Foundation of Computational Fluid Dynamics
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Lecture - 29
Part – 2

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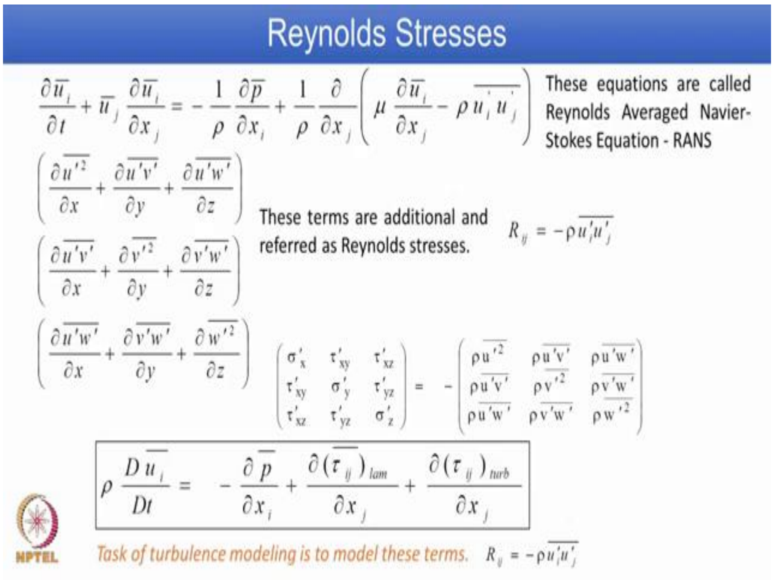

In part two, we will get some more details about this Reynolds average Navier-Stokes equation and Reynolds stress terms. We will also get explain about the closure problem. In second part, we will explain why the problem of closure still remains. So, if we write the equation and bring it to the other side, the new term if you bring the new term from left hand side to the right hand side, the same average Naviers stoke equation can be rewritten in this form. So, left hand side dous not have that new term that is brought from there to the right hand side, so left hand side remains the same as a original Navier stokes equation except that we use average quantity. Right hand side first term pressure gradient term, the viscous diffusion term is also there, but we have that additional term written and included in that derivative term. Now these equations are specially called Reynolds averaged Navier-Stokes equation in short form RANS.

Now this additional term rho u i prime u j prime; if you substitute units, so rho is the density and u is the velocity, if you substitute unit and check, it will have the same unit as mu into dou u by dou x check mu into dou u by dou x is actually a stress. So, this quantity new term rho u i prime u j prime also represent stress and that is why we brought these two terms together and group them, and the derivative is taken separately outside. Now in expanded form that is by substituting value for i and j following that index notation guideline, for example, u i prime and u j prime is substitute i going from 1 to 3, and j going from 1 to 3, we get u prime square u prime v prime and u prime w prime. And the derivative denominator, we have x j x j x j and x j index is repeating and for every value of j you get one term, but then they are added and that is why you get here summation term.

Next value of i, again j, we get another term; for next value of i and j we get another term. These terms are additional terms appearing the Naviers-Stokes equation because of the procedures of that we are followed decompositions of variable following tensor algebra performing averaging, it will result in additional term called Reynolds stress term or i j is the symbol used equal to minus rho u i prime u j prime. In terms of matrix, and I can write all these additional term of matrix as shown here rho u prime square bar rho u prime v prime bar rho u prime w prime bar and other quantities.

Now if you look at each term little more detail, along the diagonal as I am showing here, you have u prime square bar v prime square bar and w prime square bar, and these are all what is known as a normal stress. So, along the diagonal element, we get normal stress, half diagonal elements, for example, u prime v prime bar u prime w prime bar v prime w prime bar, these are three half diagonal elements on one side, similarly we have three half diagonal elements on the other side. And these 3 plus 3 - 6 - half diagonal elements are called shear stress, so following the convection sigma for normal stress, and tau for shear stress, we write in the form of a symbol for the stress on the right hand side corresponding left hand side, minus sign is retained and rho is retained.

Let us put that equation again, but in slightly different form. We know from one of the beginning lecture that left hand side there are two terms one is for time derivative, other one is for convection term. The time derivative term represent the local acceleration, convection term convert acceleration. And if we put them together, that will give total acceleration and that is what is shown here with capital D symbol rho into D by D t of u i bar. So, this represents total acceleration written with average quantity. And the right side, we have a pressure gradient term then there are two terms we know already this mu is viscosity, and mu into dou u by dou x j will give you the stress and it is contribution from laminar part. So, we write tau i j with a subscript laminar dou by dou x j, and the second part we used again the letter symbol tau for stress, but it is for turbulent flow. So, we have dou by dou x j of tau i j with a subscript turbulent. So, what it means is the equation looks similar, if we put laminar stress part and turbulent stress part together, there is only addition of turbulent stress part. And task of turbulence modelling is to model is additional stress term that is rho u i prime u j prime bar.

Now let us look at little more detail. We already mentioned turbulent flow exhibits fluctuations both in time as well as in space. And it is quite random that means, it is difficult to predict what the fluctuation will be either in the next instant of time or at the next spatial location, but what is possible is the statistical way of dealing with that those variations. Now if you look at this terms that is u i prime and u j prime; u i prime is fluctuation of one variable, and u j prime is a fluctuation of another variable. We do not know the u i prime itself then it becomes much more difficult the product of two variations, hence we have to model somehow, and replace the governing equations accordingly and that process that stage that procedure is called turbulence modelling.

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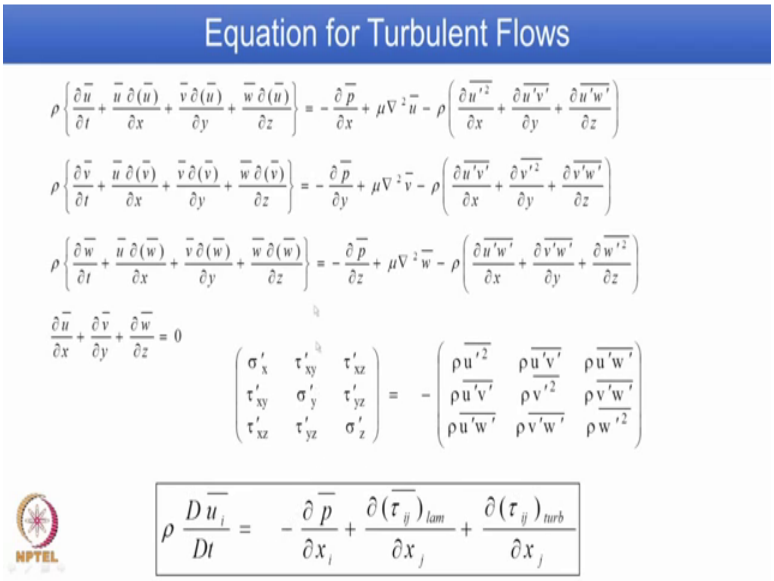

I am showing here the same equation in a much detail form without using index notation. The first set, we have Navier-Stokes equation, this is otherwise Reynolds average Navier-Stroke equation. And we have bottom continuity equations all these terms are already explained. Now you can definitely appreciate using of index notation to write terms and finally equation. So, we have three terms for convection, three term for stress, and we have three equations, all are compressed in one simple expression when use index notation. And these stresses - additional stresses are known as Reynolds stress, they are also written in matrix form, and we already mention elements along the diagonal that is u prime square bar v prime square bar w prime square bar there are all normal stress, and half diagonal elements are shear stress. And we write again that equation, now you appreciate difference between writing the index notation and in expanded form of writing equation.

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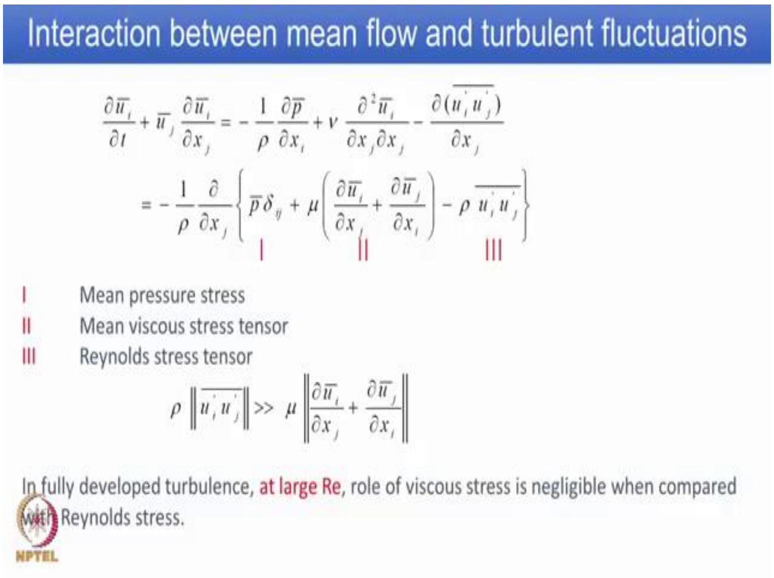

So, we write here again the Navier-Stoke equation obtain for turbulence flow as shown here. We will try to get some more explanation, we rewrite this equation in the form as shown here, minus one upon rho dou by dou x j p bar delta i j, this is the kronecker delta plus mu into dou u i bar by dou x j by dou u j bar by dou x i minus rho u i prime u j prime bar. We identified the terms as one, two, three inside this bracket. Let us get the explanation of those three terms, the first term represent mean pressure stress, the second term is because the laminar viscosity that is the dynamic viscosity mu. So, it is called mean viscous stress tensor, it is mean because we are using u i bar and u j bar. The third term is actually Reynolds stress tensor, and this is coming extra term because of the averaging procedure applied on the convection term. If the modulus of the Reynolds stress terms that is u i prime u j prime bar modulus rho is much, much greater than the laminar stress bar then that actually is the region of turbulent flow. In other words, in fully developed turbulence at large Re, the role of viscous stress is negligible when compared to the role played by Reynolds stress.

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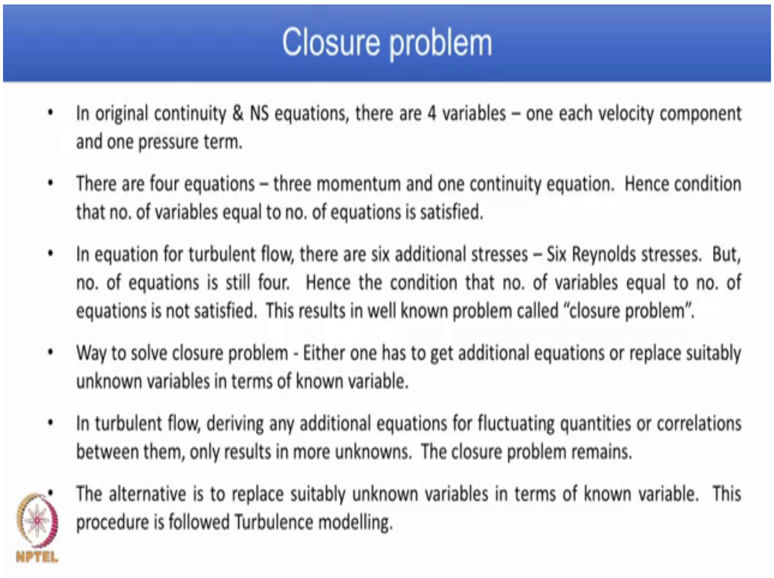

Let us tried to understand what is the meaning of closure problem. So, in the original setup, we have continuity equation - one equation, and Naviers stress equation - three equations. And in there are four variables that is u, v, w each velocity component and pressure is additional term. So, there are four variables, and we have four equations; number of equations is equal to number of variables is satisfied. In equation for turbulent flow, we just now learned, you have still four basic variables that is three mean velocity and mean pressure. In addition, we have six Reynolds stress term, but number of equation remains as four still. Hence, condition that number of variable equal to number of equation is not satisfied, and this result in well-known problem called closure problem that is we have more variables in turbulent flow, we have ten variables that is four - primary variables, mean velocity – three, pressure – one, and six additional stresses. So, we have ten variables, whereas number of equations available are only four that is three momentum equation and one continuity equation. Hence we have more variables than number of equations available and this is what is known as a closure problem.

There is way to solve this closure problem either you can get additional equations for each of those stresses or suitably replace the unknown variables in terms of known variable, hence number of unknown variables gets reduced. In turbulent flow, it is possible to derive additional equations for fluctuating quantities, and then find out also the equation for Reynolds stresses. But every time, you write additional equation, it will only end up in more unknowns, hence the closure problems remains. The alternative way is to suitably replace unknown variables, that is the stress u i prime u j prime bar in terms of known variable. This process that is replacing the unknown variable by a known variable is called turbulence modelling.

So, in today's class, we have learned in detail about equations, continuity equation, momentum equation, how to put them in index form, then how to apply decomposition principle for each variable, and get continuity equation, momentum equation for turbulent flow, momentum equation when it is applied to turbulent flow after averaging is perform is referred as Reynolds average Navier-Stoke equation. We learned while doing this process, it results in six additional unknown stress quantity, and they are to be replace with a known quantity and this procedure is called turbulence modelling procedure. In next class, we are going to talk about different models available until then have fun.

Thank you.