Foundation of Computational Fluid Dynamics

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Lecture – 18

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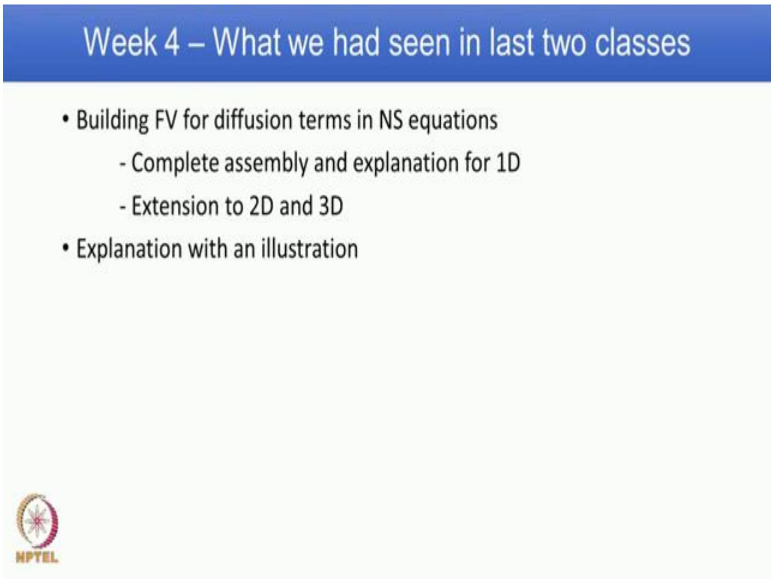

Greetings and welcome you again to this course on CFD. Last two lectures, we had seen finite volume strategy for diffusion equation. We did that in detail, we took an example problem and illustrated with the example problem including in the implementation of boundary conditions. And last class, we are also seen how to extend explanation that we did for one-dimensional to two-dimensional as well as three-dimensional. Today's class, we will particularly focus on convection as well as diffusion term included in the equation.

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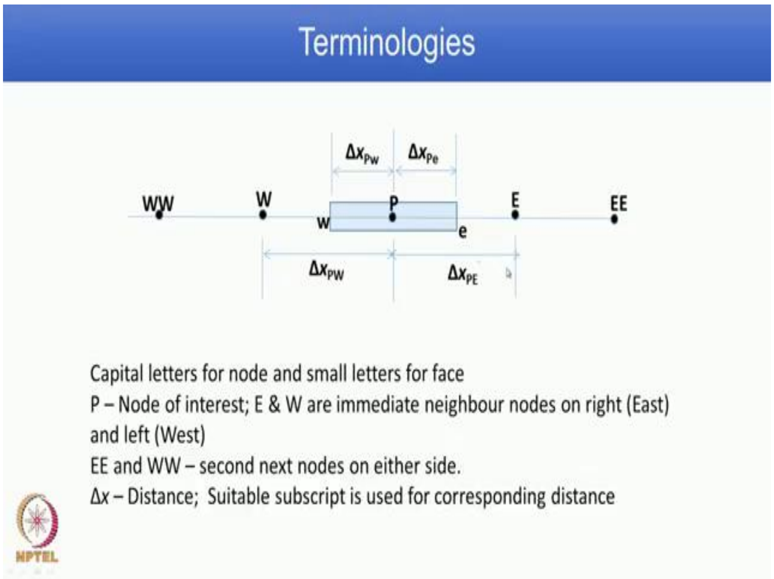

Just for the sake of completeness, we will repeat again the terminologies used in finite volume strategy. This is the mesh that we have define for one-dimensional; P is the node of interest, east node and west node, east of east node is given by E E, and west of west node is given by W W. Distance between the point of interest P and the respective nodes are given by corresponding subscript in this case for example, distance between node P and node W, because it is 1D, it is delta x and with the subscript P W, so it is delta X P W. Similarly, distance between node and the corresponding face in this case for example, the face w on the left side is given by delta X and corresponding subscript P and small case letter w. You can follow for the other face as well as for other node.

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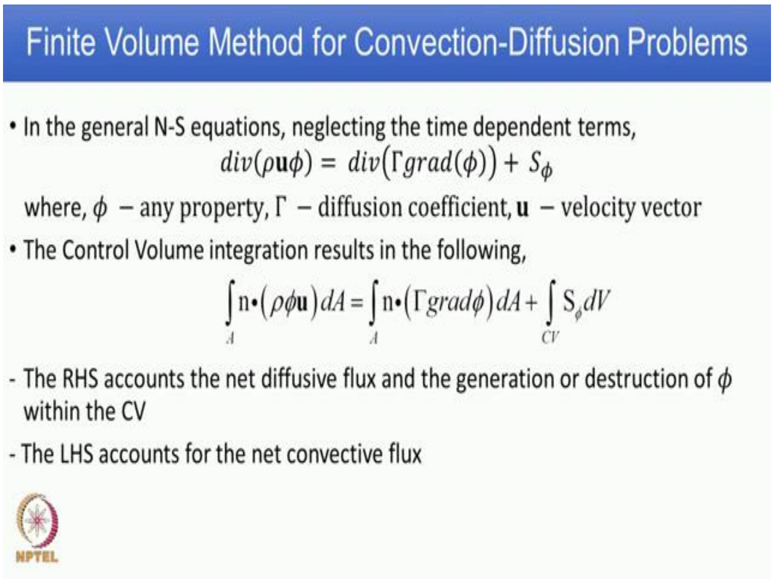

So, in the general Navier-Stokes equation, if you neglect time dependent terms, then you get convection term on the left side, diffusion term on the right side plus any source term and that is what is shown here. So, divergence of rho u phi equal to divergence of gamma gradient phi plus source term. Phi is the variable any property, gamma is the diffusion coefficient and u is the velocity vector. if you do the control volume integration for this equation, it will result in this form integration over A n dot rho phi u d A, so this gives the flux – convective flux equal to diffusion on the right hand side plus source term the second term on the right hand side. So, the R.H.S accounts for net diffusive flux and the generation or destruction of phi. The L.H.S accounts for net convective flux.

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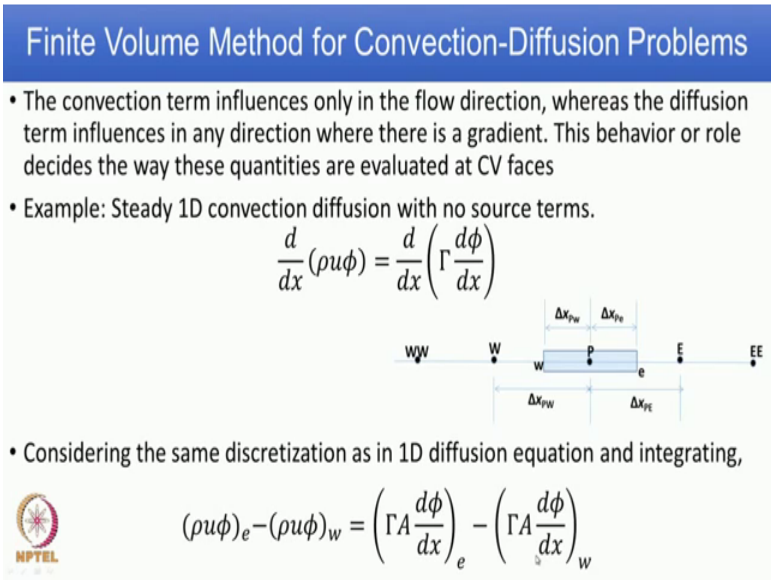

We should know how to convert the volume integral into surface integral. Main difference between convection term and diffusion term, convection term influences only in the flow direction, whereas, diffusion term influences in any direction where there is a gradient. And this difference in the behavior needs special attention when you are evaluating variables at faces and to be used in convection term differently from diffusion term. Let us take an example situation, again we take one-dimensional situation. So steady one-dimensional convection diffusion for the sake of simplicity again we do not have a source term. So, we write down the equation as given here, d by dx of rho u phi plus d by dx of gamma d phi by dx, because it is one-dimensional we write in differential form. Again we reproduced terminologies and mesh arrangement.

So, considering the same discretization as in 1D diffusion equation, integrating this equation over the control volume will result in final form like this rho u phi evaluated at the east face minus rho u phi evaluated at the west face equal to gamma A d phi by dx evaluated at the east face minus gamma A d phi by dx evaluated at the west face. If you observe here, the right side terms are already familiar to you, we did that in detail when we did 1D diffusion equation. So, these quantity are to be evaluated at east face and west face. Here on the left side, we have a convection flux that has variable to be evaluated at east face and west face, whereas, the right side, it has the derivative of the variable to be evaluated at east face and west face. So, you understand we have additional complexity when we deal with convection term. If this convection is written for say phi as the variable, u as the variable for phi then it becomes rho u phi u and rho u u evaluated at east face and west face.

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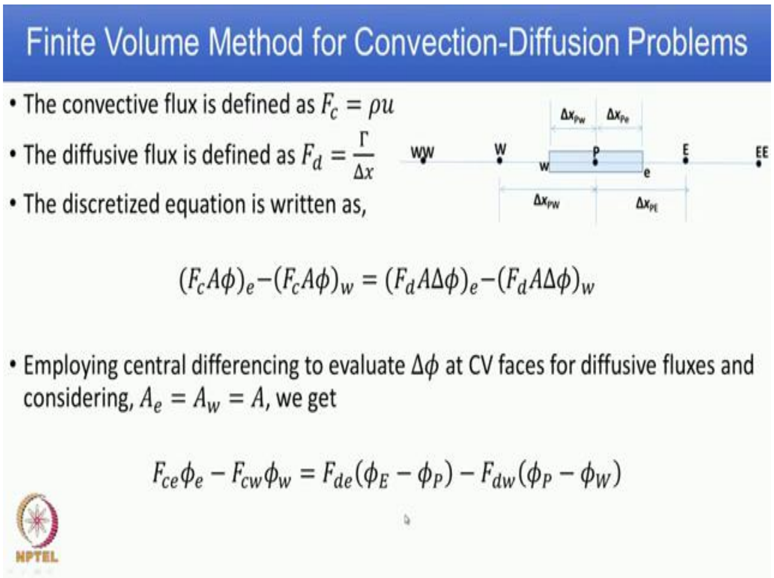

So, we define separately convective flux and use the symbol letter F subscript c for convective flux equal to rho into u. And diffusive flux F subscript d a gamma by delta x. using this symbol the discretized equation is rewritten as F c A into phi evaluated at the east face minus F x A into phi evaluated at the west face equal to similar rearrangement right side for diffusion term. we did in the diffusion equation central differencing type of approximation to get delta phi at faces, we follow the same here also. So employing central differencing scheme to evaluate delta phi at control volume faces for diffusive fluxes and for simplicity sake, we assume area is the same, so A e equal to A w is same as A, which means all the area terms from this equation will get cancelled. So you get finally, expression as shown here, F c evaluated at the east face, phi evaluated at the east face minus F c evaluated at the west face phi evaluated at the west face equal to we followed central differencing type of approximation and we already defined F d as delta x, so corresponding delta x term goes there, so we get final form as shown here.

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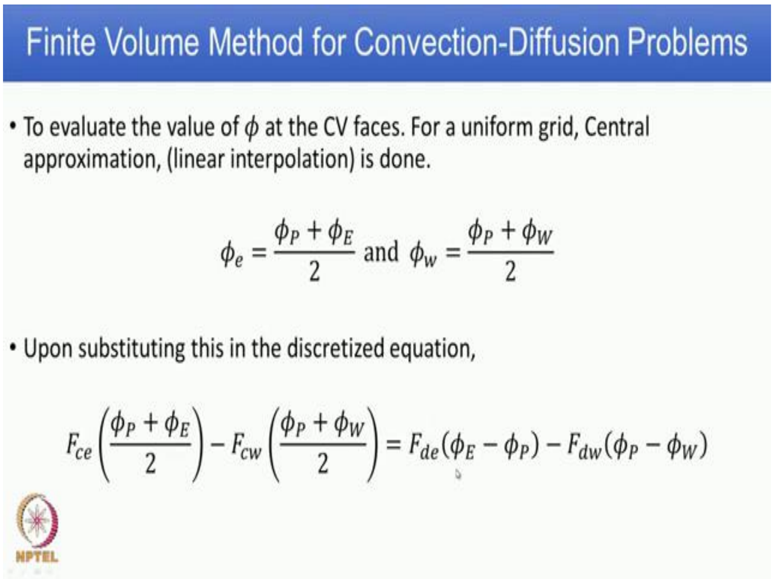

We have additional term that is the variable phi itself at control volume faces. For a uniform grid arrangement, again linear type of interpolation can be used. So, phi evaluated at the east face, phi define at the east face is evaluated from phi at node of interest P and phi at E, so it just the linear approximation or central type approximation; similarly for other quantity phi that is required at the west face is related to phi variable available at node P and W. Upon substituting this replacement in the discretized equation then we get finally, this equation as shown here. So, phi is actually replaced correspondingly, so this is for phi E and this term is for phi W; and on the right side, we have already written for diffusion flux.

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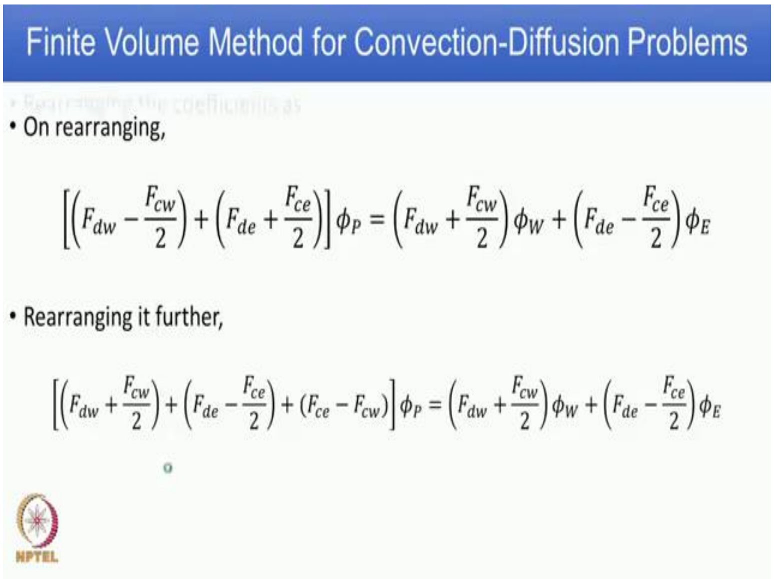

We will have to do some more arithmetic that is F d w minus F c w by 2 plus F d e plus F c e by 2 and all these are coefficients multiplying phi p. In the previous equation, we collect all coefficients that is multiplying the variable phi at p, we rearranged then we get this form. So, this is the node of interest and that is unknown and that is retained on the left hand side. Then on the right hand side, similarly for west as well as east variable, coefficients are respectively written. As we said many times before, P is the node of interest, and in this case for 1D, the node of interest will run from left to right.

We do one more rearrangement, we will understand why we are doing this rearrangement, so this F d w is retained and we would like to have F c w by 2 with the minus sign. So what we do we add F c w full term and then subtract. So, eventually this F c w by 2 positive and minus F c w which is added newly will result in minus F c w by two. Similarly, we have only F c e by 2 and we add F c e newly and subtract, so if you put this term that is minus F c e by 2 plus F c e the original positive F c e by 2 is maintained. This is for some convenient sake, you will understand equated to terms on the right hand side. if you observe terms on the right hand side, for example, coefficients for phi at w, it has a term F d w plus F c w by 2 and you are able to observe, similar term appearing as one of the coefficient on the left side, so this is F d w plus F c w by 2 that is one term on the left side, which is same as coefficient term for phi at w.

Similarly, the second term on the right side, which is written as coefficient for phi at east face at east node F d e minus F c e by 2, again you are able to observe this coefficient appears as one of the term in the coefficient for phi p as shown here. And these are new terms we are added purposely to get coefficient terms as shown here, and similar to coefficient term on the right side. Again you recall, how we did for diffusion equation, in diffusion equation, we had finally, written a form a P that is coefficient for node of interest P is the summation of coefficient from the surrounding nodes and that is the format we are getting it here also that is why we have to do this rearranging.

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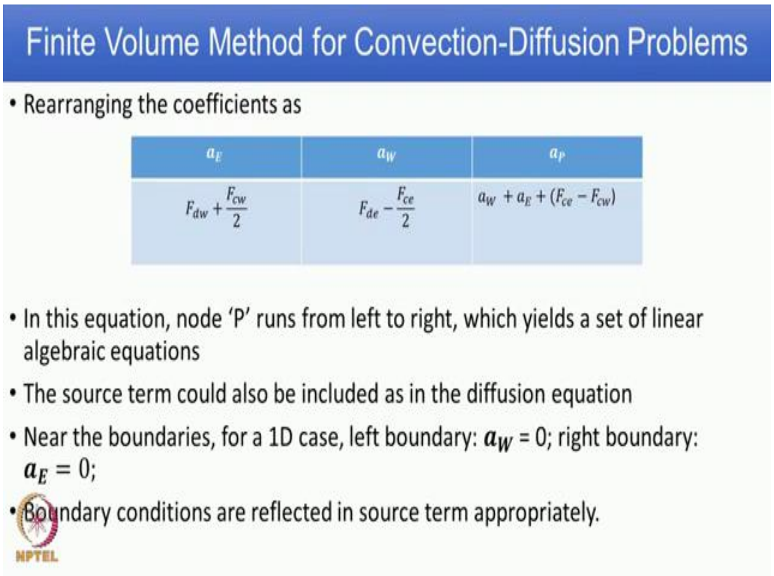

Rearranging the coefficients as we did in diffusion equation, so we have a E as defined F F d evaluated at west face plus F c w by 2; then a W written as shown here, so you are able to write coefficient for a P as sum of coefficient for west node, east node and this subtraction. So in essence, you observe, if you have a diffusion equation, and you have written a code for diffusion equation, now we want to improve the code or argument the code to include the convection term then all that you have to do is just add this extra terms for a P, extra terms for the respective neighboring nodes, so in this case, a W and a E. So, this is how the code is developed from one module to the next module.

As I mentioned before, node P runs from left to right, so if you run node P from left to right, in this case it is 1D, so we say it is running from left to right; in the case of 2D, you will also run from in the y-direction from the top to bottom; and 3D, respectively for the third direction. If you do so, then you will get set of linear algebraic equations. Then you have to have a procedure for inverting the matrix. For the example illustration that we have taken, we have considered without the source term, and it is possible to include source term and the source term will go as a known term and it will go to the right side of the equation. And near the boundaries, so in this case, for example, 1D case, left boundary node near the left boundary, there is no west node, hence coefficient a W goes to zero. Similarly, node near the right boundary, there is no east node, hence coefficient a E goes to 0. This is something that we had seen in the diffusion equation and they do not change when you include convection term also. So, the boundary conditions are reflected in the source term appropriately, this again we had seen in the illustration that we used to explain diffusion equation. The temperature term, which is enforce as a boundary condition appears on the right side of the equation.

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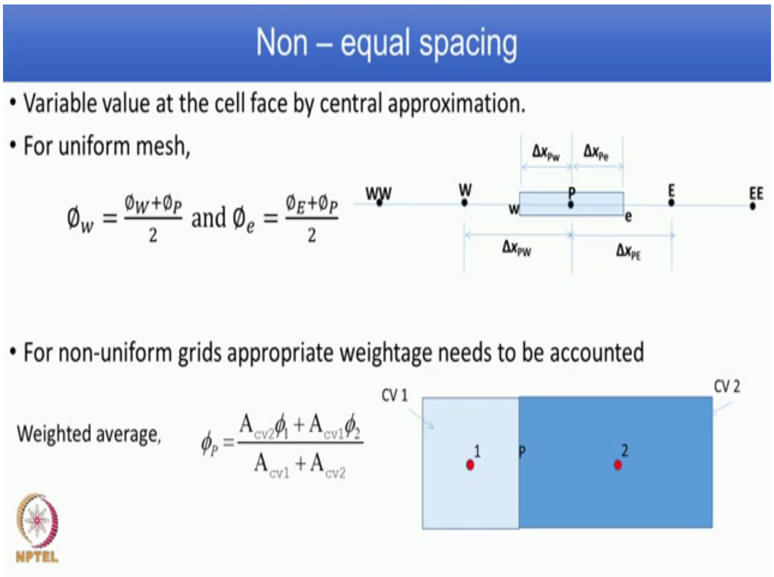

We explain all these for a uniform spacing. Variable phi appears in the convective flux and the derivative of the variable appears in the diffusive flux. So, we need to evaluate variable as well as derivative at space. And we have demonstrated with the help of a uniform mesh. in the case of non-equal spacing, we also have to know how to do for non-equal spacing, because in a practical situation, you do not have a luxury of having a uniform spacing mesh throughout your domain. So, for non-uniform spacing, so this is terminology or mesh arrangement that we use for uniform mesh. And we have phi evaluated using central approximation or linear type of approximation. For non-uniform grids, we have to have a appropriate weightage procedure, you account for contribution from respective nodes. So what is shown here, it is illustration, we have one control volume as shown with a node marked as one then another control volume with a node marked as two. Then we have also point of interest P, and the control volume for P will run from one to two budding one to two. In such case, suppose you want to find for P, say variable at P, then you can have weighted average procedure as shown here; phi at P is evaluated based on area multiplying a variable at other node, similarly area of the second or control volume multiplying variable at that another control volume and summation of area.

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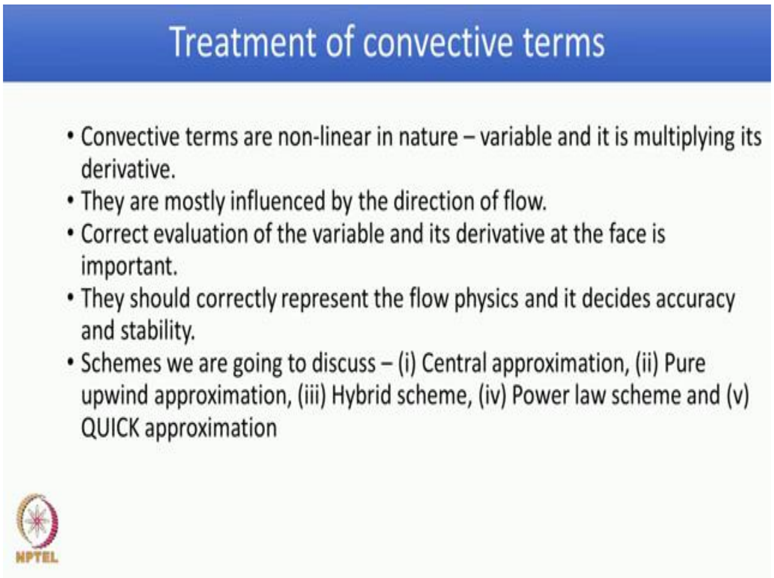

So, we have been talking about convective flux and variable phi present in the convective flux and how to get variable evaluated at that space. this convective terms are non-linear in nature, and it also multiplies its derivative. They are mostly influenced by direction of flow. So, we have to have the correct evaluation of the variable and its derivative at that face, which is very important as it affects accuracy and solution procedure. They should; obviously, represent the correct flow physics and it decides accuracy as well as stability, and we will observe with an example graph later, how it affects accuracy. We are going to discuss five approximation schemes in detail. They are central approximation, pure upwind approximation, hybrid scheme, power law scheme and QUICK approximation.

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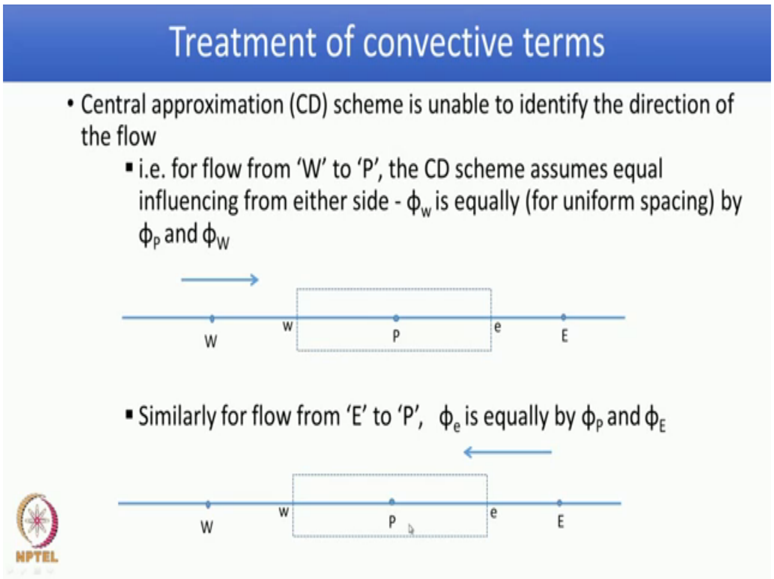

First, we will take central scheme, we have already seen the central scheme. The central scheme as we noticed earlier it takes equal weightage from the neighboring nodes, because of its nature, it is unable to identify the direction of the flow. So, for example, in this illustration that is shown here, the flow is from left to right as marked here, we know the variable value at west node, and node of interest P then the CD scheme assumes equal influencing from either side that mean phi at this face w takes the value of phi at P and phi at W and divide by two that is for the uniform spacing. Suppose it is non-uniform spacing you take into account the weightage. Similarly, for another situation, if the flow happens to be from right to left, then phi at east face is evaluated based on phi at east node and phi at P node. why do we have this flow direction changing that may be a curious question.

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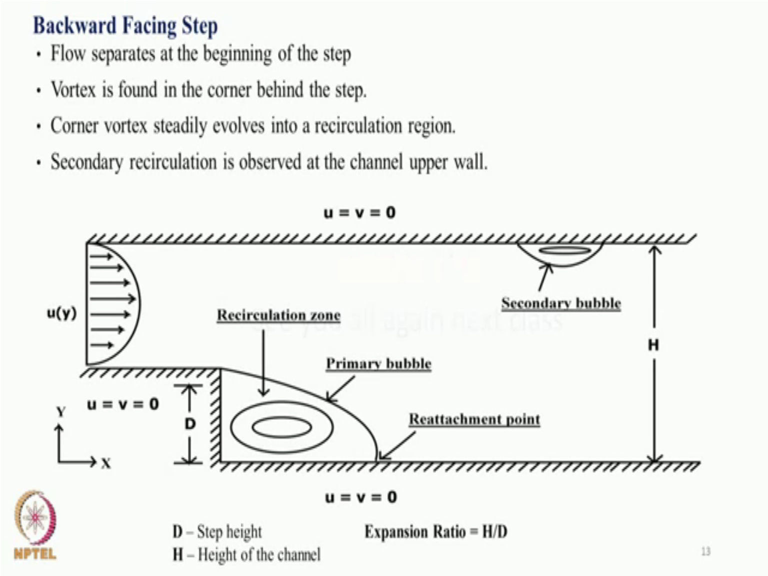

We will reproduce this slide that we had seen in week one lecture, this is the test case problem flow through a backward facing step. And in this case, the inflow is here and it is a fully developed parabolic velocity profile, and the top side is channel wall and bottom side there is step and then there is a channel wall. As you have explained before at the corner, at this tip of the corner flow separates and then it comes and reattaches on the bottom wall at some distance, where it attaches it depends on the flow condition, Reynolds number and so on. the question that is important is what is happening to this region, this is a recirculation zone, and there is a eddy and flow re circulating in this region that mean if you define x and y coordinates orientation to be as shown here then you can observe with respect to the coordinate definition that you have chosen. For some parts, u velocity is positive; and for some part, u velocity is negative. So, this is the main reason, why you have to have a scheme which takes into account the convection of the flow also.

There is a another situation for the same problem, depending on the Reynolds number and depending on the flow condition for the same geometry, you may have a situation where there is secondary bubble formation on the top wall. Here also you can observe for some part of the location, it is u that is positive that is going from left to right and for the some part of the location u is negative that is going from right to left. And this u positive or u negative depends on your coordinate definition. So for the coordinate definition that you have chosen I have explained u positive and u negative. Of course, there is a difficulty near this boundary, where it switches immediately between positive to negative. So, such difficulty or such change of flow direction is very common in practical engineering problem, hence we have to look in detail how to deal that situation. By conclude today's class here, and we will look into details about different methods of interpolation in the next class.

Thank you.