Foundation of Computational Fluid Dynamics
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Lecture - 28
Part -1

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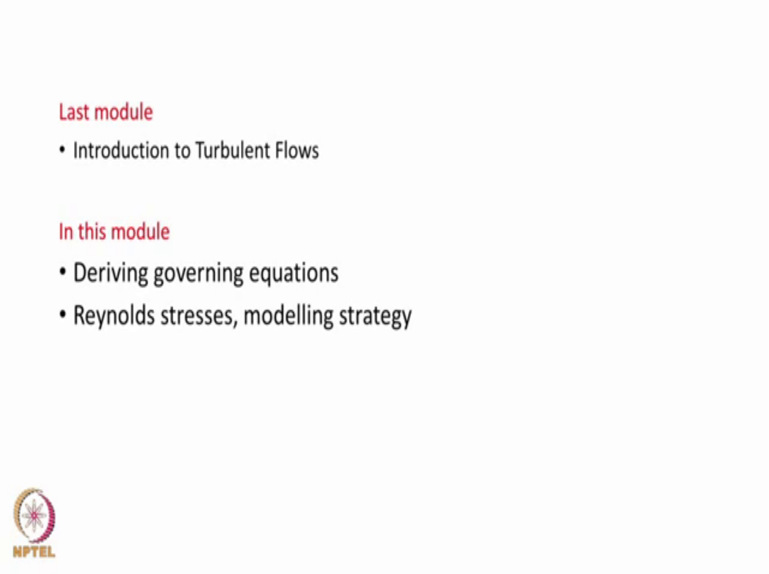

Greetings welcome, and welcome again. Today we move onto module two of this course. In the last module, we explain about turbulence flows with some introduction. We took example situation; flow passed building jet and so on. We also talked about mean velocity and deviation from mean velocity or instantaneous velocity is decomposed into mean velocity and fluctuation. We also learned a little bit about tensor notation and some algebra.

In this module, we will now focus specifically on deriving governing equation for turbulence flows. While deriving equation for turbulence flows, we end up and getting a new term Reynolds stresses. We defined Reynolds stresses and modelling of Reynolds stresses by different procedure. So, this different particular module is split into two part; in the first part, I am going to explain till we get the derivation of the Navier-Stokes equation for turbulence flows. In part two, we get some more detail about this Reynolds average stock Navier-Stokes equation and Reynolds stresses terms.

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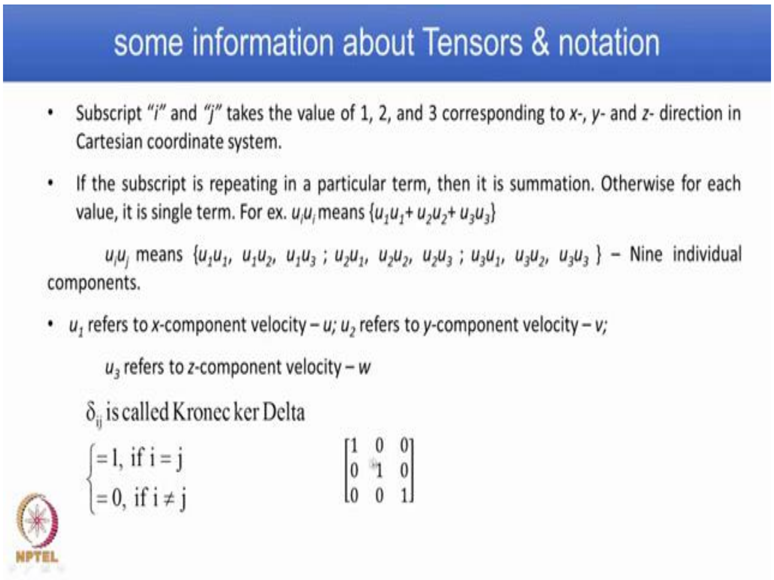

We will also get explain about the coarser problem. For the sake of completeness, I will have to repeat important information about tensor and notation used. We used subscript i and j, and i and j will take value 1, 2, 3. If the subscript is repeating in a particular term then it is a summation; otherwise it is each term is a single term. For example, u i u i is a particular term subscript i is repeating for every value of i you get one term, and then they are summed up. And as you can see here u i u i means u 1 u 1 plus u 2 u 2 plus u 3 u 3.

If you have a term u i u j where subscript is not repeating and then every value of i and j, you get u term; for every value of i 1, 2, 3, you have every value of j, so we have nine terms in total and all the nine terms are listed here as u 1 u 1 u 1 u 2 u 1 u 3, similarly the last term u 3 u 3. U 1 is refer for x component of velocity, and u 2 is used for y component of velocity v, and u 3 is used for z component of velocity w. We also learn a new term call Kronecker delta - delta i j. The Kronecker delta as a value of one, if i is equal to j; otherwise it is 0.Wwe also represented Kronecker delta in the form of the matrix as shown here 1 0 0 0 1 0 and 0 0 1. So, it is a diagonal matrix.

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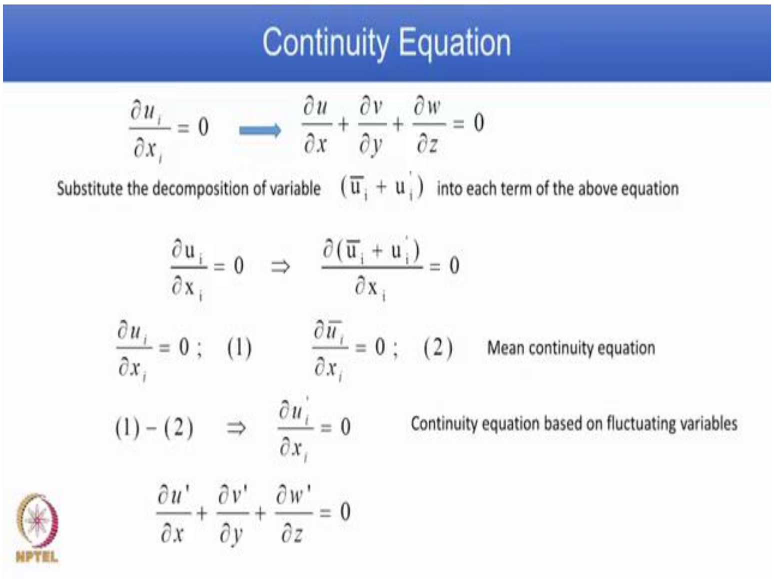

We will now start using index notation in detail and wherever possible I will repeat information about index notation. We know there are two main equations one is a continuity equation, other one is momentum equation. We will explain in detail how to derive continuity equation for turbulent flow first what is shown here in continuity equation is general we know for a study as well a comfortable situation the continuity equation is written as shown here that is dou u by dou x plus dou v by dou y plus dou w by dou z. Now the same continuity equation the form of the index notation is written here dou u i by dou x i equal to zero i mention if the subscript is repeating particular term it is summation and in this term i is repeating one in the numerator one in the denominator. As mentioned i takes a value of 1, 2, 3, and each term is obtain then they are summed up and that is what get here.

When you substitute i is equal to one you get u or u one and you get corresponding x direction i is equal to two you get you two or v and corresponding direction y i is equal to three you get w or u three and corresponding direction z. So, you understand the easiness same continuity equation with three terms in the form of index notation and it written concise. Now we substitute the decomposition, we already learn the flow variable instantaneous can be written as a summation of mean plus deviation from the mean for example, u instantaneous quantity is written as a sum of mean u bar plus deviation of the mean u prime now we use index notation it means the decomposition is applied for any particular velocity component u v w. So, we substitute this decomposition concept into each term in the above equation that is u v w that is the meaning of using this index notation i. So, dou u i by dou x i equal to zero if you substitute u i which is now in turbulent flow is interrupted as instantaneous quantity. So, dou u i by dou x i is equal to zero continuity equation it is very genetic then you apply for turbulence flow u interrupted velocity as a instantaneous velocity. So, dou u i by dou x i is equal to zero you are applied the decomposition for the velocity u i as shown here that is u i bar plus u i prime and divided by dou x i is equal to zero. So, you take that separately that is dou u i bar by dou x i as one component and u i prime is another component.

So, if you write down that way it becomes dou u i bar by dou x i equal to zero. So, this equation that equation that is mark as one is for instantaneous quantity that equation that is mark as two that is dou u i bar by dou x i is a mean quantity and if you subtract one minus two that is subtraction you perform then you get equation based on fluctuating quantity as shown here. So, equation two is mean continuity equation, and this particular equation where you use fluctuating quantity is a continuity equation based on fluctuating variable. So, what we have done this slide, we are written a generic continuity equation in very expanded form then we write the continuity equation in index form when we used continuity equation for turbulent flow we interrupted that u as instantaneous quantity we apply the decomposition for the instantaneous quantity as summation of mean plus deviation.

So, if you substitute then you get two separate equation one based on mean quantity another one based on fluctuating quantity in a expanded form you write dou u prime by dou x plus dou v prime by dou y dou w prime by dou z is equal to zero. So, you take this equation, and look at this equation that is initial written continuity equation and final written continuity equation the formal looks same, that it you have dou by dou x of u plus dou by dou y of v plus dou by dou z of w, we use quantity instantaneous other one mean other one fluctuating quantity. So, we have continuity equation written in the same form the variable are interrupted differently for different situation in the case of laminar flow it is a plain velocity component in the case of turbulent flow we get two separate quantity one based on mean another one based on fluctuating quantity.

Now, we can extend this procedure for any other equation is used to turbulent flow minimum you use quantum equation if you are having any temperature to be consider the energy equation are concentration separately we will use p c c equation. So, for if each of this equation we apply this decomposition principle and take a average to get corresponding equation to be used for turbulent flow.

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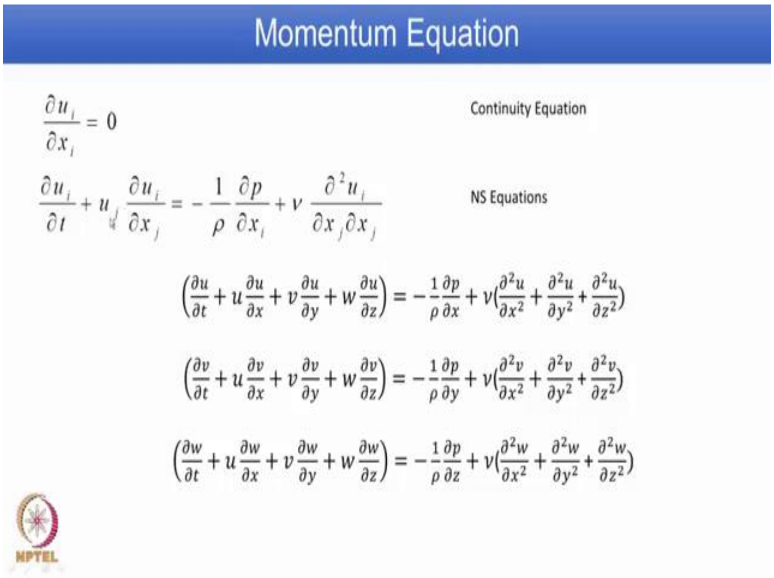

Now, let us try do it for momentum equation, I repeat again here continuity equation dou u i by dou xi equal to zero the continuity equation written in the index notation form. Now, momentum equation in the index notation form is shown here, we already know otherwise called Navier-Stokes equation dou u i by dou t plus u j dou u i by dou x j equal to minus one by pro dou p by dou x i plus mu dou square u i by dou x j dou x j. Now in this equation u i is only one term here again the next term u j dou u i by dou x j in this term u i appears only once where else j index appears more than ones - twice. So, for every quantity of j it will be one term that they are some term that is the rule we learn if the index is repeating then it is a summation otherwise each term single term.

So, i is single for every value of i you will get single term where else within the i for every value of j you will get single term, but they are some term now if you look at other term on the right hand side. So, dou p by dou x is a single term now the diffusion term numerator dou u dou square u i i is a single term there is denominator we have j index repeating. So, you get summation we already learn Navier-Stokes equation in very detailed form. So, this equation if you use index values then you get its expanded equation of the Navier-Stokes equation you can cross jet. For example, for substitute value of i to be one then you get x momentum equation and for every value of i equal to one for each term for the example second term on the left hand side dou u i is a convention term and u with the value of i equal to one is u itself whereas, j index repeating is a summation. So, for every value of j we get one u term and then we are added. So, u when j is equal to one it is u then j is equal two which is v and corresponding x j will be y when j is equal to 3 it will be a w and corresponding x j will be z.

So, if you put them together it jet this dou u by dou t plus u int dou u by dou x plus v dou u by dou y plus w dou u by dou z. So, we get three terms, but they are some term and u is only one for value of i is equal to 1. Now, you can immediately appreciate advantage of writing with in index notation in the original tensor notation form, just only one term for conjunction and in the expanded form you get three different terms on the right hand side. Once again when you substitute value of i is equal to one dou p by dou x can be appear in the second term diffusion term dou x j dou x j if you expand j with value of one two three and because its repeating summation you get dou square u by dou x square the dou square u by dou y square the dou square u by dou z square.

So, this first equation a x momentum or u momentum u momentum equation and we will now repeat for next value of i that is two. So, when you substitute i is equal to two this then you get v momentum equation or y momentum equation and that is want shown next equation similarly if you substitute i is equal to three then we get third momentum equation of w momentum equation that is want shown them on the last equation. So, these three equations in the detailed form, you are able to write very consist form using tensor notation as shown this equation, I will know you will appreciate index notation and understand the way index notation use to write that detail form of the equation.

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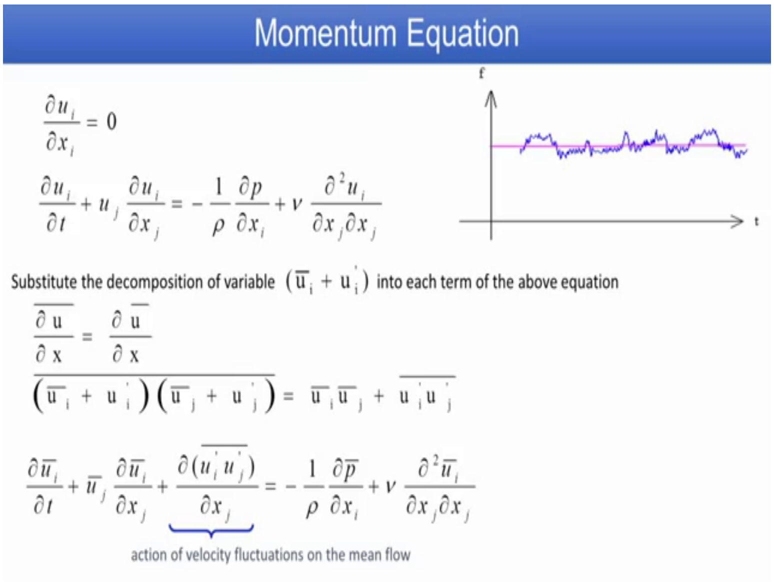

You already learned turbulence flows exhibit the random or variation, but the time and these are representation the quantity f as a function of time shows the fluctuation and we learn continuity equation in tensor form momentum equation is tensor form. Now we have taken out this equation to turbulent flows as a mention before this is generic equation. Now it depends on how we are interrupting for turbulent flow the same equation all these quantities already quantities in this equation u and u with index i 1, 2 3, the three variables that is u v w or u 1, u 2, u 3 then one more variables it pressure.

So, if you interpreted all these four variables in turbulent flow as a instantaneous quantity then you applied decomposition for each one of this quantity we know the decomposition. For example, u i instantaneous decomposed into summation of v and deviation. So, if you substitute can we get final Navier-Stokes equation to be used for turbulent flows. We learned little bit about tensor algebra some important information about tensor algebra there are some more thing that you do. So, we set time average on the derivative is a same as derivative on the time average. Now, the second quantity in the Navier-Stokes equation on the left hand side is a convention term, it has u j dou by dou x j of u i.

Now what do we do we applied decomposition into this. So, if you to do a time average, so you get u i bar plus u i prime and then u j bar plus u j prime and you take a time average on that quantity. Now why do we do this, because if you take u j into the derivative, so it becomes dou by dou f j of u i u j. So, you have a product of two velocity component now you want to apply decomposition to that product that is u i u j you want apply decomposition u i is a one instantaneous velocity u j is a another instantaneous velocity. So, if you write decomposition for the respective instantaneous velocity, you get u i bar plus u i prime the second instantaneous velocity u j because u j bar plus u j prime and we perform finally, time averaging and that is what is shown over bar covering both the quantities.

Now we follow some rules we already listed few rules based on averaging at the following rule we are not going to the details. Finally, you get a term as shown here on the right hand side. So, u i bar u j bar a product plus u i prime u j prime bar. So, if you write this term substitute this term for the conversion term you can apply similar steps for each term separately, separately for derivative, separately for pressure, separately for diffusion term and finally, you get a equation as shown here. In this, i am not showing in detail about steps it is more important to know the equation understand terms in the equation for detailed steps in the derivation one has to look separately other material it is slightly be on scope of present structure.

So, the final equation after performing decomposition for each variable following some tensor algebra performing average then you get equation I will shown here what is should be noted important thing is first term for example, is a time derivative term this term has a same form as a original Navier-Stokes equation original Navier-Stokes equation has a time derivative term the time averaged Navier-Stokes equation also has a time derivative term and this is u i bar and this is only u i it is original Navier-Stokes equation as a mention it depends on how you are interrupting for a turbulent flow the turbulent flow all these quantities are interpreted as a instantaneous, the second term in the Navier-Stokes equation u j dou u i by dou x j, the second term the time averaged Navier-Stokes equation this is u j bar dou u i bar by dou x j.

So, they can understand the second term appearance wise it is a same on the format wise it is same both in the original Navier-Stokes equation and the average Navier-Stokes equation the only difference is the variable. So, in the original Navier-Stokes equation it is instantaneous whereas, in the average equation it is average quantity then you have one new term coming into the picture that is dou by dou x j of u i prime u j prime bar this is a new term which is coming of after applying decomposition following tensor algebra doing average on the original Navier-Stokes equation then you end up getting one additional term in the time average Navier-Stokes equation and that is what is shown here.

Now, come to the right side pressure derivative term is same as the pressure derivative term in the original Navier-Stokes equation. It is except with time average similarly the diffusion is also same as the original Navier-Stokes equation except that. We use now time average quantity in the average Navier-Stokes equation the new term that is shown here dou by dou x j of u i prime u j prime bar. So, u i prime we know it is a fluctuation and bar is for average. So, this u i prime u j prime bar actually represent action of velocity fluctuation on the mean flow y it is mean flow because we have written this equation for average all the gamma appear as average. For example, u bar and p bar the only different term is this fluctuating quantity u i prime u j prime now this quantity u i prime u j prime actually represent velocity fluctuations. We known individually they represent velocity fluctuation, the relaxation of velocity fluctuation on the mean flow is actually represented by this particular term, now this term u i prime u j prime is main cause for turbulent flow modelling.

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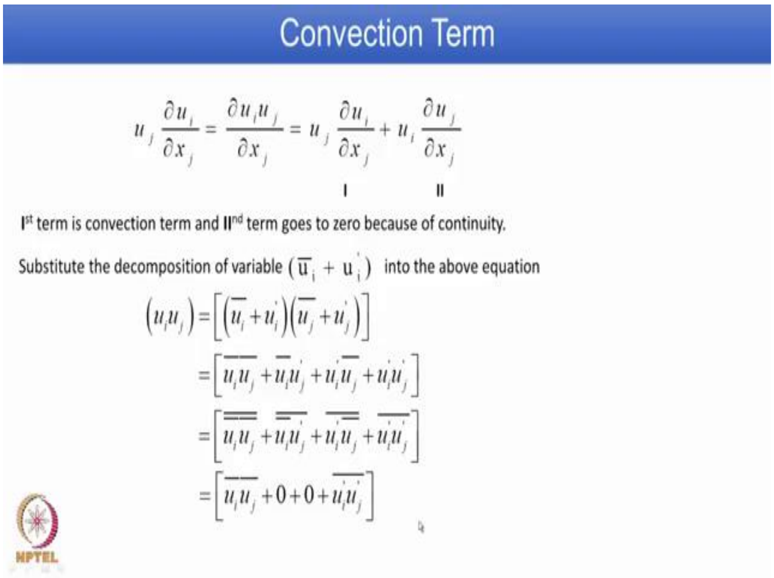

The last slide when we did the decomposition applied to convection term we end up getting a term called Reynolds stresses I did not explain a convection term in detail in this slide we will look into the convection term alone in detail. So, convection term is written I will shown here u j dou u i by dou x j equal to dou by dou x j of u i u j. So, what we have done now taken that term u j inside the partial derivative and that is what appearing here as dou by dou x j of u i u j. Now we have to explain y this can written. So, if you actually perform partial derivatives on this variables u i u j by product rule. So, you get first term as u j dou by dou x j of u i plus u i dou by dou x j of u j. So, it perform dou by dou x j on this variable u i u j apply the product rule principle then you get two different term as shown here.

Now, if you look at this term carefully the first term is actual convection term that what we have here u j dou u i by dou x j so that is the exactly convection term. The second term we have u i dou u j by dou x j and dou u j by dou x j actually represent continuity equation. We know for compressible flow, the continuity equation dou u j by dou x j equal to zero because of this it is possible to write convection term in the form I have shown here dou by dou x j as u i u j. If you substitute decomposition for every velocity variable as u i bar plus u i prime then we have u i u j written as u i bar plus u i prime u j bar plus u j prime now you perform the product. So, we have u i bar u j bar for the first term then u i bar u j prime for the second term.

Then third term u i prime multiplying u j bar then u i prime multiplying u j prime. Now you perform time averaging on this. So, they are shown separately here u i bar u j bar over bar u i bar u j prime over bar u i prime u j bar over bar u i prime u j prime over bar. Now we apply what we learn in tensor algebra. So, this time averaging on the fluctuation, here also time averaging on the fluctuation these two terms will go to zero. So, we have two terms that is u i bar, u j bar; second term and third term they go to zero then we have u i prime u j prime bar. So, the first term will be the convection term, but applied for turbulence flows the last term is an extra term and that is a Reynolds stresses. With this come to part one of this lecture, then we move on to part two.