Foundation of Computational Fluid Dynamics
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Lecture – 20

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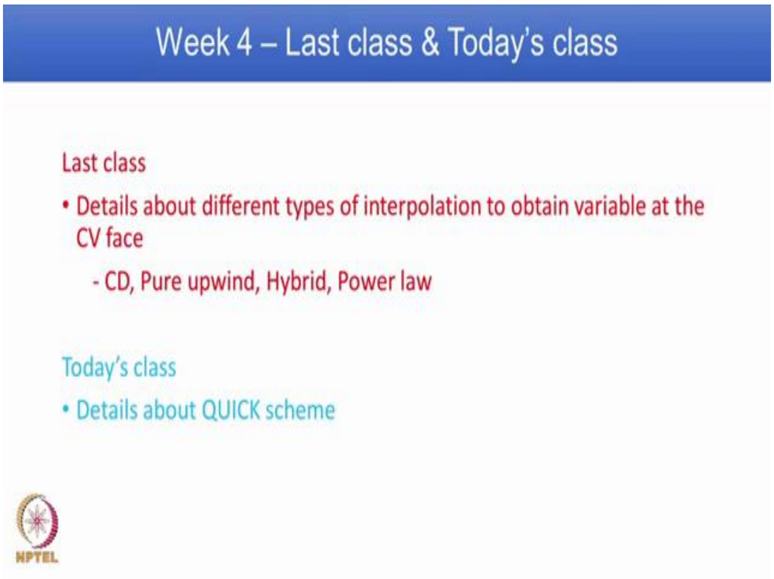

Greetings and welcome again to this course on CFD. Today is the last class for this week. Last class, we had seen detail about interpolation, different types of interpolation particularly central difference type of approximation, pure upwinding power law scheme hybrids scheme and so on. In today's class, we will focus mainly on scheme called QUICK and we will see merits and demerits of this particular QUICK scheme.

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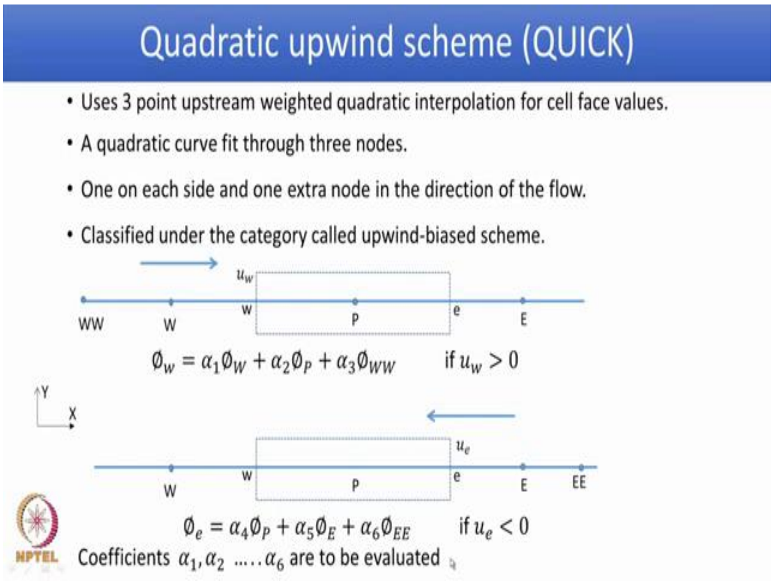

QUICK otherwise called quadratic upwind interpolation scheme. It uses three point upstream weighted interpolation. A quadratic fit it may through three nodes. And what are the three nodes, one node on each side for particular face, and one extra node into the direction of the flow, because one node extra is taken in the direction of the flow, this also can be classified under the category what is known as upwind biased scheme. Let us take this illustration. In this illustration, P is the node of interest and you construct control volume around that node and respective on the left side and right side are also marked. So, in this case, capital letter E and capital letter W, then one node more is marked which is west of west that is shown here WW. And for this illustration, the flow is consider form left to right as shown here and you are interested to find out value of the variable on this face west face which is marked here as u w, and you decide have a coordinates definition as shown here X and Y.

So, with respect to the coordinate definition, the flow is left to right and it is positive. Then phi variable value at that face is related to alpha 1 phi at W then alpha 2 phi at P, one more node in the direction flow that is WW, which is gives here as alpha 3 phi WW. So, you can observe this is the face you are interested to find out you have one node either sides and one extra node in the direction of flow.

Now, let us take the next case when the flow is from right to left as shown here. With respect to the coordinate definition that you have using, the flow direction is negative.

Now you want to find out for this case variable on the face e, such case the expression as shown here phi at e that is on this face you have two neighbouring nodes one at e another one at P; one more node upstream in the direction of the flow that is EE. So using this three nodal information, you can evaluate variable value on this face u e the corresponding expression shown here, phi at e equal to alpha 4 phi at P plus alpha 5 phi at E plus alpha 6 phi at EE; coefficient alpha 1 to alpha 6 are to be evaluated.

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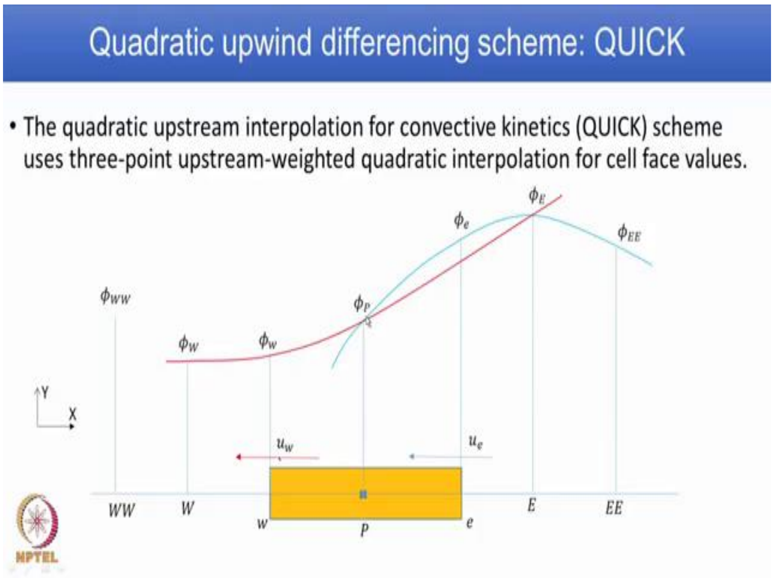

Let us also have another description; so here is the example. So, in this figure, again P is node of interest, control volume is constructed around the node of interest, and the coordinate definition is also given as X Y as shown here. If you are interested to find out u on this face that is west face which is shown here red colour then it is evaluated based on phi at W; in another words u capital letter W that is the node then another node here which these two nodes are nodes on either side then one more extra node in the direction of the flow which happens to be in this case node at E. So, we have to consider u value at E node, u value at P node and u value at W node, then you construct the quadratic fit then evaluate coefficients based on that you will get variable value on this face.

Similarly another situation for value on this face that is e, you have node E capital letter E that is on the right side of the face, and node P which is on the left side of the face. So, these are nodes available on either side for this face E, and one more extra node which is in the direction of the flow that is EE marked that is node marked by letter EE. So, if you look at the quadratic fit that is marked in this colour, which is running through these nodes phi EE, phi at E, and phi at P. Now let us look at these details with some more information.

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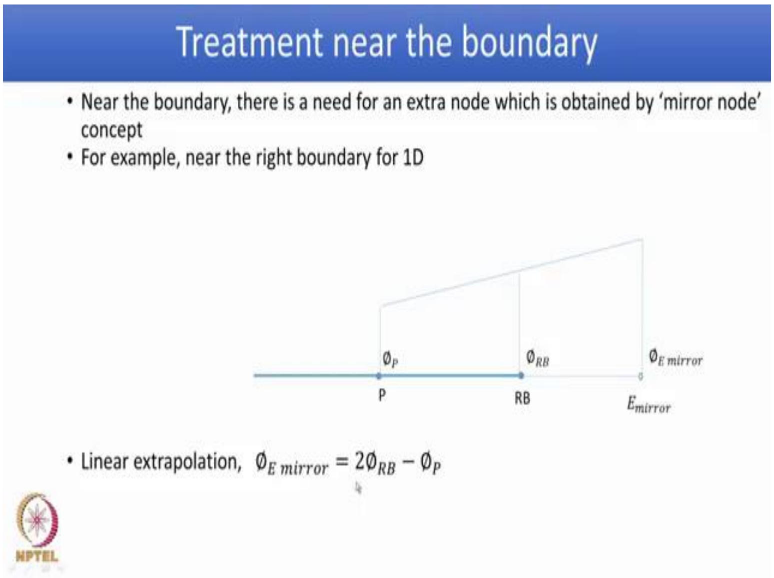

So, we have to know how to do in the case of near boundary near boundary you have a node which is marked here P then this example right side right boundary there is value given by the boundary condition that is marked here as phi at r b phi at b. So, behind this t construct another node which is known as mirror node and that mirror node for convenient it is taken as the same distance as the distance between P node and the boundary. So, this node that is the mirror node which is step at distance same distance as distance between node P and right boundary. Now we need to evaluate or get idea of value of the variable at this mirror node, because we are consider the distance same as between P node and right boundary we use linear approximation hence it is written here phi at E mirror equal to 2 times phi at R B minus phi P.

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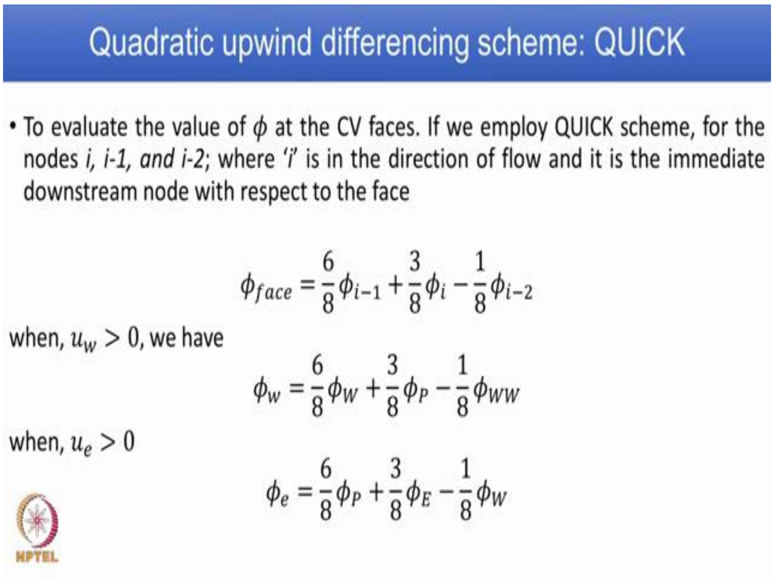

To evaluate the value of phi at CV faces, we employ QUICK scheme, and for the nodes i, i minus one and i minus two where i is in the direction of flow, and it is immediate downstream node with respect to the face. So, this particular formula is the generalized formula whether the flow is from left to right or right to left of wherever you want to find, so that is what is given here phi at any face. And for uniform spacing on in one-dimensional the formula is already evaluated and it is shown here as 6 by 8 5 times i minus 1 plus 3 by 8 phi i minus 1 by 8 phi i minus 2. So, in this, what is i, i is in the direction of the flow and it is immediate downstream node with respect to face your interested then you have i minus 1 and i minus 2.

Let us see how to actually get idea of this by taking example. So, if u W that is your interest variable value on the west face if it is happens to greater than 0, that mean if it positive then phi at W that is the interest 6 by 8 and i given here as the node in the direction of the flow immediate downstream. So, for the west face is immediate downstream is the node of interest phi at v i minus 1 is on the left side and i minus 2 is to left of W node which is phi at WW. So, this formula, which is derive for uniform equal facing is the generalized formula, and you can apply this for any side for any direction if u e is greater than 0, which is the another example. Then you can understand how to use again for u e greater than 0, which is flow left to right case i is immediate downstream node is phi at e then i minus 1 is on the left which is phi at b and i minus two which is left of P node that is phi at W.

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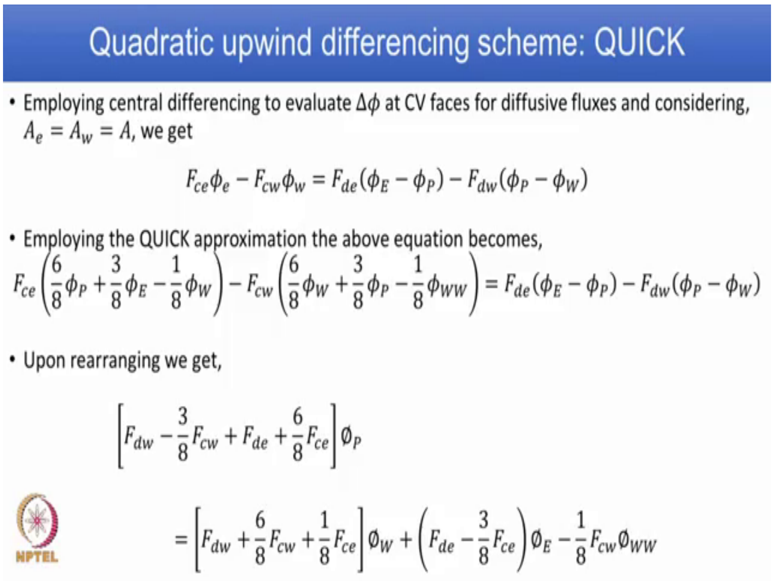

So, we already learn from diffusion equation we apply central type of difference in to evaluate derivative on the faces and for simplicity face we assume it one dimensional with equal area. So, a e equal to a W and a. So, we put convection term also then we get final discretized form as shown here. So, this F is the flux and substitute c stands for convection and e is stands for at the face e. So, F c e phi e minus F c W phi W face equal to F d e F d e multiply by phi e minus phi P minus F d W multiplying it by phi P minus phi W in this right side term is actually diffusion term, and we have learn before that also how to evaluate variable value on each face.

So, if you apply now QUICK type of approximation to get this variable value on faces you get this substituted you have just now seen. So, we can sub suited for phi at F e phi at W by quick approximation to get this expression; once again in this expression, we look at phi P is the node of interest and that term appears in the all terms here. As did before we collect all these coefficients multiplying phi P and that is node of interest are unknown put them on the left side all other quantities supposed to be known and take them to the right side once you do that kind of rearrangement you get final equation as shown here. So, you can actually verify whether they are done correctly. So, this term on the left side is the coefficient term for node of interest and the right side there are suppose to be known either form previous situation are as part of the current solution. So, this node of interest P it runs for this case of one d from left to right now when you do that system of linear equation than its setup a matrices inverse you get solution.

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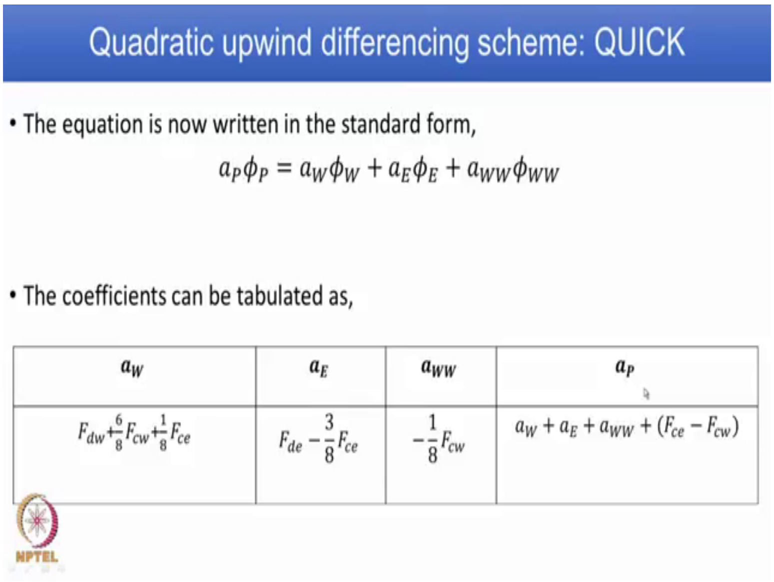

So, as we did in the case of diffusion equation, this equation is also now written in the standard form as shown here a P phi P equal to a W phi W a E phi E plus a WW phi WW. If you able to recall compare this expression, which is the expression we obtain central differences scheme or pure upwind, and we can tabulate as we did in the case of diffusion equation. So, a W, a E, a WW and a P, these are terms in the equation and write corresponding coefficients and you can again observe that coefficient for a W coefficient for a E, they appear as term in coefficient for a P. Hence, we replace them with the corresponding coefficient a at W, a at E, in addition for this QUICK formulation, we get similar expression so that is a at WW, and these are the additional terms. So, it is necessary only to calculate coefficients a W, a E and a WW, and they go into the expression of a P also. Hence only the new term that we have to calculate in F c e and F c W to get coefficient value for a P.

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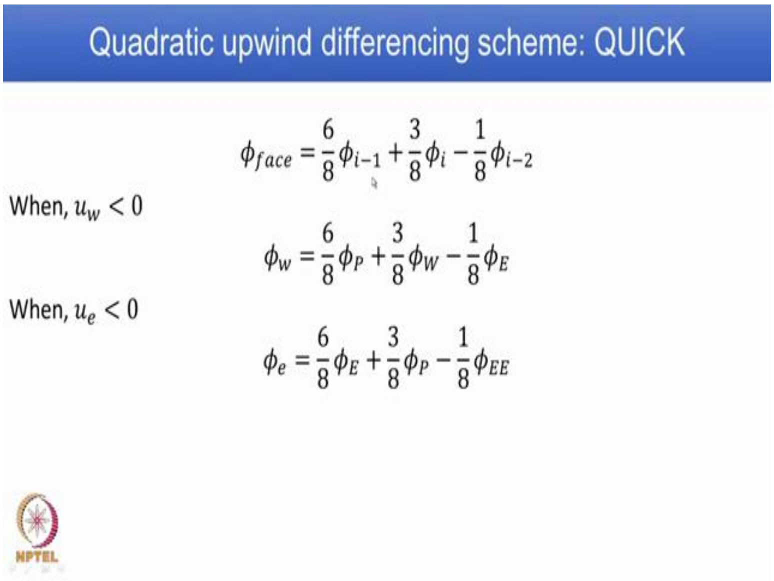

Now, all that we have done is for one side, we will try to extend for other side the generic expression for quadratic as shown. Now u w less than zero the previous case u w was greater than zero u e was greater than 0. So, we fallow now for u w less than zero and u e less than zero; that means, the flow is right to left negative. So, again try to identify for any particular face of interest i is in the direction of the flow in this case from right to left. And you are trying to find out for west face in the direction of the flow west face immediate downstream node is actually W node that what shown here then i minus 1 that is to the right of W node is P and i minus 2 that is right of P is e node. So, corresponding coefficient values used. All that you do identifying what is i for any particular face, and for the corresponding velocity condition. Second case u e less than 0, again the flow is right to left negative velocity for the face e, i node which is immediate downstream node in the direction of the flow is P node and i minus 1 is the e node and i minus 2 is u e node we have to remember always the one dimensional terminologies then you can immediately identify what is i, i minus 1 and i minus 2.

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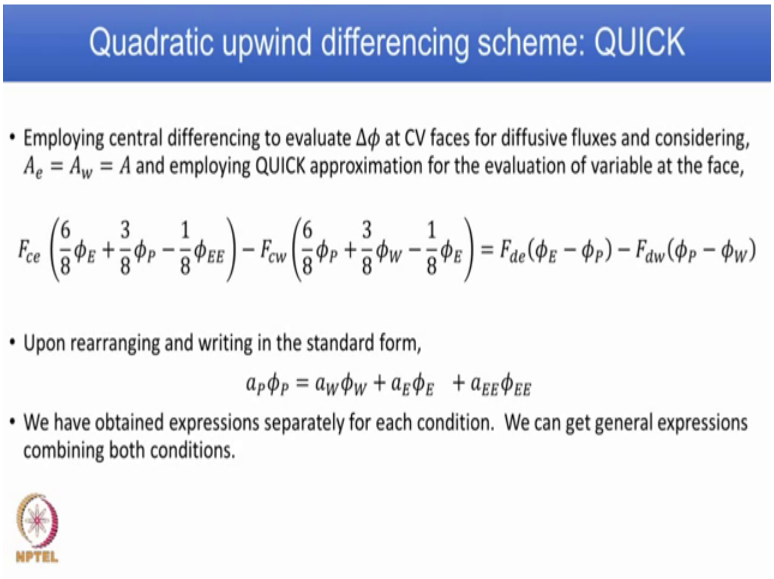

So, we follow the same sequence employing the central difference type of approximation for derivative which is used diffusive term and employing QUICK type of approximation for the evaluation of the variables at the faces then we get discretized form of the equation as shown here. All that we have done in this case, we substituting corresponding QUICK formulation for convection which is on the left side and right side diffusion. And we already know it again we can rearrange that is phi at P appear in all that term and you collect all those coefficients and arrange them together in the standard form and that is now written here as a P phi P a W equal to a W phi W plus a E phi E plus a EE, phi EE. So, this is for the case when the flow is right to left, and we have already seen separately for the flow left to right. Now, we get the generic expression because you have situation of flow coming to left to right or right to left we have no idea prior. So, we have to get generic expression. So, you have to combine both of them together and that what are going to see in the next slide, we obtain the expression separately for each condition, we can get generic expression combine in both in condition.

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Quadratic upwind differencing scheme: QUICK

- We have obtained expressions separately for each condition. We can get general expressions combining both conditions.

$$a_P\phi_P = a_W\phi_W + a_E\phi_E + a_{EE}\phi_{EE} + a_{WW}\phi_{WW}$$

$$a_P = a_W + a_E + a_{EE} + a_{WW} + (Fce - F_{cw})$$

a_W	a_{WW}	a_E	a_{EE}
$F_{dw} + \frac{6}{8}\beta_w F_{cw} + \frac{1}{8}\beta_w F_{ce} + \frac{3}{8}(1-\beta_w)F_{cw}$	$-\frac{1}{8}\beta_w F_{cw}$	$F_{de} - \frac{3}{8}\beta_e F_{ce} - \frac{6}{8}(1-\beta_e)F_{ce} - \frac{1}{8}(1-\beta_w)F_{cw}$	$\frac{1}{8}(1-\beta_e)F_{cw}$

where $\beta_w = 1$ for $F_{cw} > 0$ and $\beta_e = 1$ for $F_{ce} > 0$

$\beta_w = 0$ for $F_{cw} < 0$ and $\beta_e = 0$ for $F_{ce} < 0$

And that is shown here that is a P phi P that the node interest and a W phi W a e phi e in central approximation or pure upwind approximation we had expression up to here now when you apply quick stream you get to additional term phi at e e and phi at W w now correspondingly a P is related to neighbouring node coefficient as shown here. So, ap is a W plus a e then you also have a EE and a WW. So, this are depending on the direction of the flow plus F c e minus F c W we can also to them together in generic form in this way as shown here. So, a W coefficient is written in detail a WW coefficient written in detail and a E and a EE.

Now in this expression, if you look at carefully beta is one new variable introduces because we have flow coming from left to right as one case and flow coming from right to left as another case. So, when you are actually writing a program your incorporate in QUICK scheme inside your code, code should automatically detect in the direction of the flow and switch accordingly even it is a QUICK scheme switch between the node according to the direction of the flow and to enable that you have introduces new coefficient what is written here as beta. So, beta is the coefficient that decide the direction of the flow accordingly the switching between this side left side or right side will be taken beta is define if beta equal to one then F c W is greater than zero which means flow is from left to right and beta e is equal T p 1 then F c e is greater than zero please remember F stands for flux and c stands for convective flux. So, e stands for east face. So, F c e means it is directly related to velocity which is velocity condition on the east face when you say beat e equal to one which means the flow is greater than zero which is positive, hence it is from left to right. Another situation if beta W equal to zero and beta e equal t o zero correspondingly flux at west face and flux at east face or less than zero which means the flow from right to left so depending on the beta condition.

So, in your actual code you find out the direction of the velocity evaluate F c W if F c W is greater than zero is switch to one condition if F c W is less than zero its switch to another condition that is done in code and correspondingly it is written for coefficients. For example, if you say beta W is equal to one if you substitute in this expression for a W in the particular vanishes you can also check similarly for other term when you write a code the switching according to the direction of the flow is done automatically.

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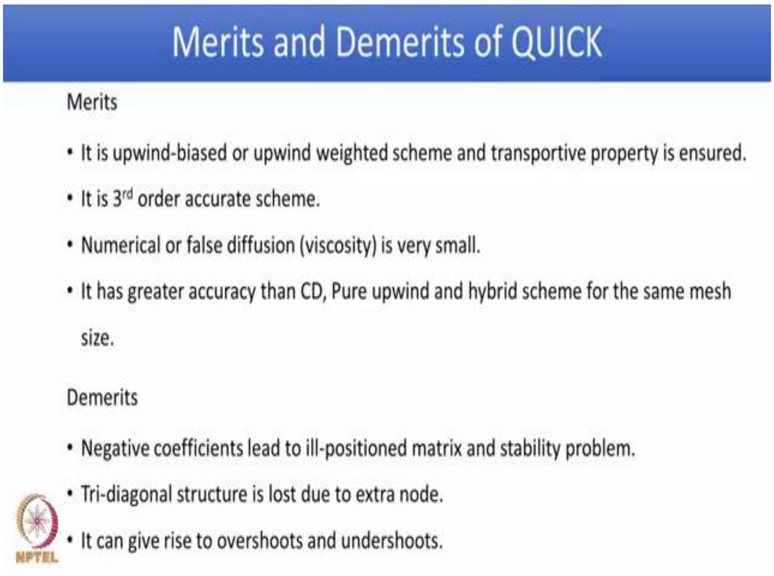

Now, the QUICK scheme appears good in terms of the direction of the flow an accuracy combining the merits of central differences scheme as well as upwind different scheme. Now let us look at specifically merits and demerits QUICK scheme, it is upwind biased or upwind weighted scheme. We have already seen there are two nodes one either side and one extra node in the direction of the flow that is way it is called upwind biased scheme because it is in the direction of the flow biased it is also possessing what is known as transportiveness property and we have done quadratic fit. So, it is becomes third order accurate again remember pure upwind is the first order accurate, central differences scheme is second order accurate quadratic scheme is the third order accurate.

So, by taking one exact node you are able to improve the order of accuracy to next level and numerical are false diffusion is also called numerical viscosity is reduces very small and it has an accuracy greater than central differences scheme, pure up winding even hybrid scheme for the same mesh size. The merits are there, it also have the demerits some time negative coefficient may happen and that may lead to what is known as ill positioned matrix because we are look only matrices at the end of the day, if the matrix is not properly form then you will not able to invert the matrix and that may result in what is known as the stability problem.

So, in the case of QUICK scheme, sometime you may have a situation of ill positioned matrix, and which may lead to what is known as stability problem. The one more important property is tridiagonal structure, we are going to see in detail what is tri diagonal matrix, and how do invert tridiagonal matrix in the case of central difference scheme had tridiagonal structure of matrices that is you have one main diagonal immediately super and immediately sub diagonal. Then there is separate procedure available to invert that type of matrices in this QUICK scheme because you consider one exact node that tridiagonality the structure is disturbed and you have a separate procedure to invert that matrices one more demerits it can give rise to sometime over shoots or undershoots. Let us get idea of what is overshoots and undershoots with the help of the illustration.

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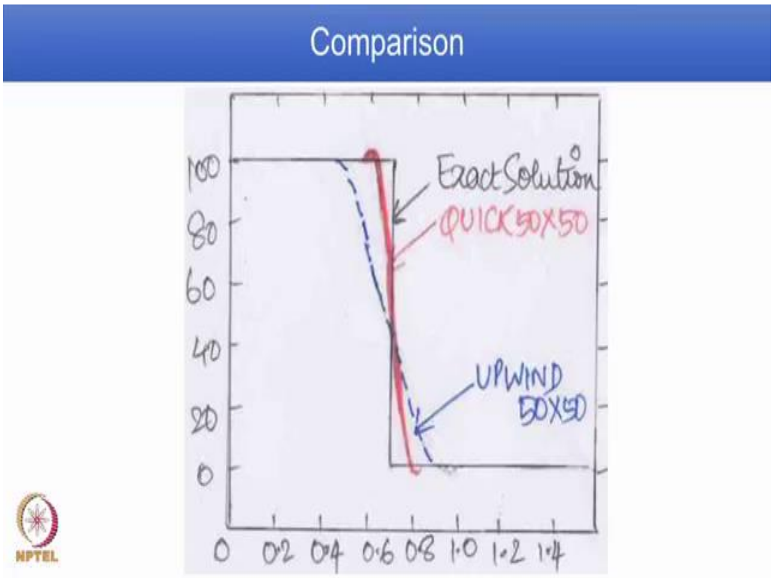

What is shown here is one solution final obtain and you see here this is a black line which correspond to solution obtain by analytical procedure or exact solution if you use pure up wind mesh size 50 by 50 and get solution as shown in blue colour and for the same mesh size if you use QUICK scheme than you get the solution as marked in red colour and you can observe between the two the QUICK solution reaches the exact solution near for the same mesh size it is also exhibits some overshoots and some undershoots that is going to below zero which is the kind of unrealistic it is not capturing exact distribution, but there is a diffusion there is no overshoot or undershoot this is the property of what is known as the numerical viscosity.

So, the numerical viscosity or false diffusion ensures smooth solution but it does not capture accurately P or the maximum value. Whereas the quick procedure for the same mesh value gets the solution very near to the actual solution; at the same time it results what is known as overshoot and as well as undershoots. In today's class, we have seen in detail about QUICK formulation and I mention in detail for two situation when the flow is left to right when the flow right left how to evaluate the variable value on east face and west f ace accordingly from a generic quadratic interpolation formula. Then we put them together, we get a generic expression and that had one extra coefficient by name beta automatically ensure accordingly the direction of the flow the switching term in coefficients. With this we come to end of description about all type approximation; in next class, we will be see illustration with an example.

Thank you.