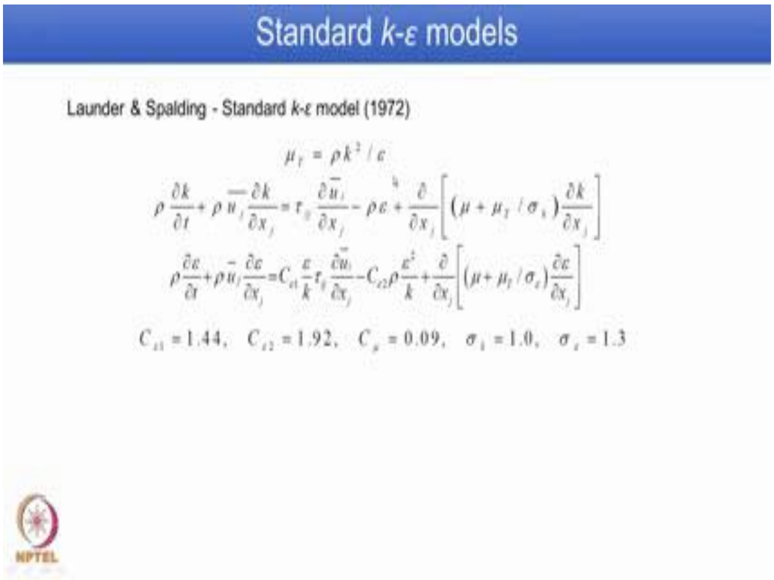

So, we have mainly two basics turbulent model that is what is known as the standard k epsilon model and standard k omega model. Standard k epsilon model was proposed by Launder and Spalding in 1972. In this the basic equation or equation for turbulent kinetic energy k, and equation for dissipation rate epsilon and mu t, turbulent viscosity is related to k epsilon as shown here rho k square over epsilon. In the previous slide, I showed how to model equation or how to model terms in the equation for turbulent kinetic energy and that is written once again here. There is a similar procedure available derive equation for epsilon and model them, we are going to do those detail. We take the final expression and that is what is shown here it has five constant as shown here c epsilon 1 c epsilon 2 c mu sigma k and sigma epsilon. So, equation for turbulent kinetic energy dissipation rate along with five constants with this relation for mu t is called k epsilon model.

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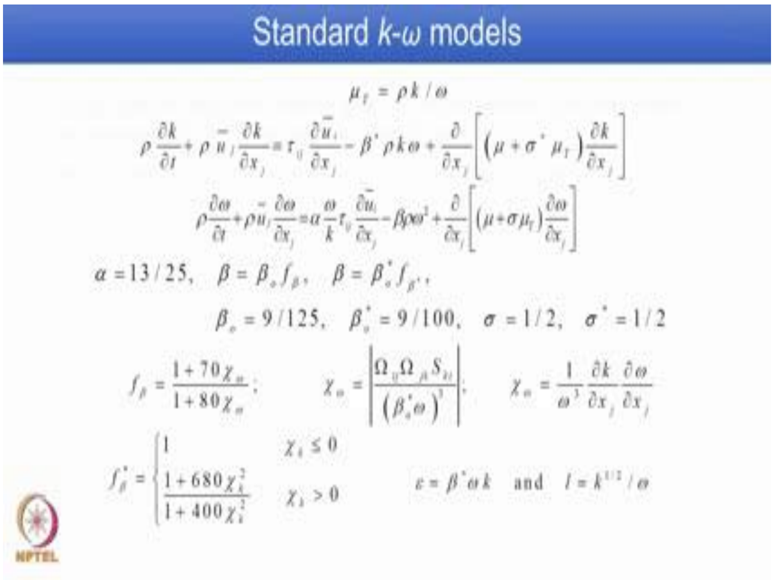

We are going to see another standard model, standard k omega model; mu t is related as shown here rho k over omega; equation for k and equation for omega are shown here. It has again different sets of constant and so on. It is not possible to go through details of the derivation. The intention here to introduce turbulence model equation and I am showing here k omega model equation. So, many commercial software, this is an equation that is solved when you choose option k omega model. Now these are constant and these are functions which are again tested and derived for standard benchmark flows.

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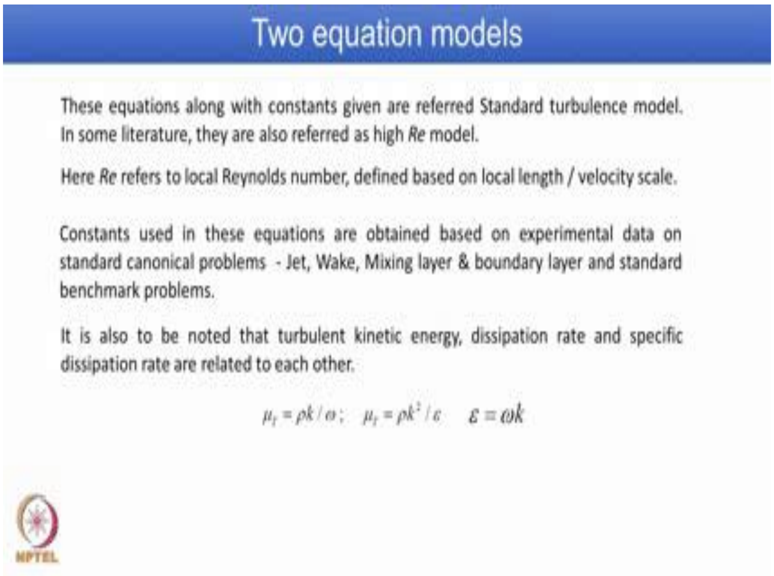

To conclude these equations along with the constants given are referred as a standard turbulence model. They are also referred as high Reynolds number or high Re model. We have to be carefully about the word high Re, when you say Re, here it is refers to local Reynolds number. Again I go back to the first definition, Reynolds number is related to velocity scale and length scale. So, depending on the choice of the velocity scale, length scale you get different Reynolds number you can for a example flow through a circular cross section pipe, you can take a mean velocity as a velocity scale or the canter velocity as a velocity scale. Similarly, diameter can be the length scale. If you take some other cross section it be the hydraulic dimension can be a length scale. So, depending on the definition of velocity scale and length scale, you get different Reynolds number.

Now in turbulent flow, it is also possible to define Reynolds number based on k as well as epsilon. As I mentioned before k when you solve it gives a velocity scale; and epsilon when you solve, it gives a length scale. So, you can define Reynolds number based on k epsilon; similarly, it is also possible to define Reynolds number based on k and omega and this high Re refers Reynolds number define based on local velocity and local length scale. So, constant used in these equation are obtained based on experimental data on some standard canonical problems. So, in fluid mechanics, we have canonical problems, they are jet, wake, mixing layer and boundary layer and standard some benchmark problems. So, we have experiment data and these constants are obtained, so that these experimental data can be reproduced. It is also to be noted that turbulent kinetic energy, dissipations rate, specific dissipation rate are related to each other in terms of epsilon is equal to omega k. The reason why I am saying here is again, we have equation for epsilon, we have equation for omega, we have equation for kinetic energy. Let us see the kinetic equations as a side and it is possible to obtain equation for omega from epsilon or it is possible to obtain equation for one quantity from the another quantity.

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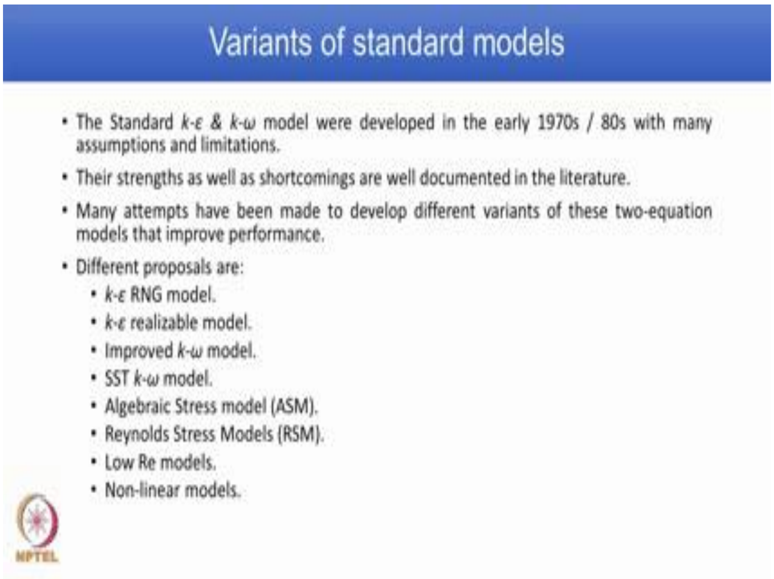

Now, we look at different variants of standard models the standard k epsilon and k omega model are develop very early 1970 or 80 with very assumptions and limitations their strength as well as shortcomings are well documented in the literature many attempts have been made to develop different variants of these two equations models to improve performance different proposal I am listing here they are for example, k epsilon r n g based model k epsilon based relaizable model improved k omega model SST k omega model algebraic stress model otherwise called a s m Reynolds stress model otherwise called r s m low re models and non-linear models it is not possible for go through all these different proposal also the equations what i am to do just descript in different in the explanations for some models and in the next class we are going to see some details about low Re models.

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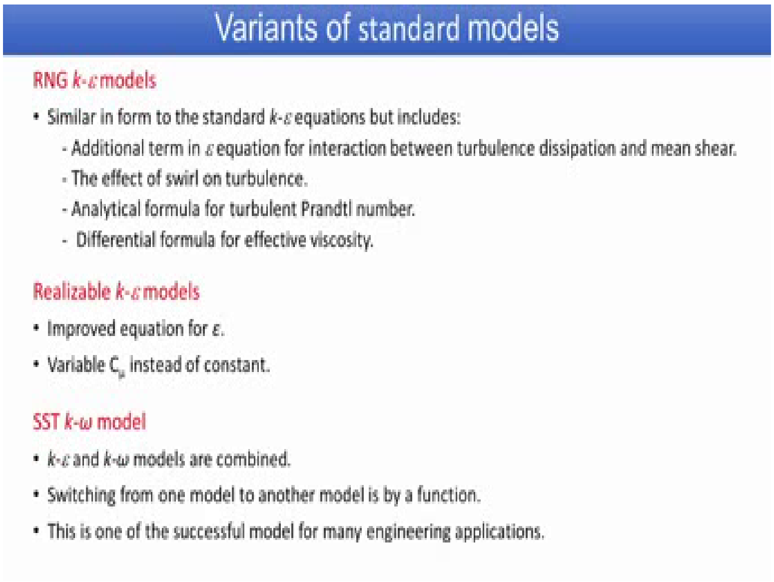

So, RNG based k epsilon model, it is similar into the standard k epsilon equations, but include in following that is additional into introduce the epsilon equation for interaction between turbulent dissipation and means share then that is accounts for what is known as a effectively viscosity on turbulence where it is gets well for example, in compression we have a methodology called spell compression and that has a influence on turbulent and that is predicted better based on RNG epsilon model analytical formula of turbulent Prandtl number was introduced and then the differential formula for effective is costly. So, as we mentioned that mu t related to k and omega it is also possible to get different expression and that is what is u string r n g k epsilon model and next step reliable k epsilon model here again k equations almost same that we have a improved equation for epsilon and then c mu who was one constant we mention there are five constant in standard k epsilon equation and c mu was one of the constant it has the value of 0.09.

So, in realisable epsilon model instead of constant value for c mu a c mu express as a function of some variables it may be velocity gradient it may be k l or it may be epsilon for different proposal on realisable k epsilon model we have a different expression for c mu next is SST k omega model; SST stands for shear stress transport, k omega model. As I mentioned before epsilon omega k are interrelated as epsilon is equal to omega into k. We have a separately transport equation for k, separated transport equation for omega, it is possible to derive one from the other or it is possible to combine together them together switch within code itself depending on a region in the flow from epsilon equation to omega equation.

I repeat, it is possible to switch from one equation to another equation within the flow based on some flow region condition from one equation to another equation by having one equation by having one transport equation combining both epsilon and omega, and that what is approach in SST k omega model. As i mentioned here also listed k epsilon and k omega models that combine and switching from one model from another model is by a function and this model happens to be very successful for many engineering application.

So, in today's class, we went through basics of modelling once again. We did few steps related to deriving transport equation for turbulent fluctuation then transport equation for Reynolds stresses. We learned that while deriving it only results in more unknown rather than closing the system of equation. However, the exercise of deriving equation for Reynolds stresses helped us that was the starting point for deriving equation for turbulent kinetic energy. Equation for turbulent kinetic energy is obtained by doing a procedure called trays or contraction on the equation for Reynolds stresses. A similar procedure is followed for equation for epsilon, and we learned two important basic turbulence model standard k epsilon and standard k omega model. We also listed few proposals available in the literature as an alternative improvement for the performance on these two models. We learned only the description about these proposals. In next class, we are going to some detail about low Re models. And with this, I close today's class.

Thank you very much and see you again in next class.