Foundation of Computational Fluid Dynamics
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Lecture - 26

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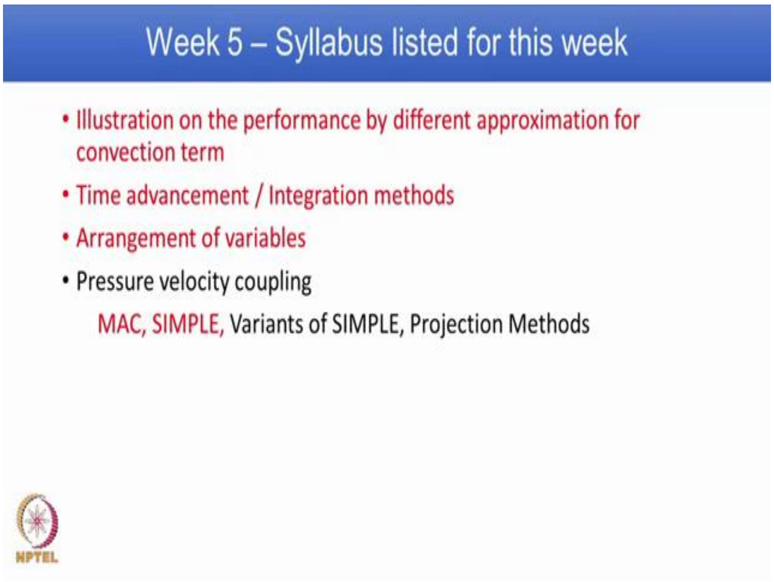

Greetings and it is my pleasure to welcome you all again to this course. We have seen so far illustration on the performance by different approximation method used for convection term, time advancement or integration methods available, arrangements of variables, needs for pressure velocity coupling. Four methods we said we will take to explain; we have already seen MAC, SIMPLE algorithm. And today's class, we are particularly going to see variants of SIMPLE that is SIMPLE R and SIMPLE C, and projection method.

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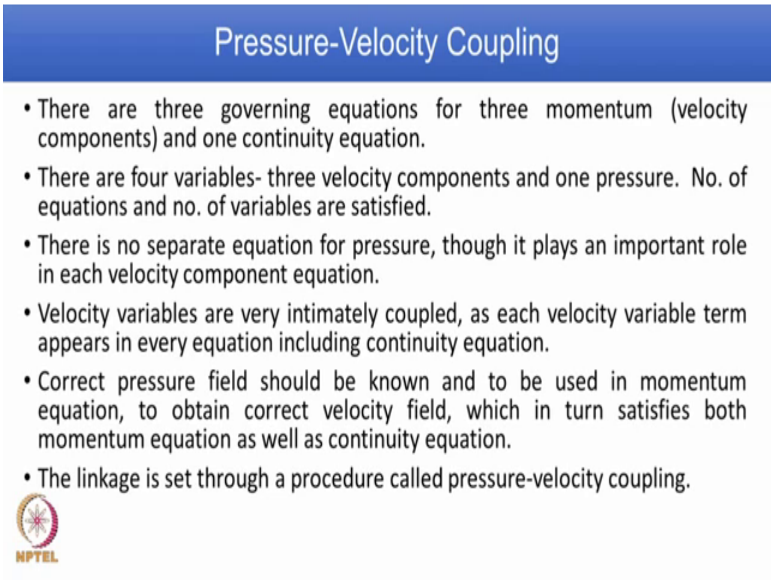

For the sake of completeness, we will go back and see what is pressure-velocity coupling; we know there are three momentum equations for three velocity components and one continuity equation. Velocity term appear in all the three equations as well as continuity equation; pressure term appear in all the three equation for momentum. There is no separate equation for pressure, though pressure gets in turn appears in all the three momentum equations; otherwise you have four variables, and four equations, number of equations equal to number of variables is satisfied. Velocity variables in turn they are coupled, because each velocity term appears in every other equation, hence there is a strong coupling among velocity components. And correct pressure should be specified in the momentum equation, to get correct velocity field which satisfies the continuity equation also. The procedure to link between momentum equation and continuity equation, the procedure called pressure-velocity coupling is used. The linkage is set through a procedure called pressure-velocity coupling.

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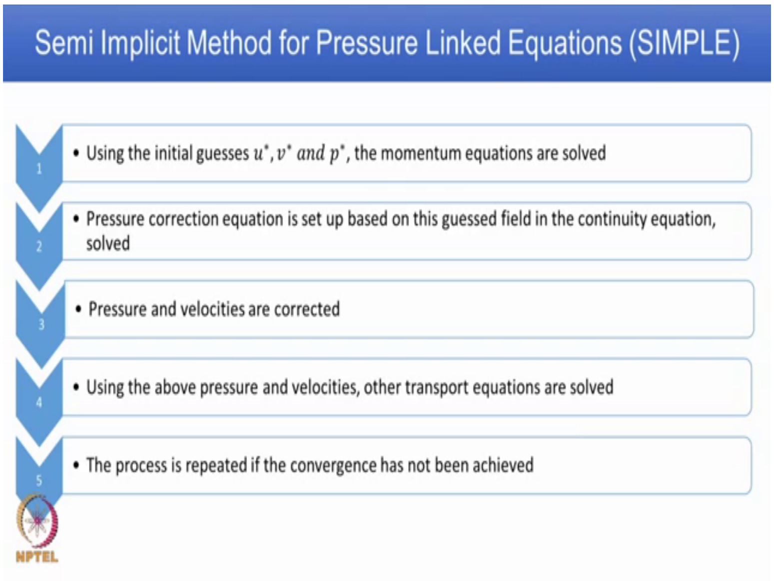

We will have a relook at simple procedure. Using the initial guesses for velocity u star, v star and pressure p star, momentum equations are solved. Pressure correction equation is set up based on this guessed field in the continuity equation, when you solve the continuity equation, we get pressure correction term. And this pressure correction value is used to correct pressure as well as velocities. Using the above pressure and velocities, one can solve any other transport equation as well. The process is repeated until the convergence criteria is satisfied.

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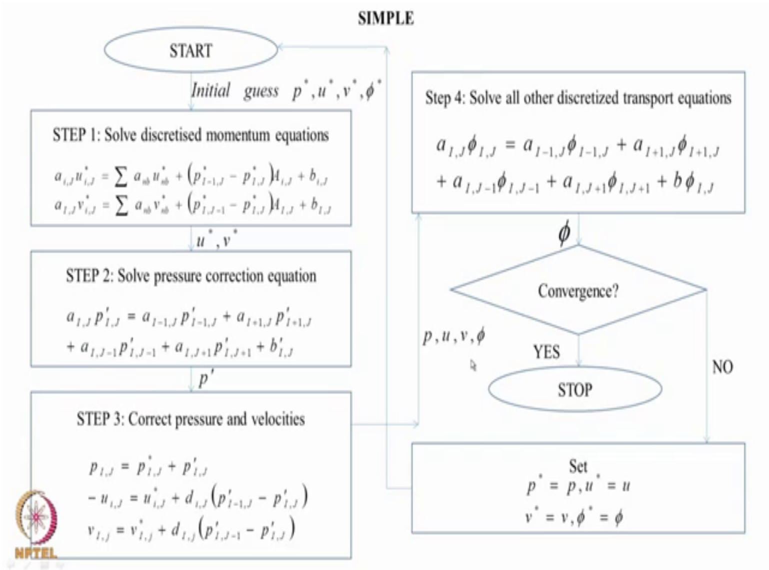

SIMPLE algorithm is shown in the flow chart as here. You start with the guess values, solve the discretized momentum equation, to get u star, v star. And this is used in the continuity equation to obtain the pressure correction equation; once you solve this equation, you get p prime and that is used to correct pressure as well as velocity as shown here. Once you solve pressure, velocities then you can take up any other scalar equation, which is given here - the symbol phi. You can follow any finite volume procedure to get discretized equation for transport equation also. So, you solve this and you get phi. And you check all of them for convergence, if convergence is satisfied you can stop the iteration there, if it is not satisfied, you need to repeat this procedure. Before repeating the procedure whatever value is obtained in this step as well as in this step, the pressure velocity as well as any scalar variable or reset as guess pressure, guess velocity and guess value for the scalar to proceed to the next iteration and the process is repeated. So, this is algorithm for SIMPLE.

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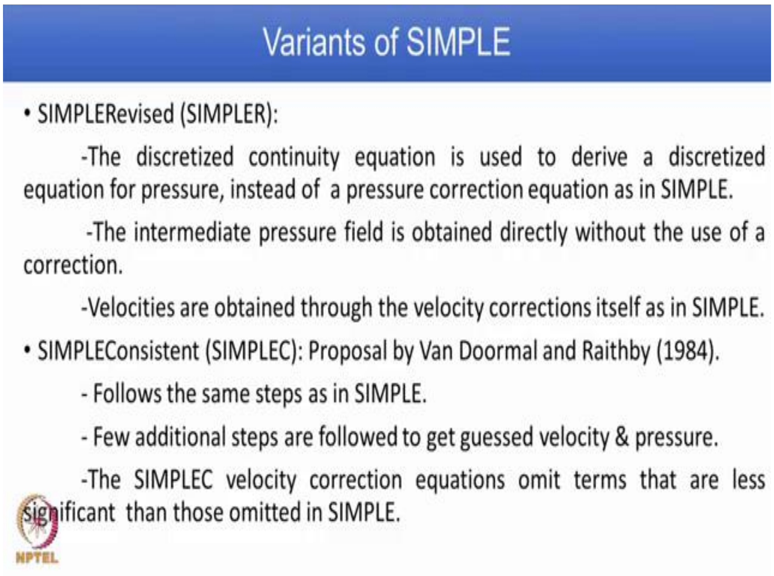

There are two versions available, variants of SIMPLE; first one is what is known as SIMPLERevised, in short form it is also called SIMPLER. In this, the discretized continuity equation is used to derive a discretized equation for pressure, instead of pressure correction equation itself as in SIMPLE. The intermediate pressure field is obtained directly without the use of a correction equation that is the difference between SIMPLE algorithm and SIMPLE R. And velocities are obtained through velocity corrections itself as in SIMPLE algorithm. Another variant is SIMPLEConsistent, in other words SIMPLEC in short form, this was proposed by Van Doormal and Raithby in 1984. It follows almost the same steps as SIMPLE. Few additional steps are introduced initially to get guessed velocity as well as pressure. We will look at those steps in the next slide with algorithm flow chart displayed.

The SIMPLEC velocity correction equations omit terms that are less significant than those omitted in SIMPLE algorithm. If you recall important step in SIMPLE algorithm, you would drop the term correction contribution from the neighboring nodes that is sigma a and b u prime n b that is the neighboring node contribution and that term is drop and that is important step in SIMPLE. Now, in SIMPLE C also, you will drop important term, which are less significant than those omitted in SIMPLE algorithm.

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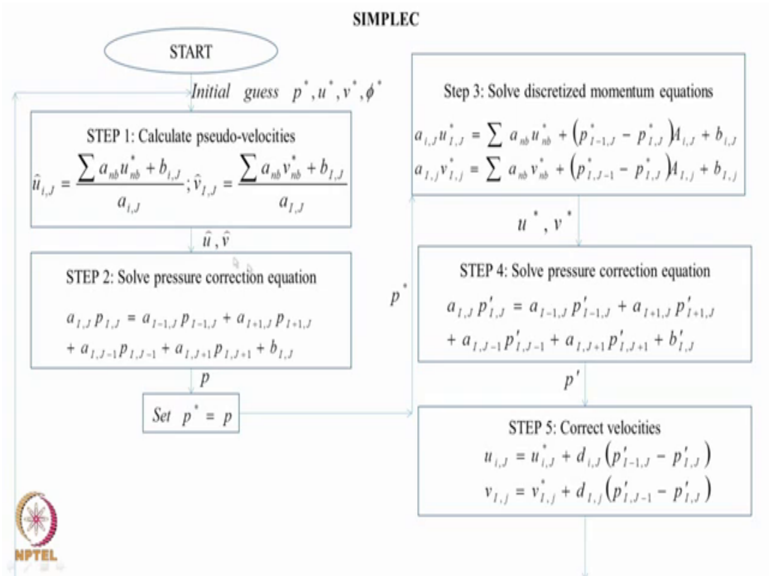

Without go into the details, we have a flow chart to explain the SIMPLE C algorithm in detail. So, here we start with the initial guess that is the initialized condition p star, u star, v star and phi star. So, we calculate what is known as a pseudo-velocity as shown here. And if you look at that expression, this is the discretized momentum equation, sum of neighboring nodes plus source term and coefficient for the corresponding interest node as shown here. Similarly, for the second velocity component v, again corresponding velocity coefficient and neighboring nodes contribution plus source term that is there in the v momentum equation. So, this is basically a discretized Navier-Stokes equation only thing it is applied for different situation. So, if you solve this equation, you get u cape and v cape that is used in the discretized continuity equation to get pressure, equation of pressure as shown here. So, once you solve this equation, you get p; now this p is said as p star. And this is the starting step for simple algorithm.

So, when you compare SIMPLE and SIMPLE C, you have two additional steps and these two additional steps are introduce to get pressure and this guess pressure is supposed to be closer to that actual pressure, because you obtain from solving the continuity equation, which is again obtain using u cape and v cape obtained from the momentum equation. Now, after this the steps are same as in SIMPLE, so you solve discretized momentum equation, you get u star, v star then you set pressure equation, then you set pressure correction equation, so p prime is obtained through that pressure correction equation solution and then velocities are corrected. So, you have two additional steps introduced when compared to SIMPLE, in SIMPLE C.

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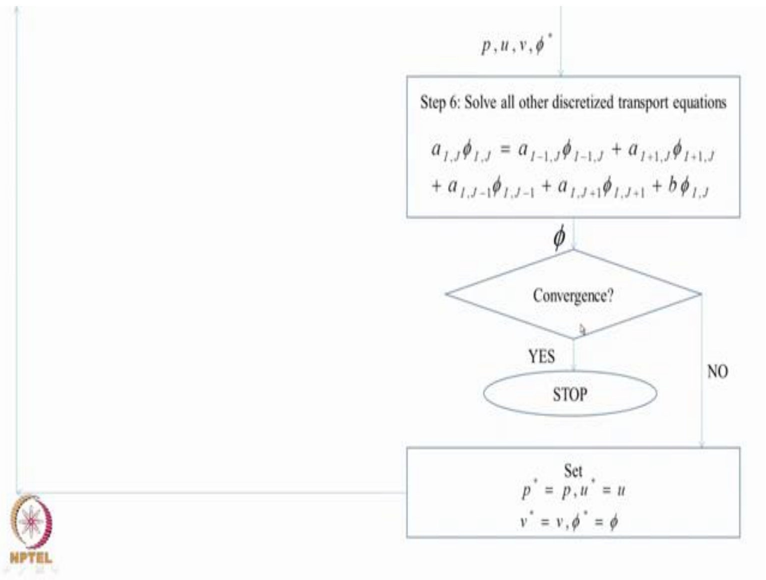

This is followed and you get p, u, v and any other scalar, any other scalar phi. Again you solved transport equation, discretized form of the transport equation for the variable phi, and you check for the convergence and you stop if convergence is satisfied. Otherwise, repeat this procedure, go back to the first step that is again starting from guess value and proceed. So, SIMPLE C, you can immediately make a guess, it takes slightly longer time in initial, but then comparatively it takes less time because the guess pressure as well as guess velocities are closer to the actual pressure and velocity. So, we have seen in detail two important algorithms SIMPLE and SIMPLE C.

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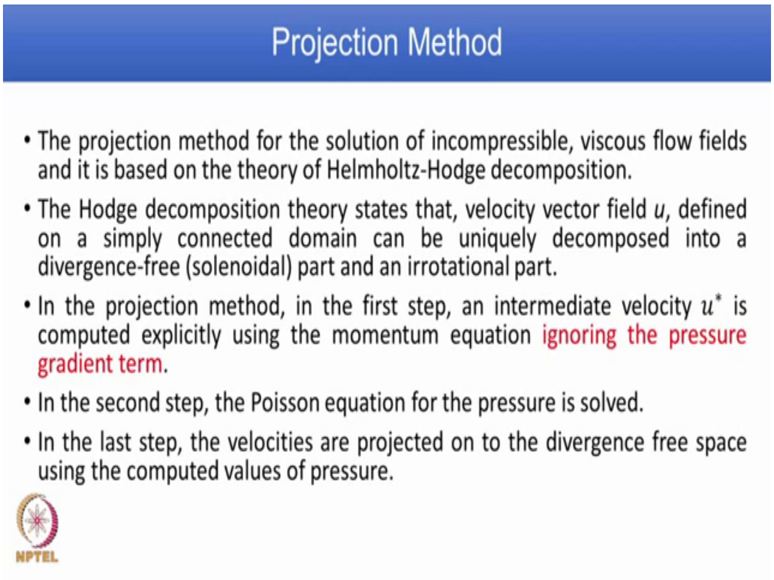

Next go to the next method what is known as a projection method, the projection method for the solution of incompressible, viscous flow fields and it is based on theory what is known as Helmholtz-Hodge decomposition of velocity field. So, this decomposition theory states that any vector, any velocity vector field u, on a simply connected domain can be uniquely decomposed into a divergence-free which is called solenoidal part and an irrotational part. So, in the projection method, the first step, as in the past is to obtain intermediate velocity, you denote it as u star and it is computed explicitly using the momentum equation with one big difference here is ignore pressure gradient term. So, pressure gradient term appears in all the three momentum equation, you drop the term then you have only convection term on one side, you have diffusion term on the other side or any other source term. So, if you drop the pressure term, whatever velocity field obtain is the intermediate velocity field and you denote it as u star.

And how do you bring the pressure into the system, you setup what is known as a pressure Poisson equation, the pressure Poisson equation is solved, you get pressure then velocities are corrected. So, pressure Poisson equation, we have already seen, it is an elliptic equation. Now, in the next slide, I am going to explain how pressure Poisson equation is obtained for two-D. So, in the last step, velocities are projected on to the divergence free space using the computed values of pressure that is you solve the pressure Poisson equation, get a pressure and that is used to correct intermediate velocity field u star and v star and that is supposed to satisfy divergence free condition that is continuity equation.

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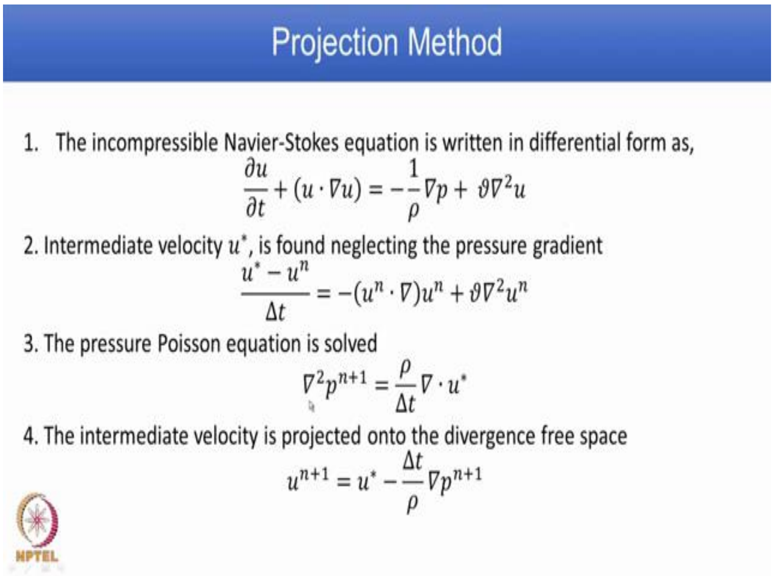

We write down here, the incompressible Navier-Stokes equation in differential form as shown here, so dou u by dou t plus u dot del u on the left hand side, equal to minus one by rho del p plus nu del square u. So, you obtain intermediate velocity field u star by neglecting the pressure gradient, so you get u star minus u n by delta t corresponding to the time derivative term equal to the convection term is now taken to the other side, so you have u n dot del u n plus the viscous diffusion term. Now, in this superscript n stands for the previous time level and star stands for the current time level. In this case, star refers to intermediate velocity field to be obtained from by solving this equation, before proceeding to the next level. So, if you solve this equation, you get u star; similar to u star, you will also obtain v star; so u star and v star are used to obtain what is known as a pressure Poisson equation as shown here. So, del square p u time level n plus one is related to intermediate velocity field as shown here. So, this is a linkage between velocity and pressure, so once you solve this, you can correct the velocities.

So, intermediate velocity is projected onto the divergence free space and that is corrected as shown here, u n plus one which is the new or corrected velocity is related to the old or intermediate velocity and with the correct pressure related as shown here. So, in this pressure Poisson equation, we have a source term on the right side, the source term is obtained by using a intermediate velocity field. If the source term goes to zero, then you have the equation what is known as the Laplace equation, with the source term appearing this equation called pressure Poisson equation. Now, in the next slide, I am going to show how a pressure Poisson equation is derived for two-dimensional situation.

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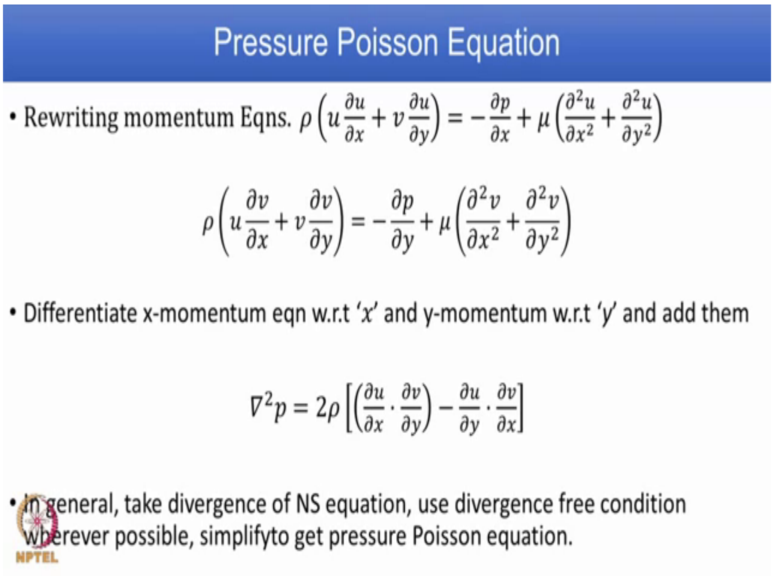

Let us rewrite the momentum equation, x-momentum equation is as shown here and y-momentum equation is as shown here. So, what we do, differentiate x-momentum equation with respect to x, and y-momentum equation with respect to y and add both of them together, so that will result in pressure Poisson term on the left side and corresponding source term on the right side. In general that is for three-D then all that you have to do is take divergence of the Navier-Stokes equation and use divergence free condition wherever possible, simplify to get pressure Poisson equation.

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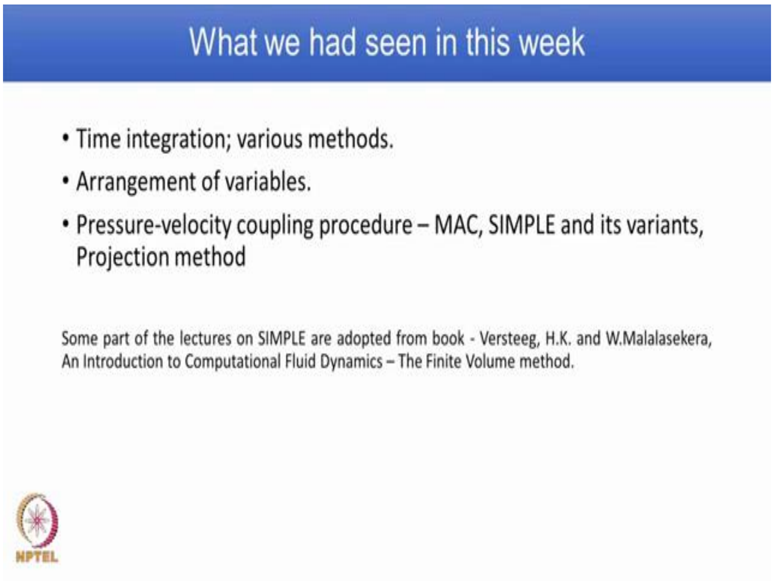

So, in this week, we have seen in detail time integration methods and various methods available Euler method, multipoint method and then how to obtain predicted-corrector to get higher order accuracy, arrangement of variables – collocated, stager, semi-stager, advantages associated with each one of them. Need for doing a pressure-velocity coupling and detail explanation for three methods that is MAC algorithm, SIMPLE algorithm, and projection methods. Some part of the lectures on SIMPLE algorithm are adopted from a book, The Finite Volume Method authored by Versteeg and Malalasekera.

Thank you and we will see you in next class, next week with some other interesting topics until then have a fun.

Thank you.