Foundation of Computational Fluid Dynamics
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Lecture – 19

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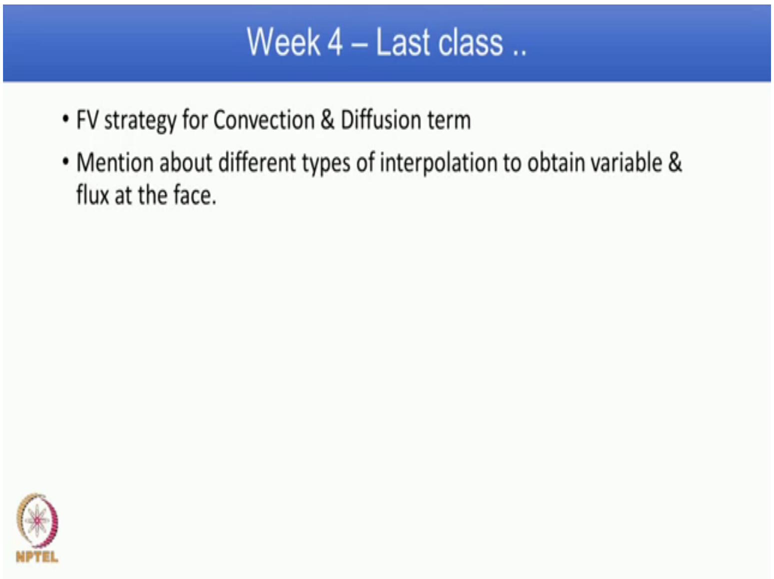

Greetings and welcome again to this course. Last class, we had particularly seen finite volume strategy for convection and diffusion term put together. And we mentioned the need for knowing different types of interpolation required for convection term and evaluating flux at control volume faces. We also looked at in detail about CD approximation, and we took an example problem of flow through a backward facing step, we understand in the same flow even in the same geometry, we have a different flow region flow going reverse direction. Hence, there is a need towards to take the flow physics represented appropriately in the evaluation of flux at control volume faces.

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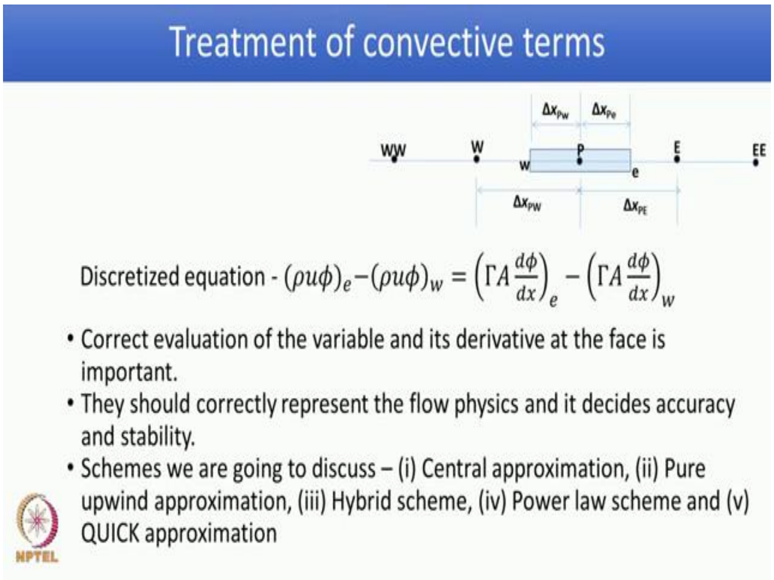

For sake of continuity, again we reproduce control volume finite volume mesh and terminologies used. The last class, we had this discretized equation including convection term. So, convection term is on the left side, and diffusion term on the right side, we noticed the convection term to be evaluated at face e and w; similarly diffusion flux to be evaluated at face e and w, and this is for 1D situation. So, correct evaluation of variable and its derivatives at the face is important. As I explained the help of illustration that should correctly represent the flow physics and it also affects the accuracy as well as to some extend stability. We are going to discuss five different approximation method to get flux or functional value at the face; they are central approximation, pure upwind approximation, hybrid scheme, power law scheme and QUICK approximation.

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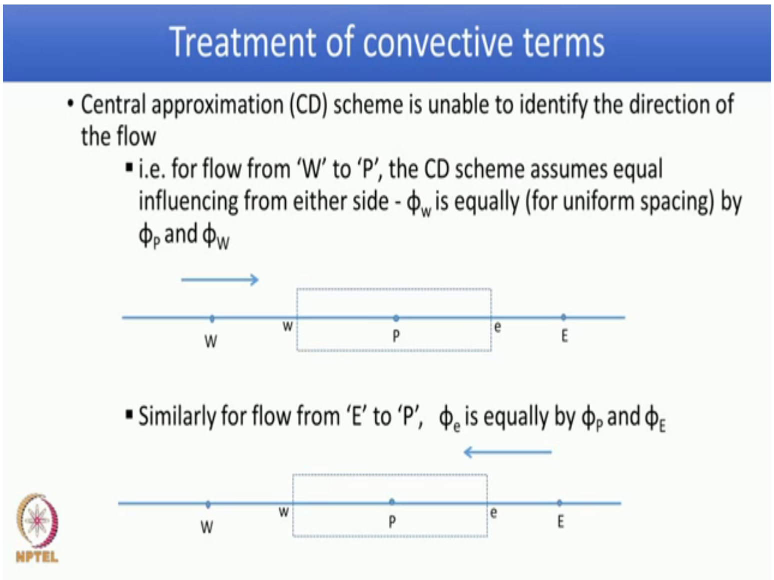

We have already seen central approximation and we noticed that central approximation does not recognize the direction of the flow. It takes equal weightage from the neighbouring nodes. For example, in this case if the flow is from left to right, and to evaluate variable at the face west then the neighbouring nodes are p and w, it takes equal weightage from both these two nodes. Similarly for other situation, if the flow is from right to left, then it take correspondingly neighbouring nodes and then gives equal weightage that is for the uniform mesh; then if it is un known information you take appropriate weightage, it does not factor the flow direction

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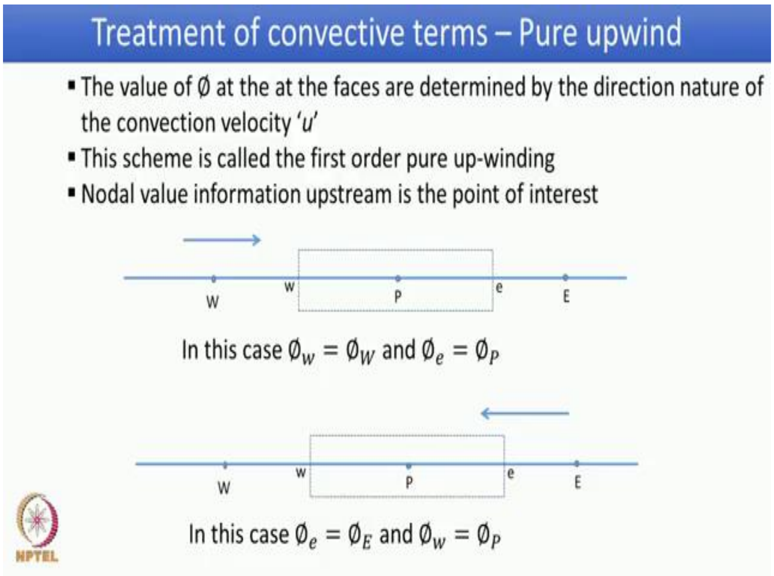

In the case of next scheme that is pure upwind then phi value at faces are determined based on the direction nature of the convection velocity u. If you can recall illustration I explained in the last class backward facing step spectrally mentioning about coordinate definition. So, with respect to the coordinate definition then convection velocity direction also is decided. This scheme which decided based on direction of the convection velocity u is what is known as pure upwinding. Here the nodal value information, upstream of the point of node of interest is considered to define value at the face. For example, we will take the same illustration that we have just shown for central differences scheme. The flow is from left to right and to evaluate variable at this face w, it is given same value as variable at the upstream of the face. So, in this case upstream of this face node is at capital W that is this node. So, variable available at this node is given and variable at this face is same as variable at this node. So, in this case, phi at w equal to phi at node W; similarly for east face, upstream of this east face is node P for the direction that you have shown that is left to right, so phi at e is same as phi at P. Now you can see very clearly the difference between central approximation and pure upwinding.

Let us take another case that is flow is from right to left as shown here. So, phi at east is same as phi at east node, because for this face for the direction of the flow that is shown the upstream node is e. Similarly if you to come to the other side for this face, upstream node is interest node P itself for the direction of the flow that we have shown that is why phi at west face is same as phi at node. So, you can see the difference between central type of approximation and pure upwinding, it is called pure upwinding, later we are going to see some kind of weighted average between pure upwinding and upstream biased upwinding.

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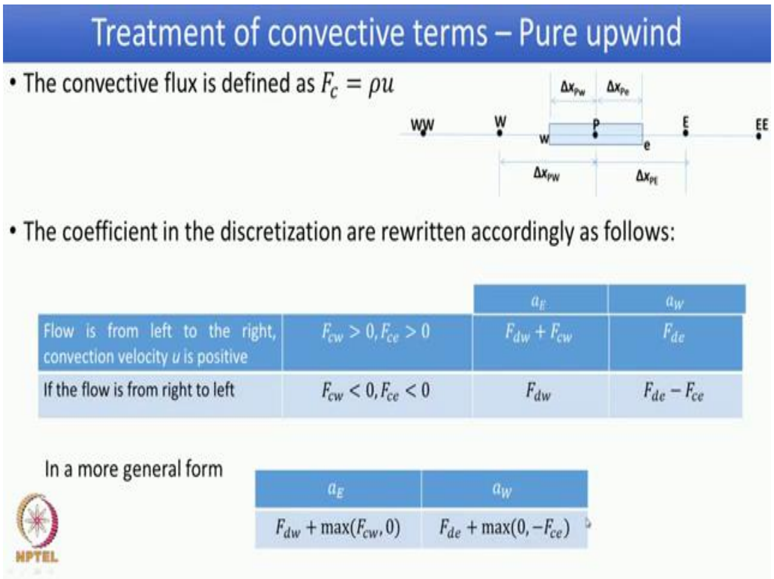

Now as we did before convective flux is defined as row u with the symbol F c now we have this switching if the flow is from left to right then we take the upstream node if the flow is from right to left you take corresponding upstream node in a problem you have some regions where the flow is from left to right in some other region you have flow right to left using the same code we should have the provident check the convictive convection velocity direction and switch corresponding upstream side and that is explain in this slide for example, it is effected in the coefficient in the neighbouring nodes.

So, it is a E and a W are the two neighbouring nodes for the node of interest P. If the flow is from left to right that is the convectional velocity is positive for the definition of coordinate that you have chosen then u is positive, so F c is also positive. So, we write F c w is greater than zero and F c e greater than 0, then you have coefficient defined accordingly for the east face and east node. And for west node, if the flow is from right to left then for the coordinate definition that you have chosen it becomes negative then corresponding F c goes to negative that is why we have condition if F c w less than zero and F c e less than zero then we rewrite the coefficient accordingly. So, this switching depending on the direction of the velocity you can actually incorporate the code also you can also have a check condition and then you can switch between one scheme to another scheme are upstream node accordingly.

So, in more general form, so to put this two things together because in a code you will have only a continuous line are one line representing all the situation we write a e as F d w plus max of F c w comma 0. So, this will take maximum of this two values zero is a limit and if it is positive for the flow from left to right F c w is positive. So, that will be taken and it added to the F d w and you get back the original coefficient expression to get other condition then we again write here if the flow is from right to left then F c e will be negative and minus F c e will be positive and greater than one. So, you have again condition max of zero comma minus F c e and the diffusion coefficient contribution. So, this condition that is defining max of zero and other value minus F c e or F c w will automatically switch inside the code you do not have to do anything manually externally.

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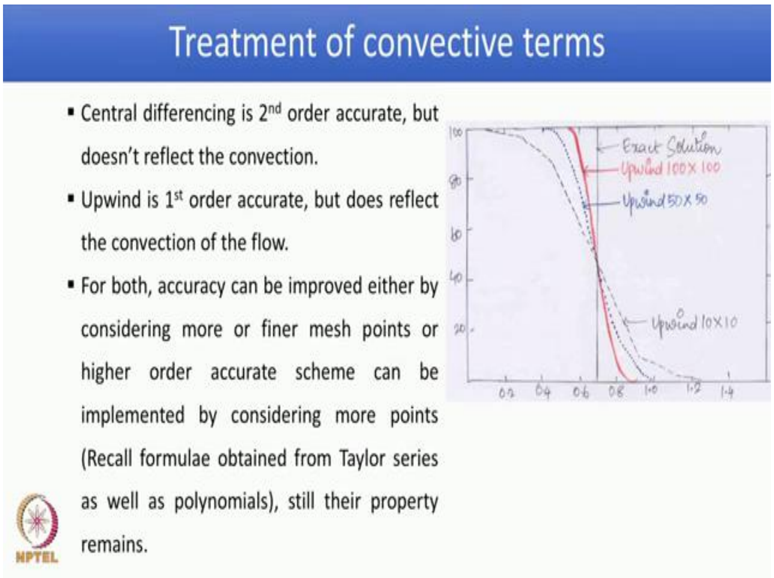

So, next we will see central differences scheme is second order accurate, but it does not reflect the convection whereas upwind scheme pure upwind scheme that we have just now seen the first order accurate as we have seen it only takes the nodal value upstream of the face for the corresponding are the respective direction of the flow were as it does reflects the convection of the flow now the obvious question either we can take or use utilize advantages of both and put it to one particular scheme. So, for both accuracy can be improved either by considering more are finer mesh points that is what we have shown is kind of second order approximation for central scheme that is you take one node on the left one node on the right and take a average you can actually extend it by taking extra node on either side. So, it can be two nodes on one side and one nodes on the other side take a average accordingly. So, that becomes next order of accuracy in central differences scheme similarly for pure upwinding instead of taking just one node immediately upstream of the face you can actually take one more node.

So, in this case for example, we for the face at w we considered node value at w in addition you can consider node available at WW. So, we can increase order of accuracy by considering more number of points; however, still the property remains the same that is they reflect convection other one does not reflect the convection, but ob order is always higher. So, I just give a illustrations just illustration for a particular problem and what is shown here is exact solution that is by one particular what is shown here as a exact solution vertically if you use pure upwinding with a mesh rise fifty by fifty then you get a solution as shown by blue colour if you have used lesser mesh size for the same pure upwinding ten by ten size then the black colour line that shown here is a result if you made the mesh much finer.

So, in this case it is by 100 by 100 then you get a solution as shown by this colour whatever it is the pure upwinding, it bias the solution. Hence you will never get to reach the exact solution in spite of the fact that you increase the mesh size twice from fifty to fifty by hundred to hundred you still have some diffusion here exact solution is not obtained were as the central scheme will try to capture this P. Hence there is the thought process whether you can combine advantage of both schemes.

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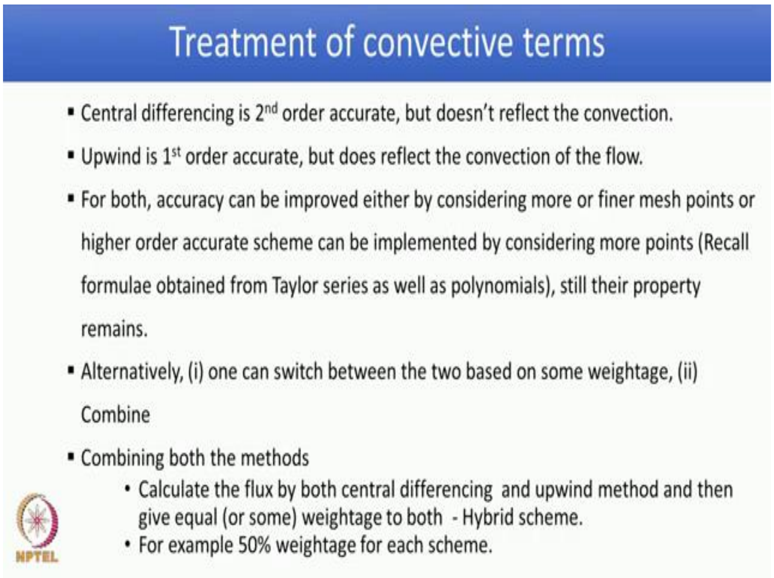

Alternatively, one can switch between two schemes that is central or pure upwinding either based on some weightage. So, you decide if the flow is less than you can switch or if the flow is normal then you can go to central differences scheme or you can take weigh you can calculate first by central differences scheme second by pure upwinding and consider equal weightage both and then combine them. So, this is what is known as a hybrid scheme combining both the methods calculate the flux by both central differences scheme and upwind method either you can give a equal weightage to both or you can decide the weightage according to the problem any one of the method falls under the category what is known as a hybrid scheme.

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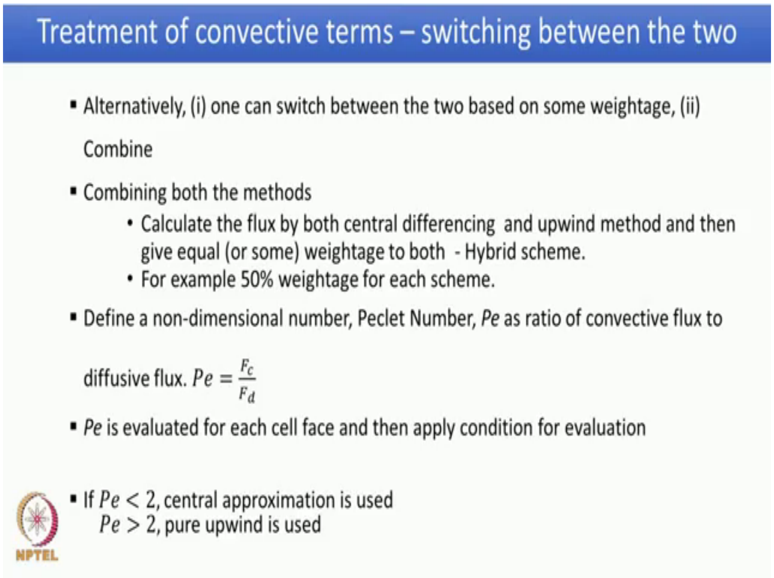

Now how to switch between the two we have a number called Peclet number which is the ratio of convictive flux to the diffused flux where is given by the letter symbol P e, it is equal to F c by F d p e is evaluated for each cell face and then it decides whether one has to switch between central differences scheme pure upwinding or the hybrid type depending on a condition that is using for a evaluation. So, for example, if P e is less than or equal to two which means it is F c and F d ratio. So, you can think of F p e is less than two then it is a central approximation that is used if p e is greater than 2, which mean the numerator is very high and denominator is very less; both these possibilities are there that mean convection is dominating hence you can switch to pure upwinding. So, this condition also you can enforce or you can incorporate your own code decide based on Peclet number and define the Peclet number has ratio of convective flux with a diffusion flux.

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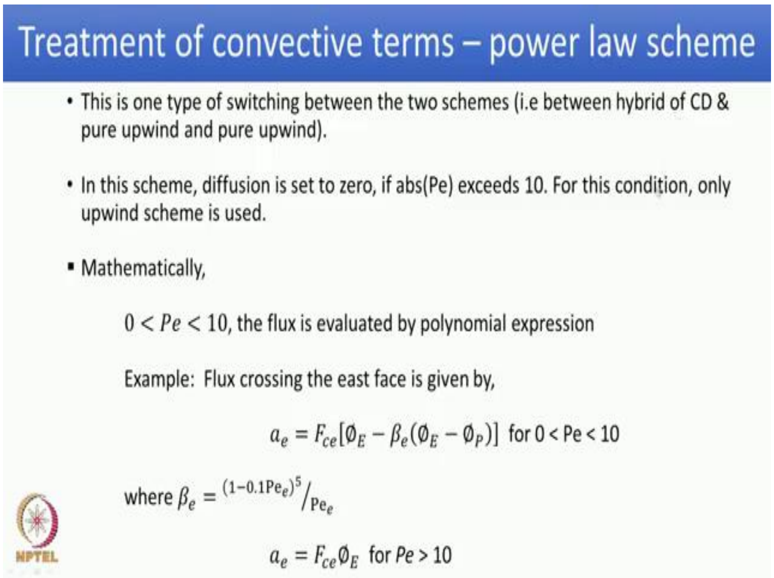

There is one more special type of treatment what is known as a power law scheme and here we have switching between two scheme The two schemes are slightly different the one scheme is a hybrid of c d pure upwinding the other scheme is pure upwinding itself. So, we will have an option you consider mix of hybrid done mix of c d and pure upwinding has one option or its only pure upwinding. So, this is what is known as a power law scheme and in this scheme diffusion is set to zero if the absolute value of Peclet number excites done. So, in the previous slide we put a condition P e greater than or greater than two then it becomes pure upwinding. Now this is condition above that condition. So, if absolute of p e that mean you take only the magnitude excites done then it is purely convection problem diffusion has no role hence the diffusion is set to zero And we have only pure upwinding scheme applied.

So, mathematically you can have the expression P e within the limit from 0 to 10, the flux is evaluated by polynomial expression as given here. For example, flux crossing the east face is given by a e equal to F c e and the coefficient phi at east node minus beta e phi at east node minus phi at p node extra and this is for condition p e between zero to ten where this beta is a coefficient which is actually the speciality of this scheme what is known as a power loss scheme there you write beta is given as 1 minus 0.1 P e. Again we are evaluating p at every face. So, here p is evaluated for east face because we are just explaining for east face. So, one minus point one p e evaluate at the east face to the power of phi divided by p e evaluated at east face just observe that if you substitute the limiting condition that e is equal to 10 then 1 minus 0.1 into 10 it becomes 0 almost so that is actually what is used here right it becomes purely upwinding right.

So, this part beta e is 0 and F c e is purely phi at e that is the pure upwinding that is what shown here for value definitely greater than ten then a e is F c e and to phi from the node e if the value of p is less than ten then you have a some fraction value here beta and that goes into this value beta has some fraction value. For example, you say 0.9 then this is pure upwinding phi e is pure upwinding first term the second term is kind of some central differencing. So, this has a combination of pure upwinding and central differences type of evaluation. So, that is why I said it is a combination hybrid between two schemes that is hybrid of c d and pure upwinding and pure upwinding.

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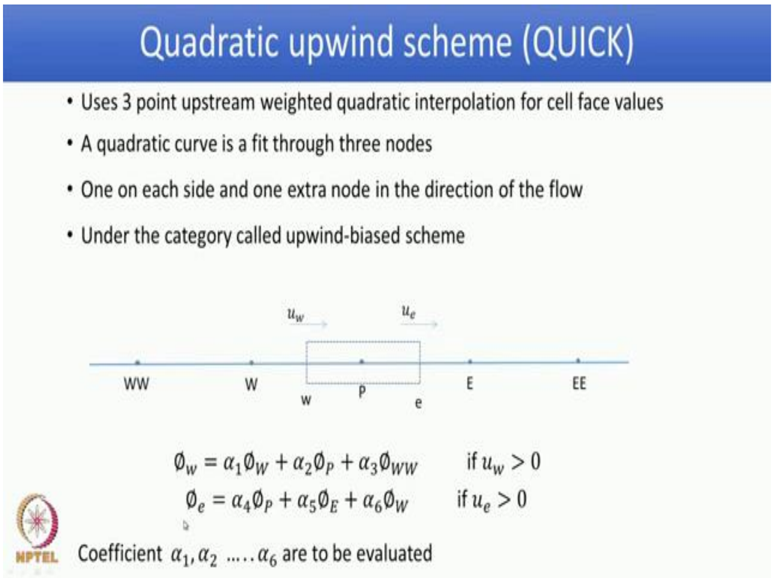

Next stream we are going to see is quadratic upwind scheme and in short form it is called QUICK. So, what it does it takes three point of stream of the node and give a weightage to the contribution from each node. So, it is use as three point upstream weighted quadratic interpolation for cell face values now why we taught as come we noticed from the previous lecture central difference type of approximation takes values from neighbouring nodes and give a equal weightage where the pure upwinding takes the values from the upstream node now if you fit quadratic among the three nodes then it is a kind of bias based on the direction of the flow.

Let us see how to do with little more detail a quadratic curve is fit through three nodes and how these three nodes are chosen is important one on each side of the interest. So, in this for example, in the case of west face for the flow going from left to right one on each side is node p and node w and one more node in the direction of the flow. So, if the flow is from left to right again west of west that is WW is additional node will explain this again this falls under the category what is known as a upwind biased scheme. So, we have a central differences scheme we have a pure upwinding scheme we have a hybrid scheme we have a power loss scheme now we have a upwind biased scheme. So, the scheme is the kind of central, but it is not pure upwinding it is the mix of both, so it is upwind biased scheme.

Let us look at this illustration for example, phi at w at this face then we have a node p immediate node on the right and node w on the left. So, these two are on either side for this particular face then the flow is from left to right. So, u w subscript w is a convectional velocity it is positive. So, for this convectional velocity condition the upstream condition is node WW. So, we consider node WW and node P and we try to fit a quadratic curve to this three nodes with some weightage. So, that is what is shown this expression here. So, phi at west face is alpha times phi at w alpha 2 times phi at P alpha 3 times and phi at WW, if the flow is from left to right that is u w is greater than zero.

Now, let us take the next case when the flow is from left to right and on the east face if u e is greater than zero then the flow is from left to right phi at e is evaluated for phi at e two adjacent nodes are east node and p node and one more upstream node in the direction of the flow. So, here the flow is from left to right for that case one more node in the upstream is actually w. So, the expression for phi at e is equal to alpha four phi at p plus alpha phi phi at e plus alpha six phi at w. Now coefficients alpha one to alpha six are to be evaluated.

So, in today’s class we have seen different kinds of interpolation mathematical expression how to implement them in details behind each of this scheme. You get a complete feeling of the effect of central differences scheme, pure upwinding, upwind biased, hybrid scheme. Now it is possible to implement all this schemes in one particular code, all they divided to a switching based on condition. Most of this commercial software has a similar provision depending on the choice that you select in the option then it for the same convection term calculation is switch the scheme. We come to end of this class; in the next class, we will explain all these schemes with a help of illustration.

Thank you.