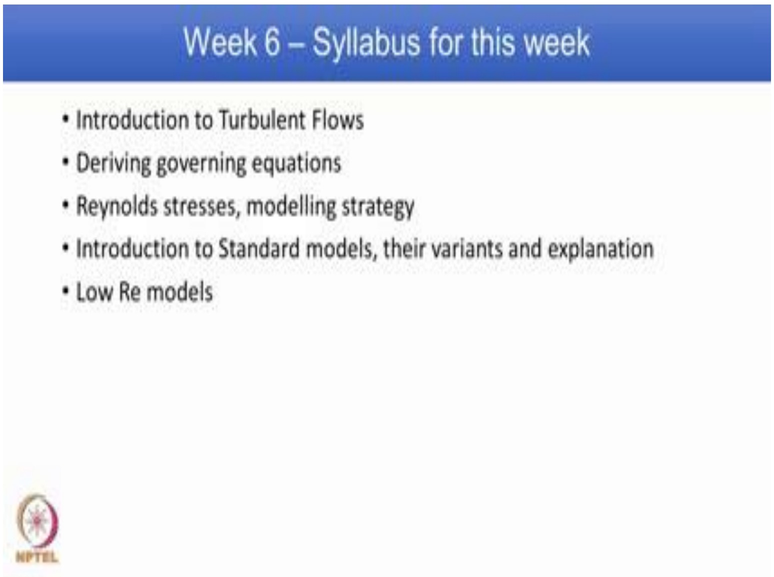
Foundation of Computational Fluid Dynamics
Dr. S. Vengadesan
Department of Applied Mechanics
Indian Institute of Technology, Madras

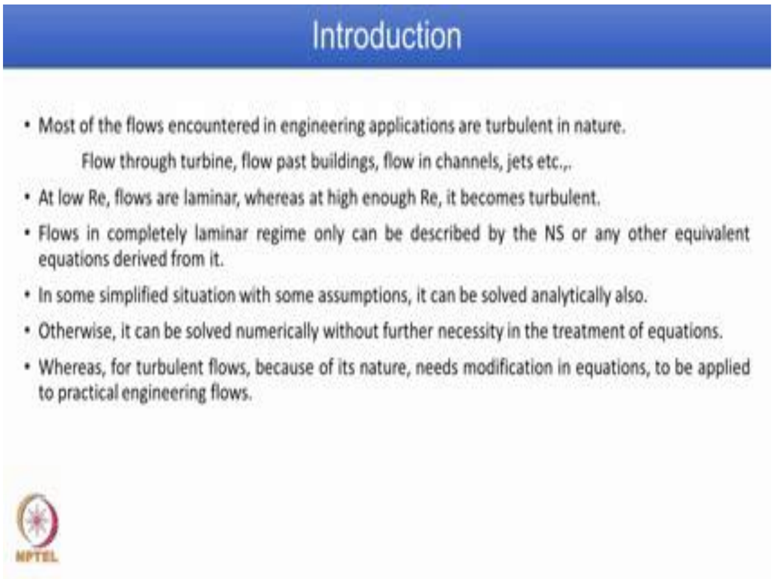
Lecture - 27

(Refer Slide Time: 00:28)



Greetings welcome again to this course on CFD. This week, we will talk about one important subject called turbulent flows. Syllabus for this week introduction to turbulent flows to deriving governing equation for turbulent flows in define a term called Reynolds stresses and how to do modelling of Reynolds stresses. And we have variety of turbulence models, we will talk about standard turbulences model, their different variants and explanation about the performance. We will also talk in detailed about another class of model what is known as a low Re models.

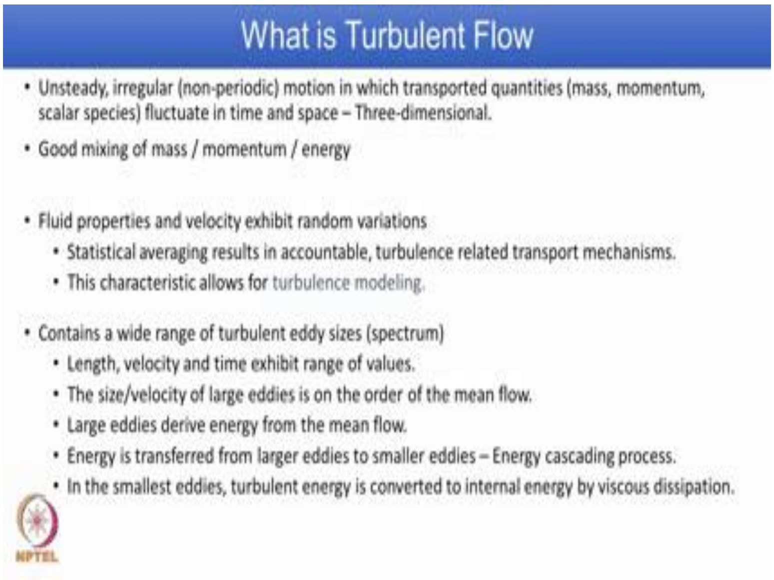
(Refer Slide Time: 01:10)



Most of the flows encountered in engineering applications are turbulent in nature. For example, flow through turbine, flow past buildings, automobiles, aircraft or any other structure, flow in a channels, jet etcetera. You can also observe agar bathi smoke or cigarette smoke, the origin and then diffusion or the spreading of the smoke, this is also due to what is known as a turbulence characteristics of that flow. We also have separate branch called as atmospheric turbulent, ocean turbulence etcetera. So, turbulence of exist all branches of engineering.

At very low Re flows are usually consider to laminar whereas, at high enough Re, it becomes turbulent in the case of laminar flow it is a very steady flow very organised flow and its possible to use Navier-Stokes equation or any other form of Navier-Stokes equation can be used. In very simple situation, it is also possible to obtain analytical solution for laminar flows. In other situation, it can be solved numerically without any further treatment in equations; whereas, for turbulence flows we will see it later what are the characteristics because of its nature it needs modification in equations, and special treatments before turbulence model can be applied to engineering problems.

(Refer Slide Time: 03:07)



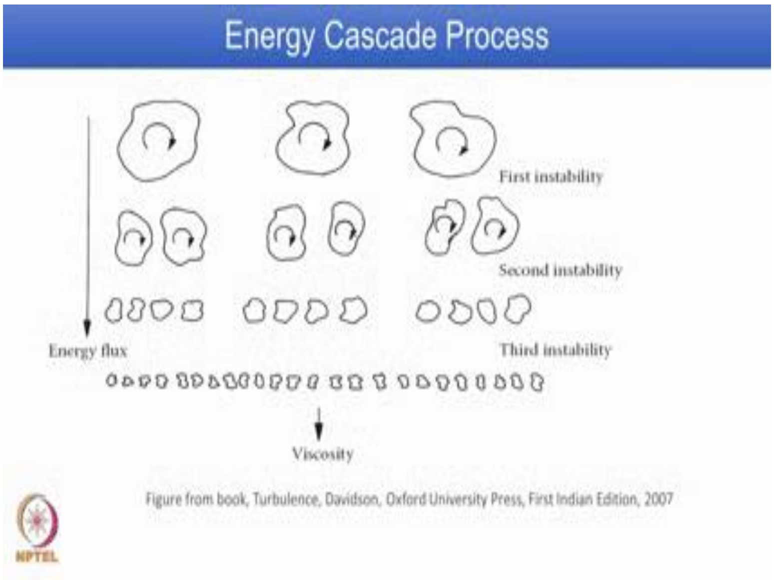
Let us start defining to turbulent flow its unsteady. So, you do not have anything like unsteady term drop in Navier-Stokes equation it is irregular. So, you do not have any predefined situation non periodic motion we are worried about are we are concern about three properties mass momentum and scalar species it may be temperature it may be density it may be concentration. So, in turbulence flows these quantities are transported and they explicit fluctuation both in time as well as in space hence it becomes three dimensional. So, we have learn three important property one is unsteady irregular and three dimensional you can have a situation of laminar flow unsteady, but it will not have a three dimensional it will not have a three dimensional t in the flow because of this there is a very good mixing of mass momentum as well as energy.

For example, you take coffee and you add sugar and to get fast mixing you stir it with the spoon or stirrer then sugar get mixed quickly and this is the example of turbulence flow mixing. Similarly as I mention before the cigarette smoke or agarbathi smoke spreading by process diffusion gets faster, because the turbulence flows. And take a bucket of water you drop a Neem it is go by itself by laminar flow, but if you stir it then its go faster the spreading is faster because of turbulence action fluid properties and velocity exhibit random variations though they exhibits random variations it is possible to apply statistical tool one can obtain average r m s or flirtation squares to statistical averaging results in accountable turbulence related transport mechanisms and it is because of this it is possible to do what is known as turbulent modelling which is a subject for this week it contain wide range of turbulent eddy sizes.

So, when we talk about turbulent flow we talk about spectrum when we say spectrum where is the range the look for three information that is velocity time and length if we look for any two the third one you can get that is from velocity and length we can get time or from velocity and time you can get length information. So, in a flow you look for anyone of any two of these information and you can obtain the third information by the dimensional an argument. So, when you say the spectrum for example, velocity information will have has as small as possible to has high as possible similarly when you say length information the eddy such as can be as small as possible as to high as possible the eddy is are fluid master and then undergo motions whenever the undergoes motions it cover some length in the flow and there is the turn over time and that is what give you the time information. So, velocity length and time related to eddy sizes it can be as small as possible to as high as possible there is a wide range available in a third flow and this is what is known as a spectrum and they exhibit range of values the size and velocity of large eddies is on the order of the mean flow.

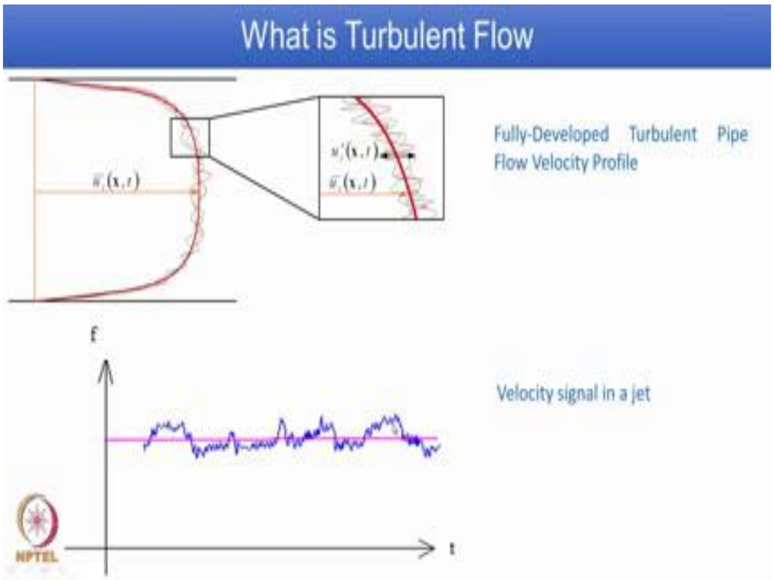
So, for example, if you consider flow through a circular cross section pipe then largest eddy possible size is a diameter of the pipe itself then there is a process called energy cascading. And now we explain that next slide with the help of the scads the large eddies derive energy from mean flow the intern passed to next small size they intern breath the next smallest size and it goes finally, at the smallest size, it get dissipated by viscous action. So, energy is transferred from larger eddies to immediate smaller eddies and then the cycle then immediate smaller eddies and then the smaller eddies is possible, this process is called energy cascading process in the smallest eddies, turbulent energy is converted into internal energy by viscous dissipation process.

(Refer Slide Time: 08:28)



This figure is an illustration to explain the energy cascade process as you can see here. These are all larger eddy is possible in flow and there is an arrow to indicate the eddies are in the motion with some angular velocity whenever the eddy comes back its position, then it is called that is a time or whenever it is moves to the next position you called that is the length information for the particular eddy. So, in this process cascading energy cascading process largest eddy transfer energy come itself to the next smaller eddy then they intern transfer energy to the next immediate smaller eddy and this stage goes until the smallest possible that is want shown at the bottom as I am shown here where the energy get dissipated because of the viscosity action. So, in Navier-Stokes equation we have a discuss diffusion term and actually plays roll here now what is an responsible mechanism for the transfer of energy from one eddy to the next eddy is by what is known as the instability process and this instability is created by initial component in the Navier-Stokes equation which is conventional terms.

(Refer Slide Time: 09:59)

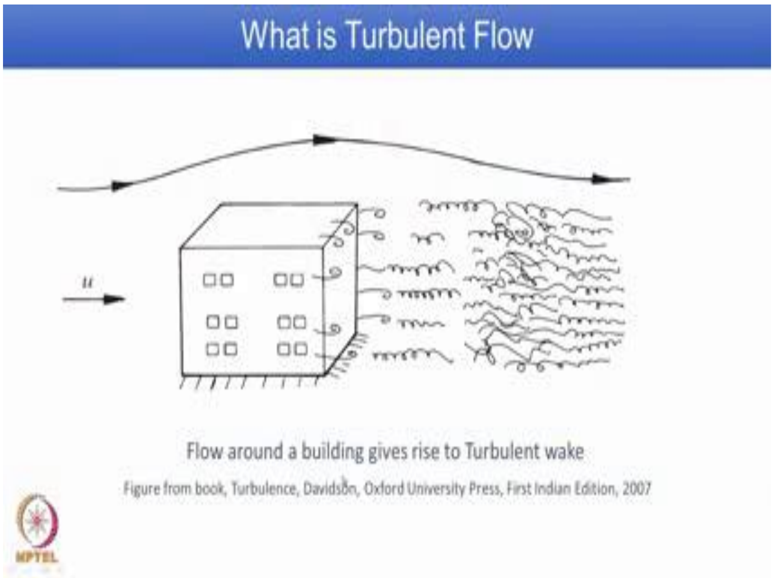


This figure is from a book on turbulence by Davidson to get some more idea into the definition of turbulent flow. I am showing here which flow through a pipe, there is a red colour profile, which is a mean velocity profile. So, we learned what is mean by in subsequent lectures now this portion alone. It is zoom, and it is shown here and you can see clearly red line profile that is a mean velocity profile then you see while variation and these variations are shown with two different colours so the mean is denoted by u sub root i and there is bar the bracket have x bold letter comma t.

Now let us understand what is a symbol so i subscript stands for tenser notation. The next lecture I am going to talk about tenser notation and some algebra association with tenser notation the bar is stands for average the x bold stands for to represent in which direction that is vector component then t stands for time. So, this u velocity profile, it is a function of both space as well as time that is the meaning of writing x comma t, now the mean is only one, but you have seeing two different colours to represents the fluctuate. And you can seeing here to represent fluctuation with respect to the mean. Now the fluctuation is function of time that is why we get a one instant one fluctuation that is shown in one colour and it some other instant, it other some other fluctuation or the deviation from the mean which is shown by another colour.

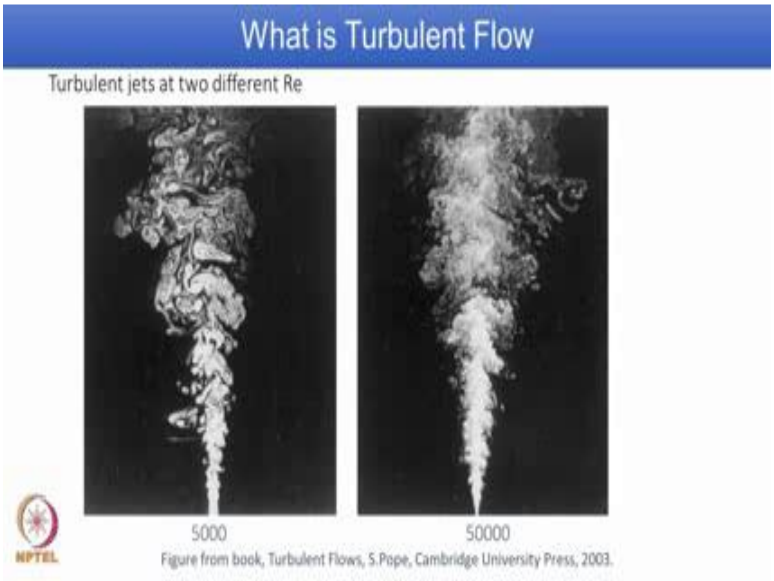
Now the deviation from the mean is represented by prime quantity because we are talking about velocity in i. So, it is u i prime and x comma t. So, fluctuation is a function of both space as well as time and that is what is shown here with notation x comma t. We explain just before that turbulent flow exhibit three-dimensional t that is the function of both space as well as time and you are able to understand with the help of this velocity profile and this is an another example this is a velocity signal it is capture by the instrument in a jet flow at one particular x location. So, you get any property, f for any property and t for time instant and you see one horizontal line to represent its average and if you take the signal at a blot as a function of time you get a wide variation. For example, if you open a tap and get jet coming out at centre of the jet if you insert any velocity pickup instrument and you record the signal this over it appear.

(Refer Slide Time: 13:35)



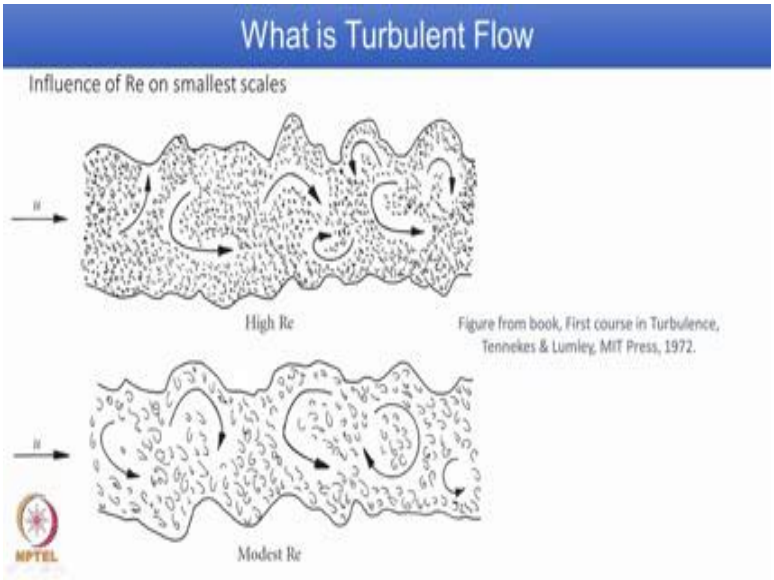
This is another example to understand what is an turbulent flow. So, this is representation for flow past to the building here is the building consider to a three-dimensional geometry flows from left to right it will flow around as well as it will flow over now if you observe immediate downstream here. These are lines to represent see here shutting and what is that shut from to geometry and if you go slightly for enough then you see the motion it becomes the turbulent. So, flower on building gives rise to what is known as the turbulent wake. So, wake is a region of the flow immediately begin the geometry. So, in this case the geometry is building and this is a way region. So, the way becomes turbulent this is important we have already mention turbulent flows exist in all works of engineering, in case of building also there is a turbulent flow what we see in just representation, in terms of isolated building, but in a setup we have varies high and immediately located adjacent with each other then you need to steady flow through building.

(Refer Slide Time: 15:08)



Here is an illustration to understand though it is turbulent, how it is behaves for two different Reynolds number. There are two pictures shown this is turbulent jets at two different Re on the left Reynolds number is five thousand on the right Reynolds number is fifty thousand different between them is one order of magnitude jet is suit at the bottom as i am shown here and the same situation on the other side now you can get Reynolds number differently either by increasing the velocity at the source or changing the geometric dimension at the source. Now immediately after the exist, there are no problem flow appears to be laminar in both situation, but as you move down stream. So, if you go little for either here are here at the same location for both situations, you see different structure for two different Reynolds number we will have a close look at one region in the next slide.

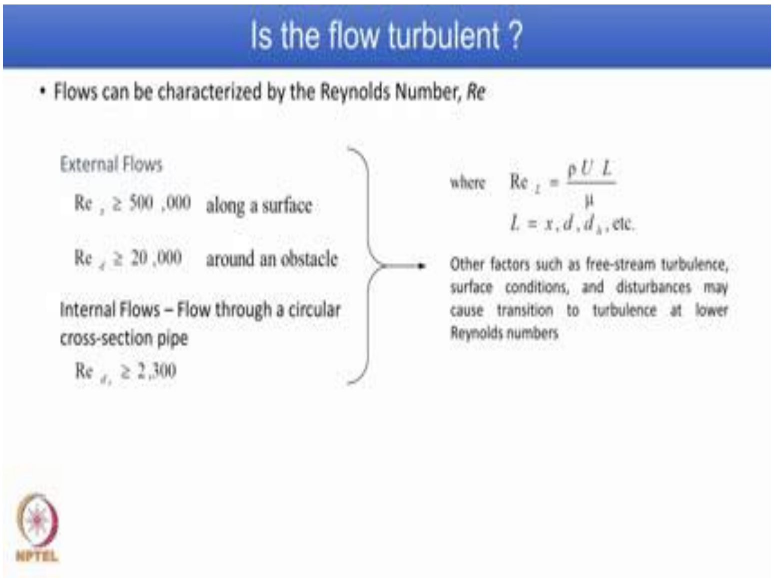
(Refer Slide Time: 16:19)



So, the same jet we take one small portion zoom try to understand how the turbulent flow at a leave two different Reynolds number the bottom one is modest Re, and for the top one high Re the flow appears to be left to right. Now you needs figure, we have a big arrow mark in also have a small curve line marks big arrow that is mark to represent large d motion we already mention larger d is limited by geometry dimension of the flow in this case the jet runs the jet is issued the geometry dimension is actually spread. So, in this case for example, from this side to this side this is a geometric dimension spread that is a limitation larger d will move between this limit and that is a motion that is it possible.

So, both in this case as well as in this case the large d is almost same. Whereas if you look at small curve in line what you see on the top which is for the high Re you will see many small curvy lines, and this curvy line sizes are smaller then what you will see in the modest re. So, modest Re also shows curvy line and these curvy lines represents smallest small eddy motion in the smallest eddy is mark by the smallest curvy lines. So, the information from this sketch is as you increase the Reynolds number the large eddy size is almost un effected where are the smallest eddy is possible becomes smaller and smaller. As you go on increase the Re number you can observe once again at modest Re the curvy line which are represent smaller eddy are only are having default length that the same curvy line becomes smaller when you increase the Reynolds number from modest Re to high Re. And this is very important because when you model turbulence in flows latter you have to know what is the smallest merge size that you have to describe smallest merge size capture smallest merge size the d size motion and the prediction becomes better. So, I repeat as you increase the Reynolds number the size of the smallest eddy becomes smaller and smaller. And that puts the limitation are careful choice of the smallest mesh size that is possible.

(Refer Slide Time: 19:13)



Now, flows can be characterized by the Reynolds number we already know and the Reynolds number definitions what is shown here row u l by mu 1 has to carefully interpreted by what is u and what is l u is a velocity scale an l is length scale for example, flow through the circular cross section pipe the well-known example if we consider velocity scale u it can be a centring velocity or it can be a average velocity bulk velocity similarly if you consider l length for example, if you consider flow faster flat a the distance from the origin can be a length if you consider smoke spreading from the cigaratte or from agarbathi, but from agarbathi stick then distance from that origin wherever you are that becomes a length.

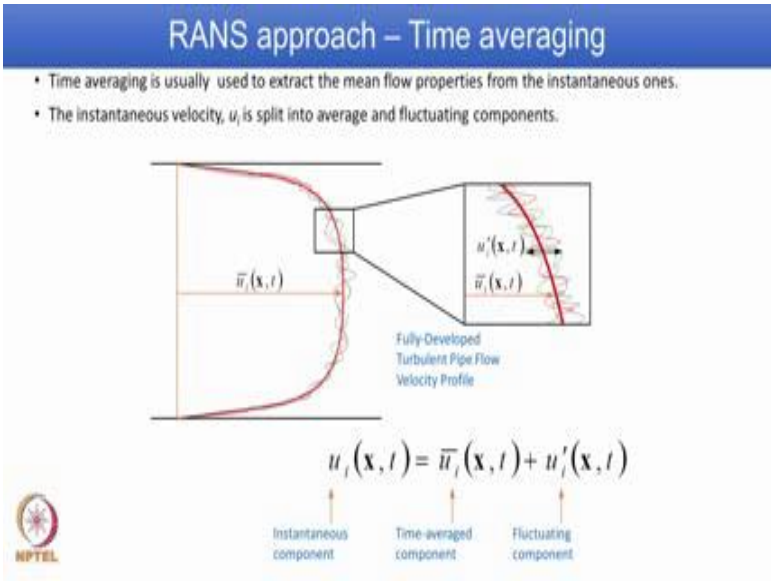
So, the Reynolds definition we know the ratio of the discuss flows after substituting formula for initial force and viscous force finally, arrived this formula. So, we have to carefully interpreted u and l u stands for velocity scale l stands for length scale lets by using this procedure it can be x which is distance from leading h r from the source it can be diameter of the pipe or it can be hydraulic diameter of the pipe depending on the definition of u and l for the same flow you can have the different Reynolds number and you have carefully used the circle. So, in this case for example, l is used then you write Re subscript

l. So, we have two situation commonly encountered either external flow or internal flow. So, external flow for example, flow as for a flat plate flow over an air fowling flow for the q obstacle buildings etc and flow for along a surface, we defined length scale l at the distance from the origin of the source and that why Re x. Now that flow becomes a turbulent if the Reynolds number defined based on the different greater than or equal to five hundred thousand.

Next situation flow past on obstacle or building or square cylinder surface etc then the Reynolds number defined accordingly the dimension for example, if it is a building it can be cross section or it can be height you have to define accordingly then Re d. If it is greater than or above twenty thousand then the flow can be characterize is a turbulence flow. Next situation is a internal flow well known example flow through circular cross section pipe, it can be any cross section, it can be a sudden convergence section or it can be enlarging section. So, Reynolds number it depends based on hydraulic diameter and it that value can be greater than or equal to 2300 when the flow comes turbulence now this values are only a guide line they did not be same at all situation they are also functions what is known as free term turbulence surface condition.

For example, if a consider flow through a circular cross section pipe we assume the internal walls are perfectly smooth it is possible that not have introduce on walls then the surface condition changes there roughness is supposed to clear roughness plays the role of enhancing the turbulence. So, for the same geometry for the same velocity scale for the same length scale if you introduce the roughness then the flow may be come terminal even before what is defined as the two thousand three hundred similarly any disturbance upstream creator any disturbance may cause transmission turbulence even at lower Reynolds number

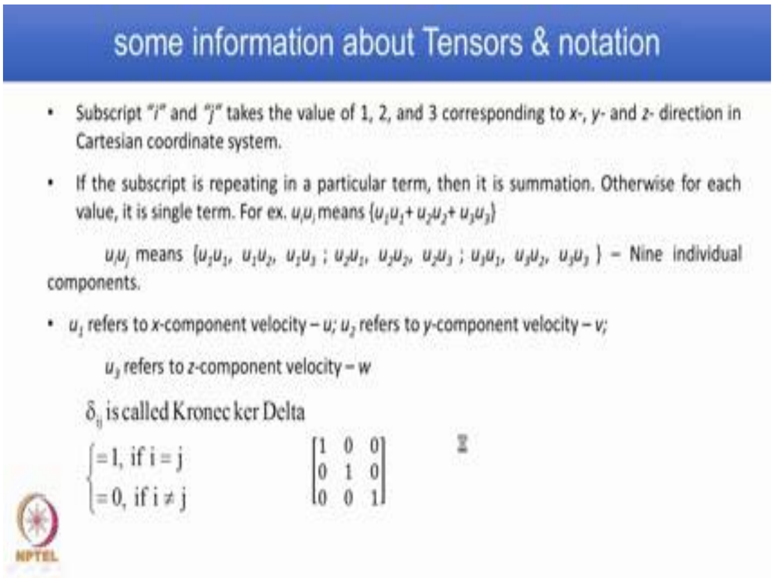
(Refer Slide Time: 23:41)



Now we look at little more detail, we already mention though flow exhibits randomness it is possible to approach the turbulent flow by what is known as the statistical methods and one such a quantity is a averaging because we mention it is varying both in space as well as time. It is possible to do both spatial averaging as well as time averaging for now we will limited to only time averaging and explain turbulent flows with help of only time averaging quantity. So, time averaging is usually used extract the mean flow from instantaneous once. So, we will any quantity that is varying with time we refer it has a instantaneous. Once you do time averaging then we get a mean quantity if you subtract one from the other then you get fluctuation in other words the instantaneous quantity u for example, and if you subscript i to representing in one direction or it a tensor quantity u i is split or decompose into average as well as fluctuating component if you already learn with the help of this figure.

So, this is a figure for flow through a i and velocity profile is marked with the red colour and we take a one small portion zoom and see the velocity profile. You already mention we have u bar x comma t and that is shown here as a mean velocity profile and you have fluctuation the two fluctuations marked just understand that you get different signal for different time level and the deviation from the mean is marked by prime. So, in this case, it is u i prime x comma t. So, you can put them together and that you give you the instantaneous quantity and that is want to show the expression here u i x comma t equal to u i bar x comma t plus u i prime x comma t, where u i is instantaneous component u i bar is a time average component u i prime is a fluctuating component. We are explaining all this with the help of one quantity that is velocity if you are involved with temperature density or any concentration each one of them represented in the same bit that is decomposition instantaneous quantity equal to mean average sorry instantaneous quantity is equal to mean plus deviation from the mean or the fluctuation

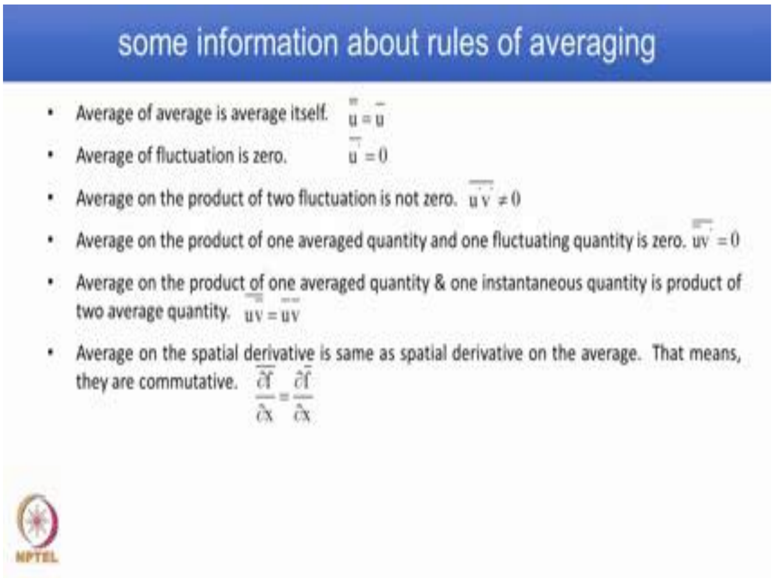
(Refer Slide Time: 26:41)



Before going to further details that is have some information about tenser notation and some algebra I would like to caution here whatever I am doing in this course is only very limited and aided very expected to take further references to understand more we use subscript i and j each one of them take the value of 1, 2, 3, 1, 2, 3 corresponding to x y direction respectively in Cartesian coordinate system if the subscript is repeating in a particular term then it is summation if it is repeating then for each value you get single term for example, if you like u i u i as one term i subscript is repeating in both then it becomes the summation. So, in this case, as a mention before i takes a value of 1, 2, 3. So, you write u i u i very explicitly as u 1 u 1 plus u 2 u 2 plus u 3 u 3. So, this is the short form of writing this quantity u i u i if you write u i u j if there is a term u i u j then the subscript is not repeating i gets a value independently 1, 2, 3 and j take the value independently one two three. So, we write complete form for every i value j will take value 1, 2. 3.

So, we write here u 1 u 1 which mean i is 1 j is 1 u 1 u 2 which means i is 1 j is 2 u 1 u 3 which means i is 1 and j is 3. So, you can observe each term is independent similarly i now takes the value of two j will take again the value of one two three separately another three component u 2 u 1 u 2 u 2 u 2 u 3e third possibility i takes a value of three and j takes a value of one two three. So, you have u 3e u 1 u 3e u 2 and u 3e u 3e. So, you observe when you write a term u i u j i takes a value independently 1, 2, 3 and for every value of i j takes a value independently 1, 2, 3 in total in get nine components nine independent component and all the nine independent component are displayed here now you see advantage of writing in tenser notation to represent nine independent components it is so easy to write the help of subscript notation u i u j. We also learn that index repeating that index is called dummy index if the index is non repeating this called free index. Now what is the meaning of u 1 u 2 u 3, we usually write x velocity has u y velocity has a v and z velocity has a w inside of writing that way we can also write u 1 refers x component and u 2 refers y components and u 3e refers to z component velocity we going to use one u term called kroncc ker delta which is given by the symbol delta with the circuit i j this kronecker delta has a value of one if i equal to j or zero if i not equal to j and this also is a matrix which is along the diagonal has a value 1 1 1 as I mention here. Then i is equal to j it takes the value of one. So, you can observe this is delta 1 1 and this is delta 2 3 and this is delta 3 3.

(Refer Slide Time: 31:20)



We will try to learn more information about rules of averaging average of average of average itself in this case u bar is average and you take the average of is that u bar that u bar itself average of fluctuation is zero. So, u prime is a fluctuation if you take average that will go to zero. So, fluctuation it may be positive and it may be negative if you take the average it will go to zero average on the product of two fluctuation; however it is not zero. So, if you take u prime which is a fluctuation for u velocity v prime, which is fluctuation for v velocity and you have a product of these two fluctuation. And you applying time averaging on top of it they will they need not v zero averaging on the product of one averaged quantity and one fluctuating quantity is zero expression y u bar is an one average quantity and v prime is fluctuation you have the product of them u bar v prime. And you take the average on that is equal to zero average of the product of one averaged quantity and one instantaneous quantity i a a product of two average quantity. So, u is an instantaneous quantity v bar is a average quantity if multiply of both of them can you take the average that will result in u bar and v bar.

Average on the spatial derivative is a same as spatial derivative on the average; that means, they are commutative in term of expression dou f by dou x is a derivative spatial derivative of the function f. After you perform a spatial derivative and you do average that is the meaning of average on derivative spatial derivative is same as you take a derivative on the average. So, dou f by dou x f bar average of spatial derivative is same as spatial derivative on the average, so they are commutative. I am only giving limited information about statistical way of dealing, these thing will be extremely helpful when we going to the detail of turbulent flow modelling. So, in today’s class, I try to give information about what is turbulent flow with the help of illustration, some information about tensor algebra, index notation and rules of averaging. Next class, we will go into detail about equations.

Thank you.