Foundation of Computational Fluid Dynamics
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Lecture – 02

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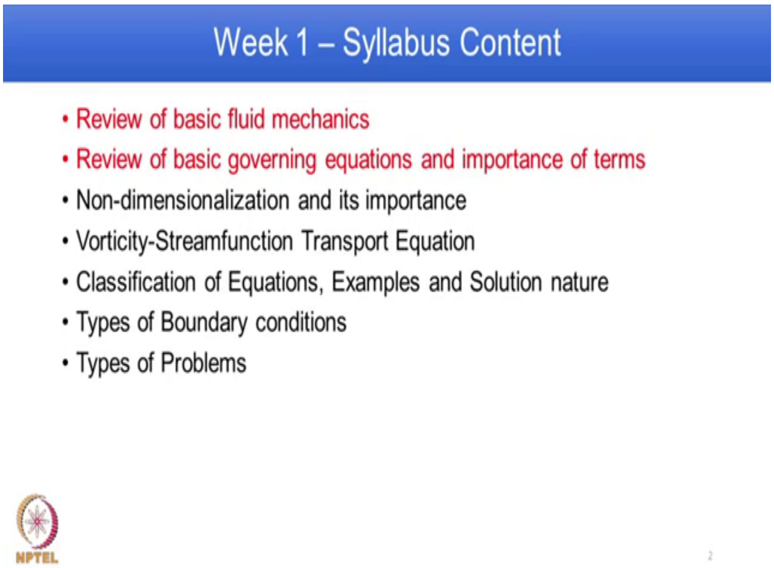

Welcome you all again to this course on Foundation of Computational Fluid Dynamics. We are on to module two of the first week. Last class, we basically did we put out the syllabus for the course, and then outcome - what you obtain from this course, some of the mathematical operations, then definition of velocity field, vorticity, viscosity, definition of fluid based on viscosity. And we still have to do review of equations, and there are other topics like non-dimensionalization, vorticity-streamfunction and so on.

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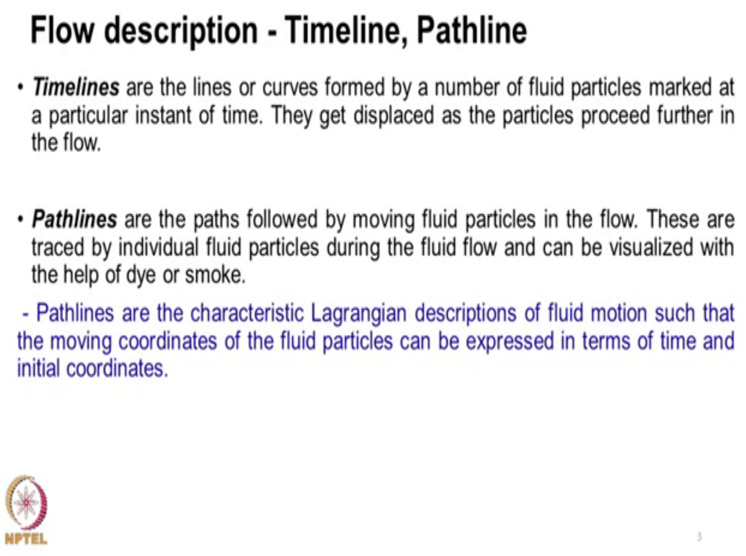

Whenever a fluid is moving, we have to characterize them, visualize them, so this is given by flow visualization and there are four such characteristic lines defined for flow description. They are time line, path line, streak line and stream line. So, we will see the definition of them and later when take an example, we will see that how these are used to define a flow. So, timeline are the lines or curves formed by a number of fluid particles marked at particular instant of time, this is very important, so it is unsteady flow it changes with time and you are looking at one particular instant how different fluid particle at that instant behave and if you are able to connect all the particles at that instant then similarly you proceed with the time, you are able to get timeline for that flow.

So, they get displaced as a particles proceed further in the flow. Pathline is another definition this is a contour or path followed by fluid –moving fluid particles. They are traced by individual fluid particles and in experiment also in CFD, we can inject a dye then it traverses when if you connect all the contour then you get pathline. So, pathline are characteristic Lagrangian descriptions of the fluid motion such that the coordinates of the fluid particles can be expressed in terms of time as well as initial coordinates.

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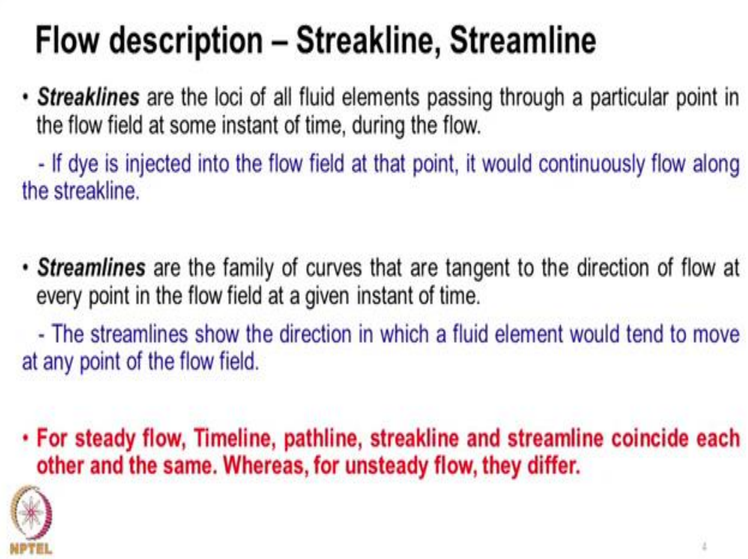

There are other two definitions streakline and streamline. So, streakline are the locus of all fluid particles which passes a particular and after some instant whatever particles that is crossed at that particular point and you connect all of them, then you get a streakline. So, in experiment, if dye is injected and it would continuously move only along the streakline. Streamlines, this is the standard definition which all of you know, a family of curves which are tangent to the direction of the flow at every point at a given instant. So, if you draw a tangent to streamline, then you get velocity component, velocity vector at that particular instant. The streamlines show the direction in which a fluid element would tend to move at any point of the flow field. We know flow can be steady or unsteady. So, in steady problem, all these four descriptions pathline, timeline, streamline and streakline they all will be same. They will be different for unsteady flow, and this is to be noted very carefully.

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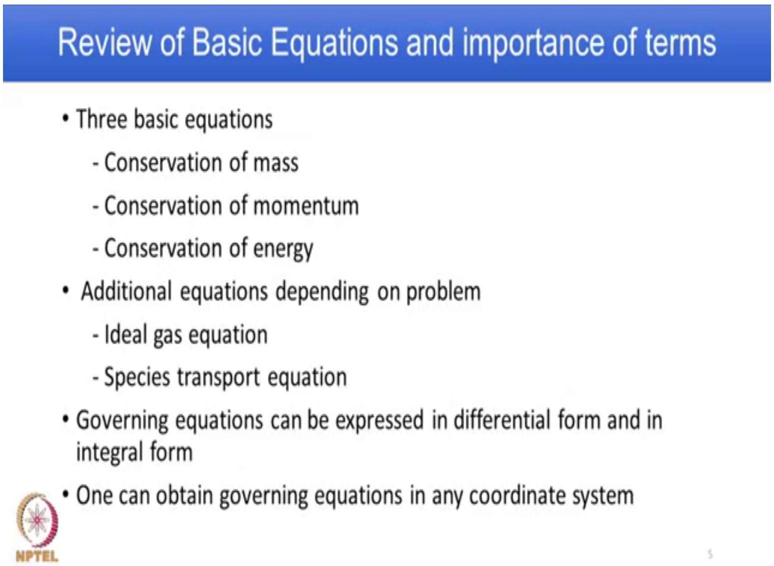

Now, next important topic, reviewing of basic equation; we will not do detail derivation, we will only touch few steps and explain importance of terms; detailed derivation is available in any standard under graduate fluid mechanic text books. In general, there are three basic equations, one is conservation of mass, which is also known as continuity equation; conservation of momentum and then conservation of energy.

Primarily conservation of mass and conservation of momentum are necessary for any fluid mechanics problems and conservation of energy is used whenever you are interested in compressible flow or to do with heat transfer problems. These are only primary equations. You also need to have additional equations depending on problem. For example, ideal gas equation is used in compressible flow; and if you are interested to find out concentration distribution for example, pollutant disperse a problem, then you interested to find out particular concentration then you solve additional species transport equation.

Governing equation can be expressed in two different forms; one is the differential form and integral form. Similarly, we already seen last class, different coordinate systems, Cartesian coordinate system, cylindrical coordinate system and spherical coordinate system, so one can able to convert or derive governing equation in any coordinate system and convert from one coordinate system to other coordinate system.

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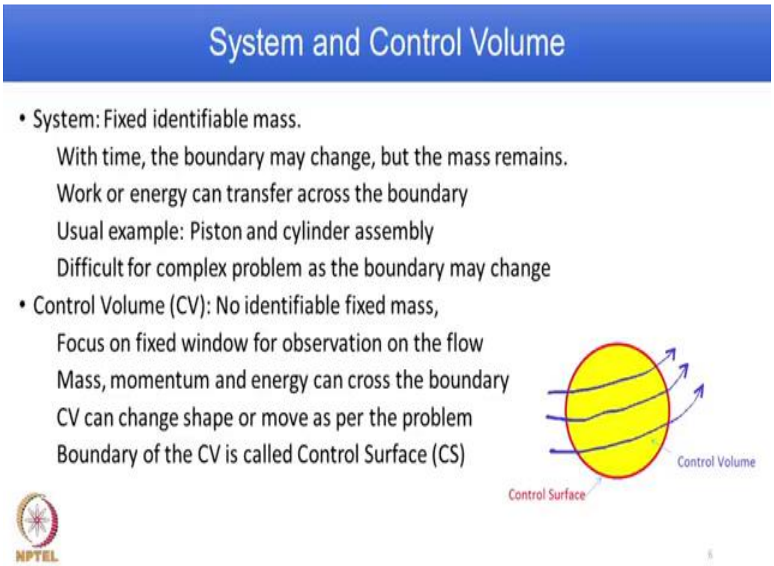

So, we before going to the derivation, one important concept that we have to learn in fluid mechanics what is known as system and control volume. System is basically related a fixed identifiable mass. So, with the time, the boundary may change, but the mass remains; the boundary which contains that mass may change, but the quantity of mass that remains same. So, across the boundary, we can have a work or energy transfer. The usual example given is piston and cylinder assembly. Certain quantity of gas is inside the cylinder and piston moves front and back, the mass that is remains the same. And you have an energy transfer happening across the boundary and piston moves back and fro. Now, though it looks simple for a problem where there is continuous change of the boundary, it is difficult to apply idea of system or concept system and get a solution.

So, there is alternate what is known as control volume, where we do not really focus on a fixed quantity of mass, only the boundary focused on fixed window and then observe what is happening to that window. So, we can have a mass transfer also, in addition to momentum as well as energy transfer crossing the boundary. Control volume can change shape or move, it need not be fixed in a flow, it can also move. For example, you want to find out flow pass in aircraft or flow moving over a four-wheeler and you define control volume around the aircraft or four-wheeler as aircraft moves or as the four-wheeler moves, the control volume also moves. Similarly, if you take a balloon then the balloon has deflating, it changes its shape. So, the boundary which is defined here as a controlled volume, it can change shape. This is helpful for example, we have a elastic material and you want to find out flute structure interaction problem later then there is a boundary it is not fixed it undergoes changes as a function of time; control volume study is very helpful.

The boundary which is enclosing the control volume is called control surface, so there is a schematic available. So, here this red boundary is the control surface and fluid crosses through the control volume, and you find out what is happening to the control volume, because of the flow that is happening through this control volume. So, for example, you want to find out in the case of aircraft, what will the lift generator, what is the drag force exerted on the aircraft then you find out a net get a drag estimate.

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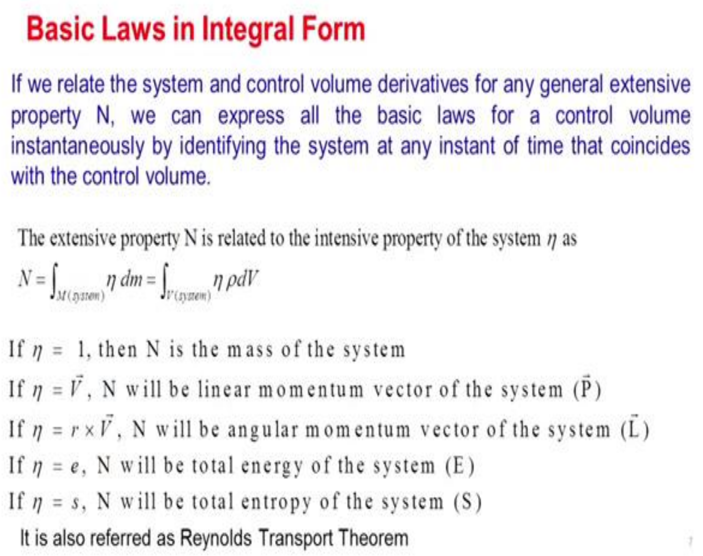

So, as I mentioned before one can get all the governing equation either in integral form or in differential form. We will see sum up them in integral form then later we will move to differential form. So, to take an integral form, we can relate the system and control volume derivatives for any general extensive property N, then one can get all basic laws by instantaneously by identifying the system at that coincides with the control volume. This is theory what is known as a Reynolds transport theorem; right now, we are not going to details of that derivation. In mathematical form, it is expressed and given here; the extensive property N is related to intensive property eta and it is related here as N equal to integral over mass, which is for actually system eta into dm is related to volume, which is for the control volume into eta p into dV.

Now, you can see by substituting different values for eta, you get different governing equations. For example, if you say eta equal to one, so if you substitute eta equal to one here, and then the resulting quantity corresponding extensive property is actually the mass. So, if you put one eta equal to one, it is integral rho into dV; you know rho is the density kg per meter cube and dV is meter cube, product of them will give you mass, so that N is actually mass and that will give you conservation of mass equation in integral form.

Similarly, if you define eta - a intensive property as V, then if you substitute here V, so V into rho into dV, you know mass into velocity will give you the linear momentum, and similarly on the left hand side extensive property N will give you the linear momentum. So, we can substitute different value for eta and they are given here. Next one is for angular momentum, then total energy and total entropy. So, one can obtain from this general relationship, it means extensive property and intensive property to this integral relationship, one can obtain basic laws in integral form. As I mentioned before, this is what is known as Reynolds transport theorem.

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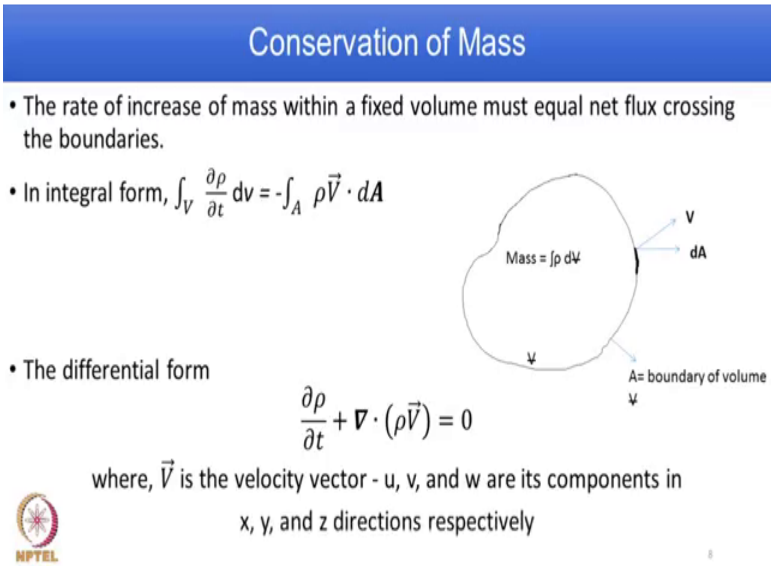

Now, we will see how to get expression in detail for conservation of mass. We know this equation stated as rate of increase of mass within the fixed volume must be equal to net flux crossing the boundary. So, if you consider a control volume, the mass that is insider the control volume is rho in dV, V is the volume that is given here, and then if you define at any point dA is the normal vector, and v is the velocity vector, A is the area of the boundary volume. So, if you express conservation of mass in integral form, so dou by dou t of rho into dv over volume v is equal to rho V dot dA, because area is also vector, velocity is also vector, you do the dot product, you get a flux crossing and here it is dA is continuously changing for this illustration figure that is given so you find out net flux crossing the boundary.

It may be coming in some place, it may be entering the volume, in some place it may be going outside the volume and sum them up you get net that why we do the integral. The integral is correspond to summing and you get that sum and that is related to rate of increase of mass within that control volume. Now, the differential form of the equation is dou rho by dou t plus del dot rho V equal to zero. We have already seen this different operator. So, V is the velocity vector and in Cartesian coordinate system, you have u, v, w as a component to define of the velocity.

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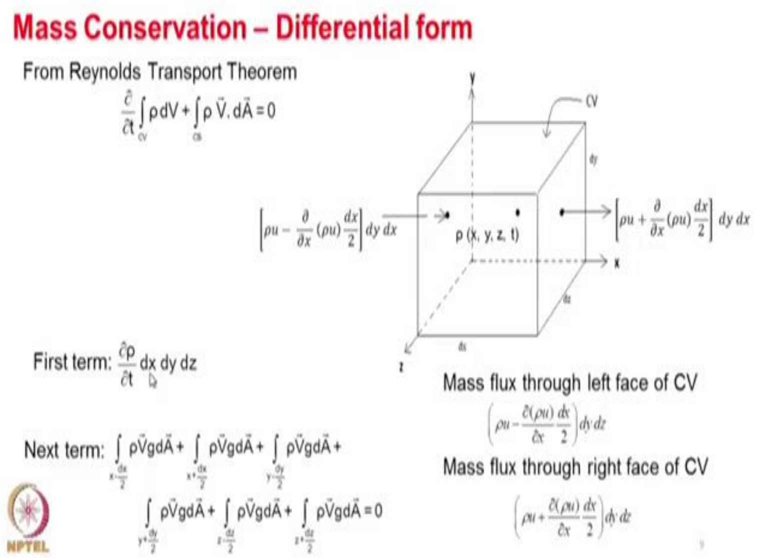

So, we already seen, we will now see in detail. So, let us take a volume three-dimensional representation here, fluid element; and with the elemental length define in each direction, for example, dx is the length of that element in x-direction, dy is the length of the element in y-direction, similarly dz is the length of the element in z-direction. So, Reynolds transport theorem, we already seen; so if you substitute eta equal to one, then you get N on the left hand side to be mass, so that equation is put here again.

And volume is taken inside, so dou by dou t of control volume rho into dV will give the mass and this will give you the flux crossing the control surface. And you find out flux crossing at each control surface and then you sum them up. So, in this case, if you consider this is the rectangular volume element, and you define for example, this is one face, and this is another face, front face, back face and so on, so there are six faces here, top and bottom. So, one has to get explanation of this term on although control surface.

So, the first term is dou rho by dou t into dx into dy into dz; dx, dy, dz corresponds to volume and dou rho by dou t is multiplying the dx dy dz will give you the rate of change of mass. Now, if you consider the next term, second term in this equation, so as I mentioned, if you consider rho, if you consider center of this volume x, y, z then from the center, if you go to the left in x-direction, it becomes dx by two; similarly, form the center, in the x-direction if you go to the right, that elemental length is dx by two positive. Similarly, you can go from the center come front and go to the back, there will be dz half; similarly, top and bottom that will be dy by two, half in either direction. So, this term, which is accounting mass flux through control surface, and you will find out the term for each of this face, so x minus dx by two will be the left face; x plus dx by two will be the right face. Similarly, y minus dy by two will be the bottom face, and y dy by two will be the top face; similarly, from the front and back. So, there are six terms for this cube bar element that you have consider.

And if you write for example, only for the left face, mass flux to the left face rho, x, y, z, t that is defined here. Assume that is a density available at center of the element and you go left hand side, and this will give you the flux that is crossing and this is flux that is leaving. So, we can say mass flux through the left face of CV, and mass flux through the right face of CV. So, you get term define like this for each face and put them in this equation. And finally, in all, you have dy, dz and dx term, which is actually the volume.

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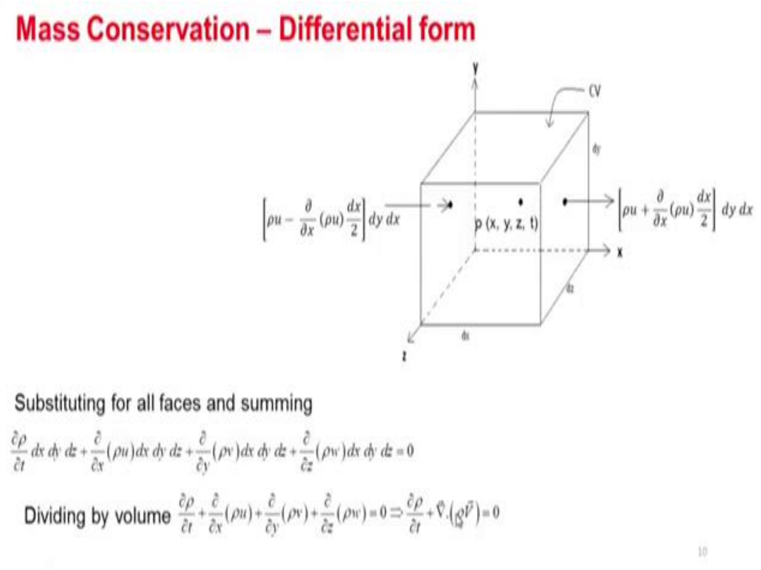

So, if you do that arithmetic substitute for all faces and sum them up; if you do that arithmetic and divide by volume dx, dy, dz that terms appearing in all the three terms of the spatial as well as on temporal. If you divide by the volume, the you get dou rho by dou t plus dou by dou x of rho u plus dou by dou y of rho v plus dou by dou z of rho w, and you also know how to convert or write in other form, this dou rho by dou t plus del dot rho into V equal to zero.

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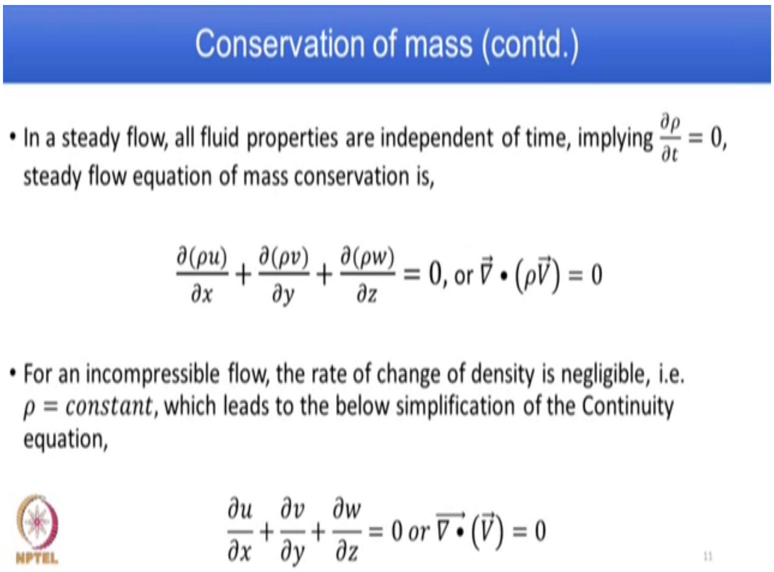

Now, what we have done is a generic, not particular to any situation; generic equation gets reduced or simplified for different situation. For example, in steady flow, we know the properties do not change with the time, so any term with the time derivative is there equal to zero. In this case, it is dou rho by dou t that first term dou rho by dou t goes to zero, and mass conservation equation gets reduced with only a spatial derivative term that is given here, and in terms of vector, del dot rho V equal to zero. Another simplification, if it is incompressible flow, we know density is the constant, so any derivative, any spatial derivative of density will not be there and that term is also removed and we get only the velocity component, so dou u by dou x plus dou v by dou y plus dou w by dou z equal to zero; and in terms of vector, it is del dot V equal to zero.

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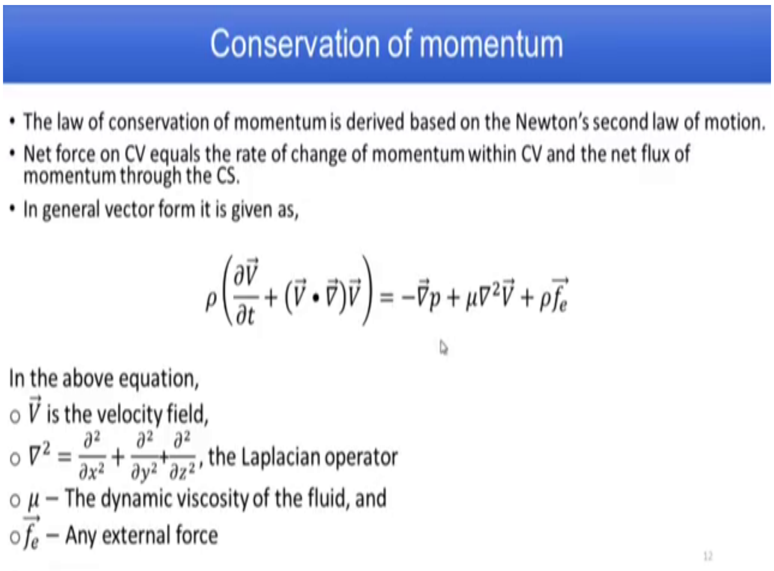

So, the next equation we said we will do three equations – conservation of mass, conservation of momentum and conservation of energy. The second important equation is conservation of momentum, which is based on Newton second law of motion F is equal to ma. Like we did for conservation of mass, here also we need to consider elemental volume, find out net force acting on the control volume and then net momentum flux crossing the control volume. You account for both of them then you get finally, conservation of momentum equation. So, it is stated as net force on the control volume equals rate of change of momentum within the control volume and net flux crossing the control volume.

So, in vector form, here again we are not doing the detailed derivation that is there in any standard fluid mechanics text books or any other open source materials or other NPTEL course. So, in vector form, conservation of momentum is given here, rho into dou V by dou t plus V dot del into V, which is on the left hand side equals pressure gradient, viscous force and any other force. Now, V is the velocity field; and this del square V, which accounts for shear force, it is the Laplacian operator and given here as del square by del x square plus del square by del y square plus del square by del z square. And mu, we know it is the dynamic viscosity of the fluid. So, if you divide equation by rho, then you get mu by rho, mu one upon rho into del p and only the external force.

So, if a problem, you are applying the problem on say involving the magnetic field then this external force - f e represent additional magnetic force. Similarly, a problem involving gravity then you have gravity force appearing on additional source term. So, this is given as a source term, and for different problem, you will have a different definition of the source term or external force. So, if you look at this equation, on the left hand side, you have one component for time derivative, and this del three components as a spatial derivative and this derivative of time, you know that gives acceleration, similarly this term also supposed to be acceleration, we will get the definition clarity in next slide. So, the left hand side is acceleration; and right hand side, pressure is the force, viscosity of the shear force or any other force so that is why I said Newton second F is equal to m into a is actually the first principle based on which conservation of momentum is derived. And you are able to see here the same acceleration term and force term on other side.

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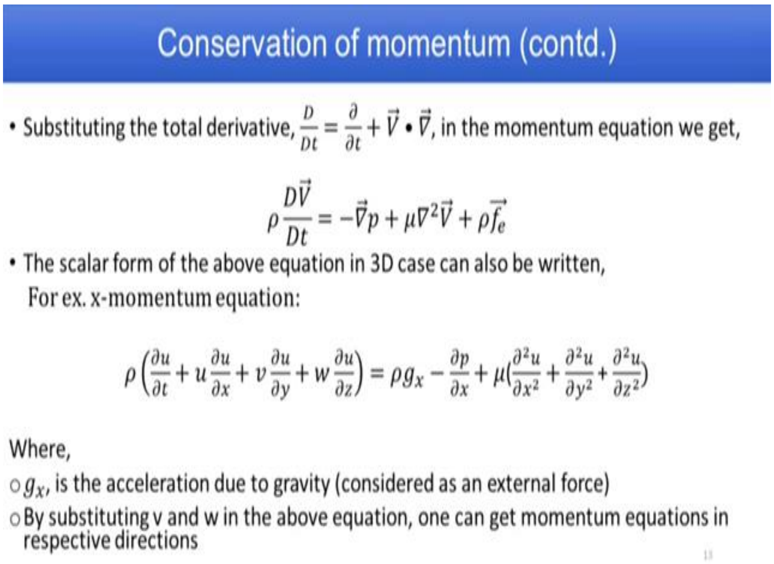

We have already seen in the first class, different review, we have seen total derivative which is capital D by Dt equal to time derivative dou by dou t plus V dot del. So, if you substitute the definition of material derivative or substantial derivative in the momentum equation then it is rewritten as given here. So, D by Dt of V on left hand side; the right hand side is the same pressure force, viscous force and any other force. It is convenient to write in vector form, sometime it is easy to write equation in scalar form, so we will also get to see how to write momentum equation in scalar form. So, for example, we know three-dimensional representation, x- momentum equation; the detailed expression of the same momentum equation for x- momentum equation. We know the velocity component along the x is u, so dou u by dou t plus u into dou u by dou x plus v into dou u by dou y plus w into dou u by dou z, and this corresponds to expansion of this material derivative D by Dt of V on the left hand side; and pressure term dou p by dou x is appearing here for pressure, viscous force is appearing here and then external force is consider as gravity force. And we take a component, you get rho into g x.

So, where g x is acceleration due to gravity, which is consider here as an external force. One can also obtain similarly by substituting for V vector, in terms of velocity component for y as v, and w for z, we get momentum equation the corresponding direction, y- momentum equation and z- momentum equation.

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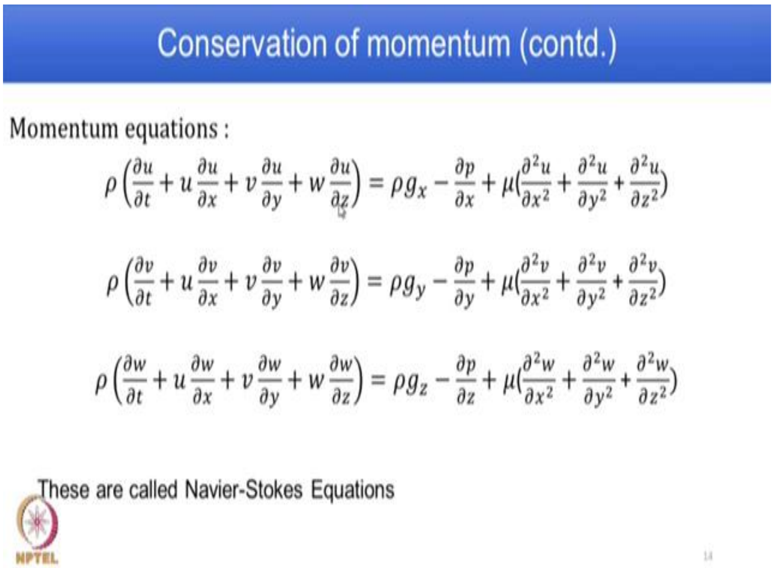

So, if you do that, you are able to write a complete momentum equation, and you are able to see here corresponding v- momentum equation or y- momentum equation and third direction w- momentum equation or z- momentum equation. Now, advantage of writing in vector form, writing in Taylor form; and in vector form, these three equations with so many terms or written in elegantly in simple one equation with only two terms. Whereas in scalar form, you are able to write in detail and one can actually locate what component responsible of what. So, while writing a code also you write in detail form then write a code, it is easier to decode later. And this set of equation that is all the three momentum equations with full component written, all the terms written, we generally called Navier-Stokes equations. Navier and Stokes are two scientist, who independently developed these equations and as a credit to them, these equations are named as Navier-Stokes equations.

And you can again understand in this equation, if you take left hand side, all the time derivatives, which is given as dou u by dou t, dou v by dou t and dou w by dou t, they are all called local acceleration. And u into dou u by dou x plus v into dou u by dou y plus w into dou u by dou z, those three terms in x- momentum equation; similarly three terms in y- momentum equation, three terms in w- momentum equation, they are convective acceleration. And you put them together all the four terms will give you total acceleration.

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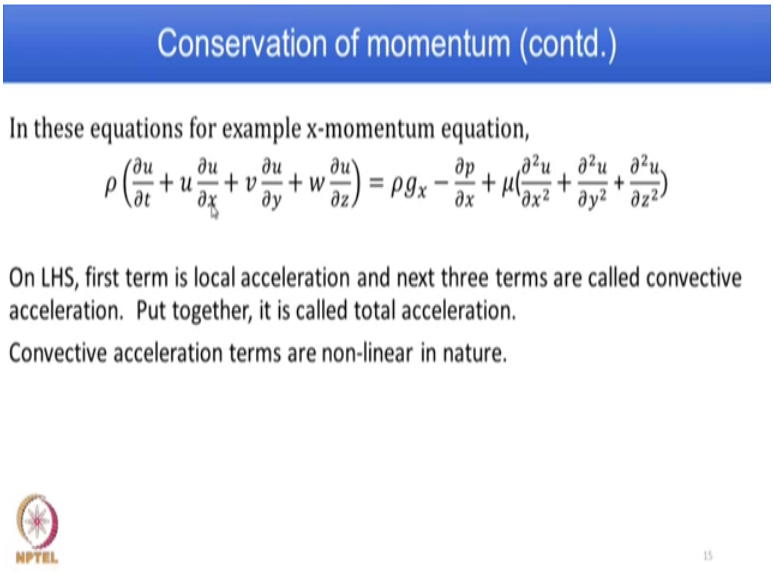

So, that what is explained, left hand side, first term is local acceleration that is this term and remaining three terms are called convection acceleration. Put together, it is called total acceleration. Now, if you look this particularly convection acceleration in closely, we see here u is a velocity field, which is the function main quantity in the x- momentum equation, and dou u by dou x is the derivative of the same velocity. In other words also v into dou u by dou y, v is the velocity field in y component, and there is velocity derivative. So, function multiplying the derivative of the same function is result in what is known as a non-linear in nature. So, such non-linear nature of this convective acceleration actually attracts a special attention, because the behavior of how to solve or how to represent u, multiplying dou u by dou x in discretization is important and we are going to focus a special attention on this convective term, because of its non-linearity. The behavior and the treatment of this convective term can change the solution results in some in stable also accuracy to some extent.

So, we are going to see in detail treatment of this non-linear convection term later. So, in this class, we did important topic flow description, conservation of mass equation, conservation of momentum equation detailed in differential form and writing the momentum equation in vector form and scalar form. And important of non-linear convection term and why one should focus on convection term discretization, and how to represent different external force for different problem. So, next class, we are going to see little more detail on this momentum equation, and go onto the next equation, conservation of energy equation.

Thank you.