Foundation of Computational Fluid Dynamics

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Lecture – 25

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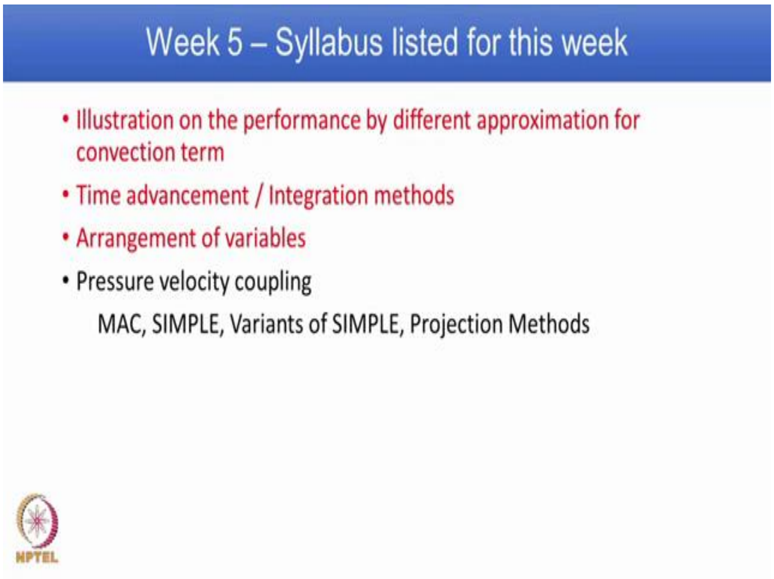

Welcome, welcome again to this course on CFD. This class is module five for this week. We have seen so far illustration on the performance by different interpolation scheme for convection term, time advancement, different methods in time advancement, arrangement of variables in particular, three different arrangements we have seen. And then need for pressure-velocity coupling, we described why one has to go for this pressure-velocity coupling procedure. We listed four procedure, out of the four procedure, last class we have particularly seen a method called Marker and Cell method - MAC. So, this class, we will focus on another method what is known as SIMPLE method.

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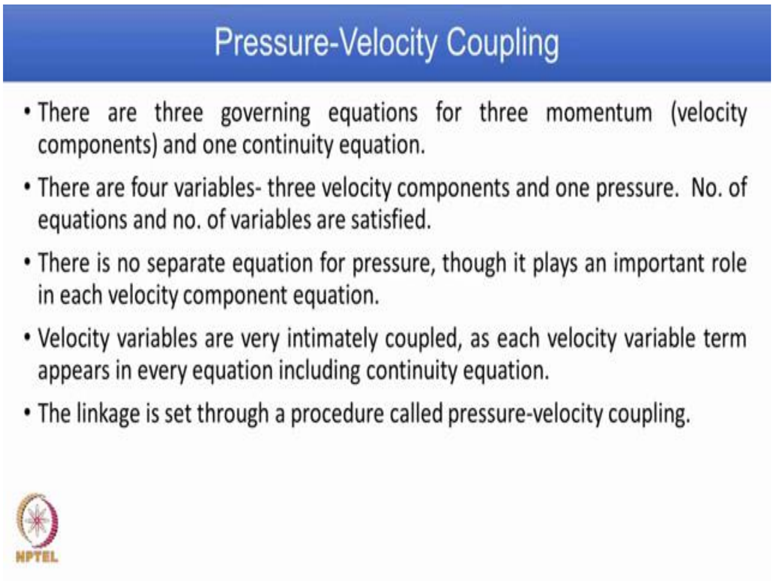

For the sake of continuity, we will try to understand once again what is pressure-velocity coupling, so in governing equations there are three governing equations, one for each velocity component, and we have one continuity equation. There are three variables for three velocity, and one for pressure, so in total there are four variables number of equations four, number of variables four, hence it is satisfied. Though pressure appears in each term, there is no separate equation for pressure, and pressure derivative plays an important role in each momentum equation. However, velocity terms appear in each equation, and they are strongly coupled. So, the linkage to connect between pressure and velocity through these four equations is established by procedure what is known as the pressure-velocity coupling procedure.

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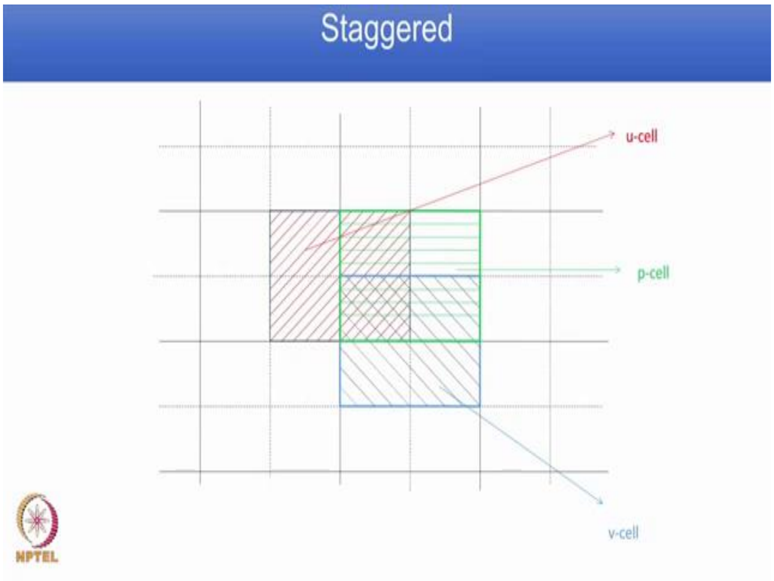

So, in this particular class, we are going to look at a scheme called SIMPLE scheme. Before going to the detail, let us first understand what is again staggered arrangement. In this figure, you see vertical lines - thick vertical lines, and then you are also able to see dash vertical lines in between two thick vertical lines, you are able to see dash vertical lines. Similarly, horizontal lines, you get thick horizontal line, and there is another thick horizontal line, and there is a dash line in between horizontal lines. Here also you can see the dash line in between two horizontal lines; on other side also you are able to see. We have already seen staggered grid, velocity variables itself or stored at staggered location. And pressure or any other scalar, you stored in some other location. So what is shown here is a first u cell, that is marked in the red color and that is at one location; then v cell is marked here, and then p cell is marked here. So, each variables are stored at different locations and pressure cell is at different location. SIMPLE algorithm that we are going to talk about this class is specifically explained for staggered arrangement. this figure is repeated in the next slide the nodal information also included.

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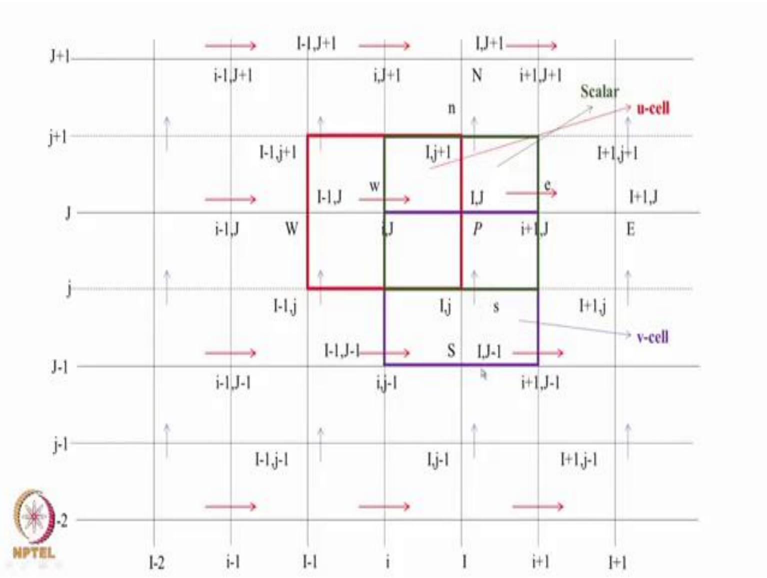

So, in this figure, which is repeat of the previous figure that with the nodal location also marked. For example, you have a mesh and nodes marked, there are two different types of letter, one is capital letter and another is small case letter. I is for the x-direction, and j is for y-direction; so in the x-direction, all the thick vertical lines are marked with the capital letters. So, in this case, I, I minus and I plus 1. All the intermediate lines are marked with small case letter i and i plus 1 and i minus 1. U velocity sits between two vertical lines as shown here. if you go in the j-direction, J is capital J is used. And Next thick horizontal line is marked as J plus 1, similarly the adjacent thick horizontal line is marked here as J minus 1. And small case letter, for example, here small j is referred for dash horizontal line, similarly for j plus 1, and j minus 1.

So, to locate a node that is given as P here is a pressure node, we have capital I and capital J. The face on one side face is small i and capital J, because you are on the same horizontal line. And you go to the other side, right side, you have small i plus 1 and capital J. if you go vertically, capital I comma capital J is here, and you go vertically up capital I is retained, because you are on the same vertical line and small j plus 1 and I, J minus 1. And you can get idea of all other important points. And in this figure, u-velocity is from left to right and that is shown in the red color; and v-velocity is from top to bottom that is shown in this color. And as we did before, u-cell is shown between two vertical lines in the staggered arrangement; v-cell is shown between two horizontal lines in the staggered arrangement. And pressure or scalar is marked as shown here in the original grid line between two horizontal and two vertical. Now this index, please make a note, because we are going to use this extensively in explaining the SIMPLE algorithm that is small case letters and capital case letters.

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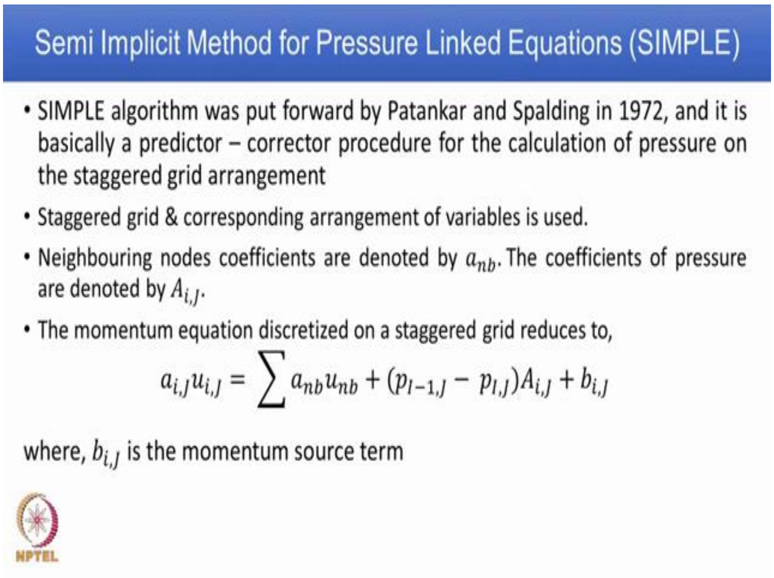

So, in some situation instead of using small case and capital, they may use half, so for example, i, i plus half, i plus 1, i minus half and i minus 1; similarly j, j plus half, j minus half, j plus 1 and j minus 1, any method you can follow. SIMPLE algorithm was put forward by Patankar and Spalding in nineteen seventy two. It is basically a predictor and corrector procedure for the calculation of pressure on the staggered grid arrangement. Staggered grid and corresponding arrangement of variables is used, and we explained just now through two slides staggered arrangement of variables. Neighboring nodes coefficients are marked as a n b – subscript, and n b stands for neighboring nodes, and pressure can be discretized. Now we have the Navier-Stokes equation, so you discretized all the terms, convection term, viscous diffusion term, and pressure term. Pressure is also discretized by following procedure. And coefficient of pressure after discretizing is denoted by A i comma j.

So the momentum equation after discretized on the staggered grid is rewritten in this form as shown here. So, a i comma j U i comma j equal to sum of all the neighboring nodes coefficients plus pressure terms – discretized pressure terms plus any source term that may be there in the momentum equation. So, this equation is the generic equation, in the sense, if you run the variable i and j appropriately from left to right or from bottom to top then you are able to write this equation for the all nodes in that domain. And this is the neighboring node, and it is brought to the right side, because it is suppose to be node either from the previous iteration or from the guess value.

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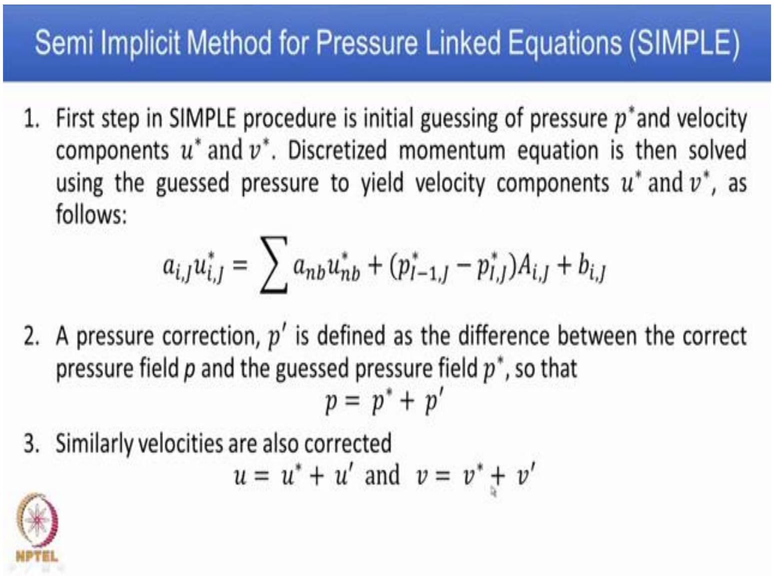

The first step in SIMPLE procedure is initial guessing of the pressure as well as velocity components. As I mentioned in the last class, we have a domain defined then you have a grid defined, you also decide choice of variables location, boundary condition is defined. You have to start the calculation or simulation. To start the simulation, we need to make a guess, this step is called initialization. The same is followed even in the SIMPLE procedure and we call this as a guess pressure denoted by p star and guess velocity denoted by u star and v star. We are explaining here for two dimension, and it can be extended to include third dimension as well. So, discretize momentum equation, and we have just now seen in the previous slide is then solved, so once you solved discretized momentum equation, using the guess pressure then it will result in next step what is known as a guessed velocity component u star and v star as shown here.

Now we can setup a equation for pressure correction p prime, and this p prime pressure correction is defined as shown here. Actual pressure is p, guess pressure is p star, correction pressure is p prime, so if you add the correction pressure to the guess pressure that will result in corrected pressure field. We follow the similar procedure for the velocity component, so u star is the guess velocity and u prime is the correction in velocity; if we add them together, you get correct velocity field, similarly for the other component of velocity.

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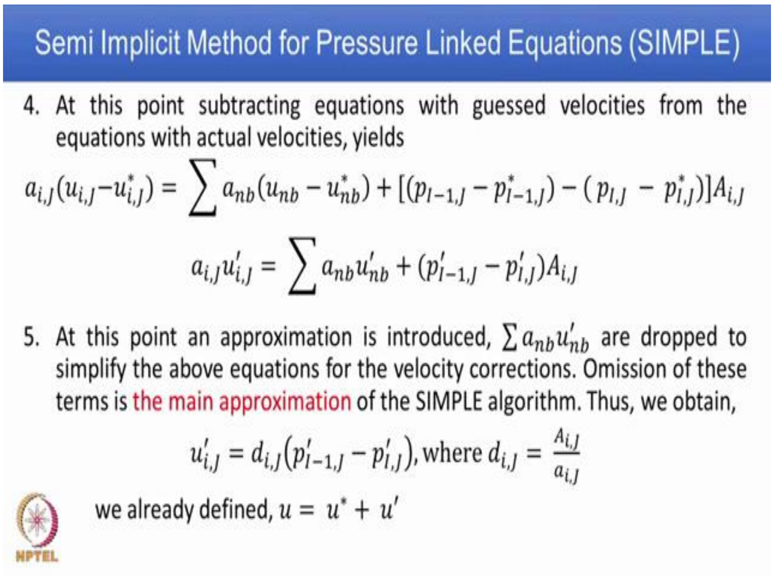

So, at this point, if you subtract equations with guessed values from the equation with the actual velocities, which means we have actually started with guess value, discretized Navier-Stokes equation is the same. Suppose, you had used the actual velocity as well as pressure field in the discretized momentum equation, and you have now discretized momentum equation written with the guessed value pressure and velocity field. So, if you subtract one from the other, then you get correction pressure as well as correction velocity and that is what is shown here. So, u i comma j is the actual velocity and u star is the guessed velocity; if you subtract the actual velocity from the guessed velocity that will result in correction in velocity value and this is repeated for all the terms inside, so this is for the neighboring node and remaining for pressure. And we already defined, we use the symbol u prime for the left side u i comma j minus u star i comma j is u i comma j prime, so this is correction velocity; similarly on the right side, from the neighboring node, correction velocity term and pressure term.

There is one important step in this equation if you look at, you are doing the summation of all the neighboring nodes coefficients with the correction velocity. Suppose you had actually solves with the original or the corrected actual velocity field then this contribution term should go to zero. In other words, if we do not have any correction, there will not be any contribution from the neighbor, actual velocity is directly related to the pressure field, so that is what is important approximation made. At this point, I repeat that approximation; and at this point, approximation is introduced that is summation of a n b u n b prime are dropped simply from the above equation for the velocity corrections. And omission of this term is the main approximation of SIMPLE algorithm.

So, once you drop this particular term, once you drop this term, summation of the neighboring node contribution, the resultant equation is shown here. And we take the coefficient a i comma j that supposed to be known to the other side and you define another variable, you define another coefficient d i comma j equal to A i comma j by a i comma j and that is multiplying correction in pressure. So we have obtained one such expression correction in velocity u prime. You can repeat this procedure for same u prime at different nodal location, for example, u prime at i plus 1 comma j; similarly, for other component of velocity v prime. We already defined u is equal to u star plus u prime.

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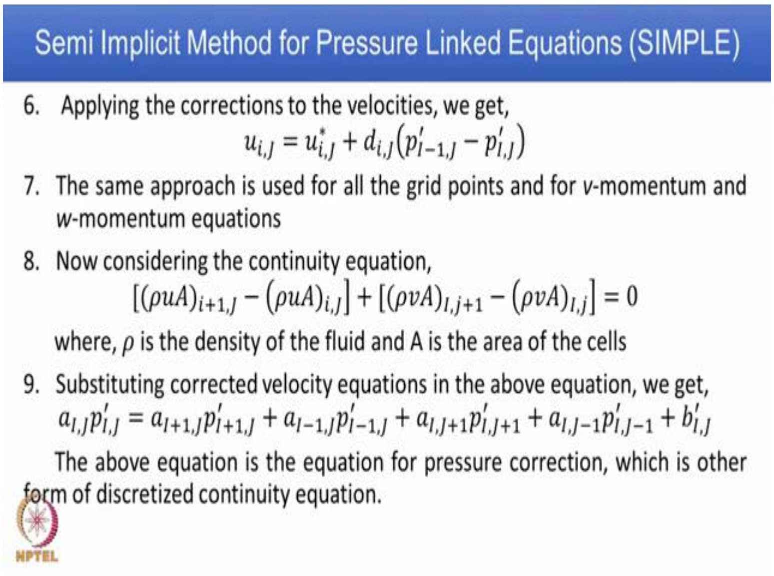

In the last slide, we got expression for velocity correction. Expression for velocity correction had pressure correction term inside. Now, we get the corrected velocity as shown here u i comma j that is the corrected velocity equal to the guessed value that is u star plus the correction velocity as shown here. We follow this procedure for all the grid points and also for velocity component v as well as w. So, we get all expression, we get expression for all corrected velocity and we use that in the discretized continuity equation. What is shown here is the continuity equation for two D and discretized form with the corrected velocity incorporated and you get finally, this expression. Now if you substitute the corresponding expression in terms of pressure correction, into the discretized continuity equation then we get equation as shown here. This equation is equation for pressure correction and the node of interest where pressure correction needs to be obtained is on the left hand side, and all other terms are brought to the other side. So, this equation has a form similar to other discretized equation. The above equation is the equation for pressure correction, which is other form of discretized continuity equation, this step is what is known as pressure-velocity coupling stage.

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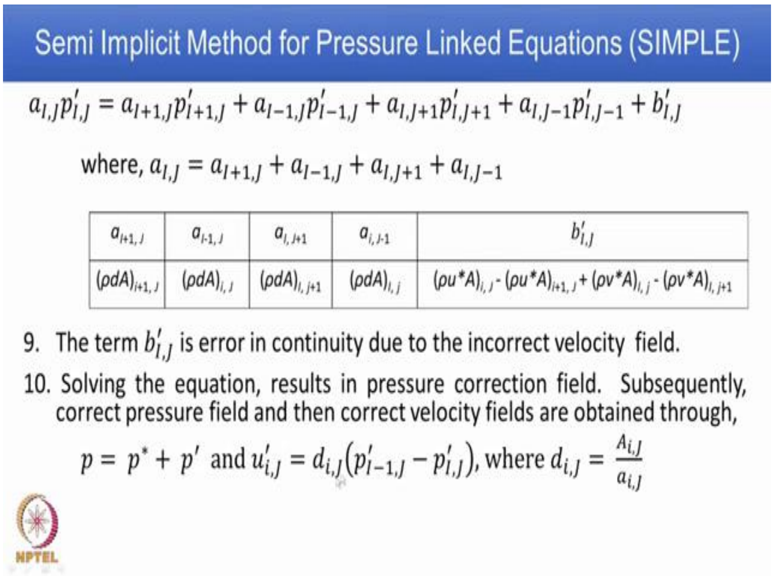

I rewrite that equation in the standard form a i comma j as shown here. And this is the procedure that we have followed, whenever we did finite volume procedure, the neighboring nodes are, neighboring nodes coefficients are separately written and the point of interest source terms appears as shown here. We have used u star expression here; the u star and v star as we explained before is the guessed velocity field. The b prime is the error in the continuity, which happens because we have used guess velocity field in the continuity equation. When you solve this equation, you get what is known as a p prime, which is the correction in pressure. And you can subsequently get correct pressure field then velocity fields are obtained by this expression p is equal to p star plus p prime and u prime i comma j is as shown here. And this is repeated for other velocity component v and at all nodal location.

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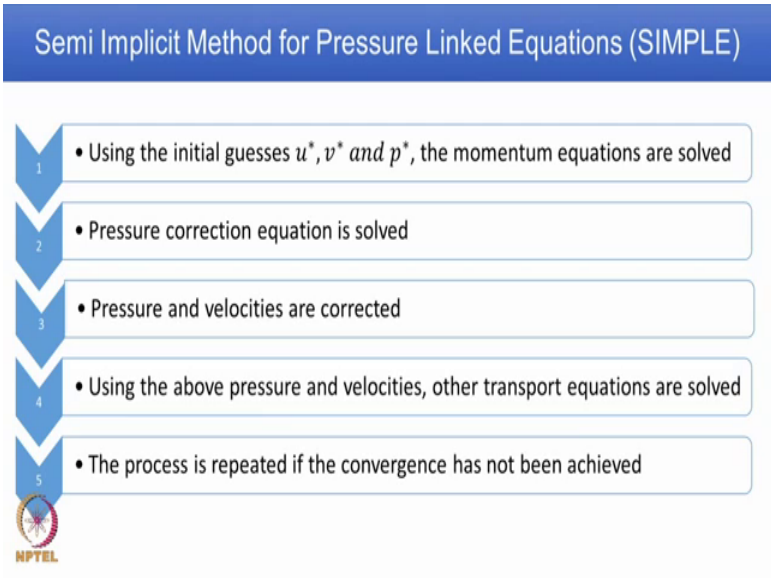

So, if I put all the steps in one slide, we start with initial guesses for u star, v star and p star, the momentum equations are solved. Pressure correction equation is setup and that is solved. Then from that step you p prime and that is used to obtain pressure as well as velocity. Once you use the above pressure and velocity, then any other transport equation can be solved, for example, temperature or concentration. The process is repeated, all the steps are repeated until convergence criteria that you have set is satisfied. As we did in MAC algorithm, here also we set up a condition for convergence, it may be ten power minus three or ten power minus five, it depends on problem to problem. Once the convergence is satisfied, then we quit the iteration procedure.

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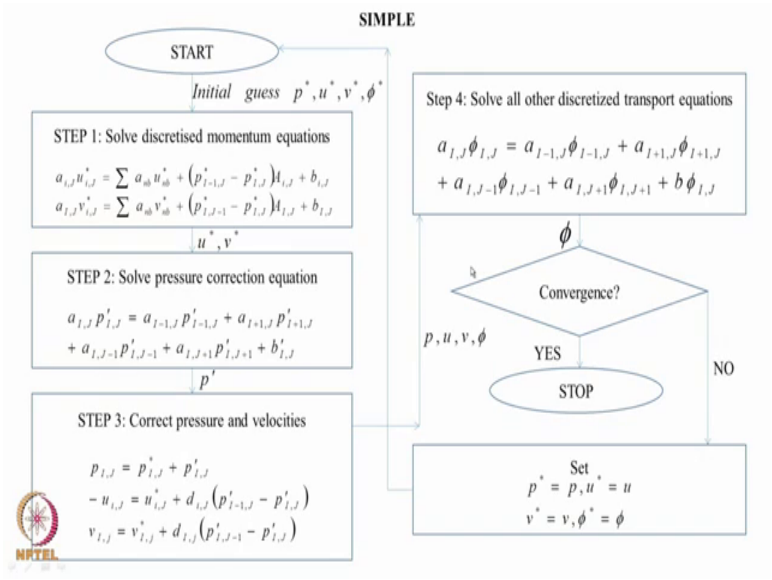

In terms of flow chart, it is shown here. So, we start with the initial guess p star, u star, v star and any other scalars that is generally written as phi and phi star. So, you solve the discretized momentum equation, we get u star, v star, solve the pressure correction equation, you get p prime then you correct pressure and velocity as shown here. Then solve all other discretized transport equation, which is meant for any other scalar variable. We are still following the same procedure as we did for pressure or velocity variable. So, following the finite volume procedure, you get generic form of the discretized equation.

So, you get phi then you set up a convergence criteria, you check whether it is within convergence limit that you have defined; if so, you can stop SIMPLE step or iteration, move to the next step; if not, whatever velocity pressure that is obtained in this step is now used as a guess pressure and guess velocity for the next iteration that is what it is shown in this set p star as p, u star as u, v star as v and phi star as phi. So, this values are obtained guessed know iterated solution and if it is not satisfying continuity, they are reset to guessed value and procedure repeats. And you can see here, go back and repeat the procedure.

So, in this class, we have seen one more method of pressure-velocity coupling, what is known as a SIMPLE method. It is very popular and powerful method, people have used it extensively, it is reported in many literature. Next class, we are going to see two other procedure what is known as a SIMPLE consistent and projection methods.

Thank you and have a fun until then.