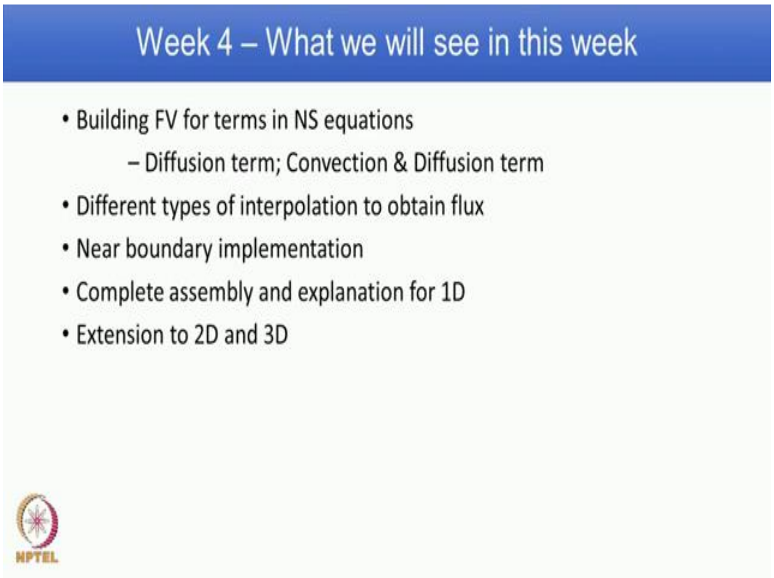
Foundation of Computational Fluid Dynamics

Dr. S. Vengadesan
Department of Applied Mechanics
Indian Institute of Technology, Madras

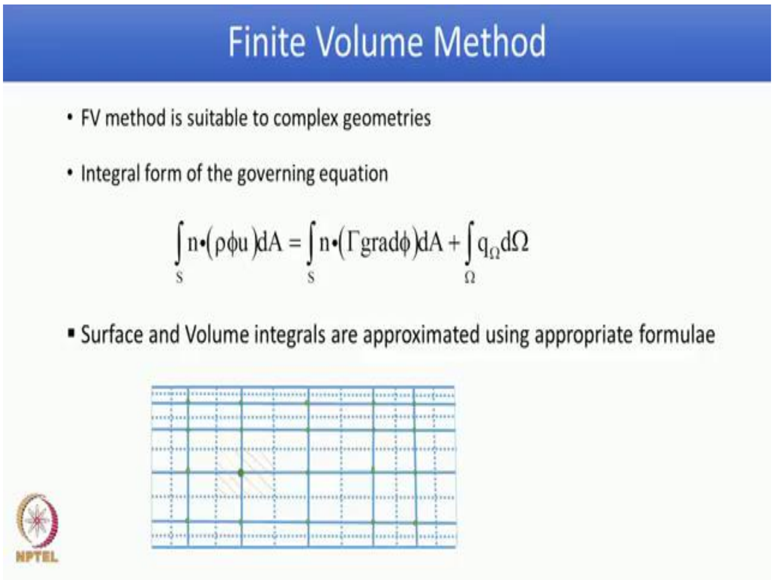
Lecture - 16

(Refer Slide time: 00:40)



Greetings to all you, I welcome you all again to this course on CFD. Today is the starting of the week four we will be touching up on important topic what is known finite volume method of discretization. This week we will particularly see diffusion equation in detail, convection and diffusion term in Navier-Stoke equation in detail. We will do initially these in 1D form then we will get to know how to do in two-dimensional as well as three-dimensional. Then we will have a separate detailed working on convection term alone, we learned before convection term is non-linear in nature hence a separate treatment is required for convection term. We will talk about convection term treatment in detail with different schemes. Then we will also illustrate with example these methods and show comparison wherever possible. There is a special treatment for near boundary implementation, near boundary nodes we will have to account for boundary condition and we will see how the finite volume formulation gets rewritten for near boundary accounting. This will complete in detail the finite volume formulation without time derivate term.

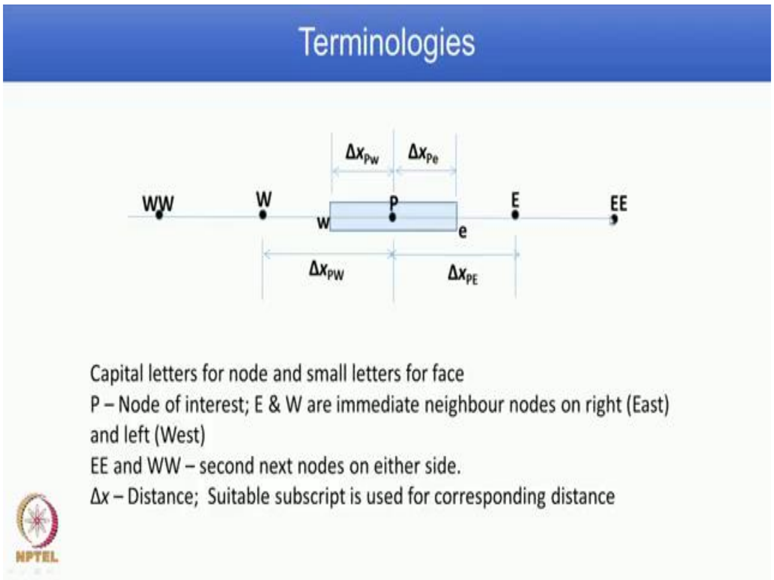
(Refer Slide time: 02:16)



So, in general, we know finite volume method is very much suitable for complex geometry. And what is shown here is a just a simple structured mesh arrangement. You see here horizontal lines as well as vertical lines and wherever they meet shown by a dot that is supposed to represent node, and here you are seeing a node with control volume surrounding that node which is shown by dotted horizontal line as well as vertical line. And this hash line incline hash line supposed to represent the volume surrounding that particular node. And you can extend this everywhere and the procedure is same or almost similar for unstructured grid also.

In finite volume formulation, we use integral form of the governing equation, and it is given as integral over surface flux for a particular surface and equal to first term on the right hand side is for diffusion, and second term on the right hand side is for all the source term accounted. And this is integrated over the volume and this is integral to the surface. Now when we are writing finite volume formulation, the surface integrals as well as volume integrals are approximated using appropriate formula and we are going to see them in detail in subsequent lectures.

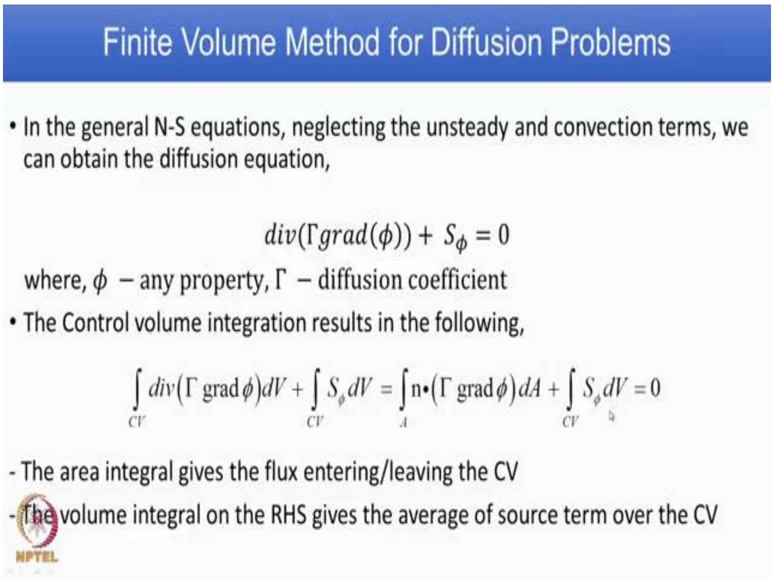
(Refer Slide time: 04:07)



Before we go in the details, we have to get familiarize with terminologies with regard to finite volume mesh, and they are same for 2D as well as for 3D. What is shown here is a line, which represent one-dimensional mesh and we have always dotted circle these are the nodes along that one-dimensional grid; P is the point of interest and W E that is capital letters W E, similarly capital letter W W and E E are to represent nodes. And then you construct control volume between nodes. So, for example, between P and capital W, there is a control surface and this is referred as small case w, similarly between node p and capital letter E which is on the east side node, a control surface is constructed and that is referred by a small case letter E.

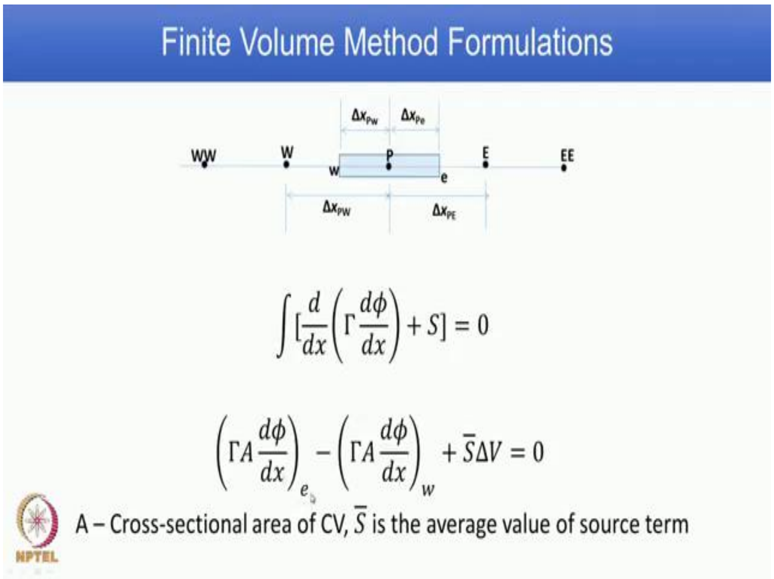
Now, we have to know distances between node and face, and node and surrounding nodes. So, because it is one-dimensional, we have delta X, and delta X this is between node P and west face W. So, we use delta X and subscript P capital and small case w. Similarly the distance between node P and the east face E is given here as delta X subscript capital letters P and small letter e. The same way, distance between node to node, so for example, in this case between node P and E is given as delta X subscript capital letter P and capital letter E; similarly for other side. Now this west of west is referred as W W; similarly, east of east is referred as E E. These are required in case you are deciding to have higher order accuracy scheme.

(Refer Slide time: 06:50)



In general, Navier-Stoke equation if you neglect unsteady term and convection term then we obtain diffusion equation and it is given here are diversion of gamma gradient of phi source term equal to zero. So, this gamma is a diffusion coefficient depending on equation. Suppose if it is the temperature equation and then this corresponds to diffusion coefficient correspond to temperature equation and phi is any property. The control volume integration of this equation results as integration over the control volume divergence of gamma gradient phi dV plus integration over control volume source term dV equal to and this we are converting volume integral surface integral and that is what you have seen on the right hand side. The source term is left as if it is because if it is integrated over the volume, and we take average. So area integral gives the flex entering or leaving the control volume. So, in this case, this n dot dA, dA is the area vector, n is vector normal to that area. So, this give actual flux either entering or leaving. And the volume integral on the right hand side gives the source term over the control volume, so that was shown here. Now let us see how to actually apply this equation for the mesh details that we have shown.

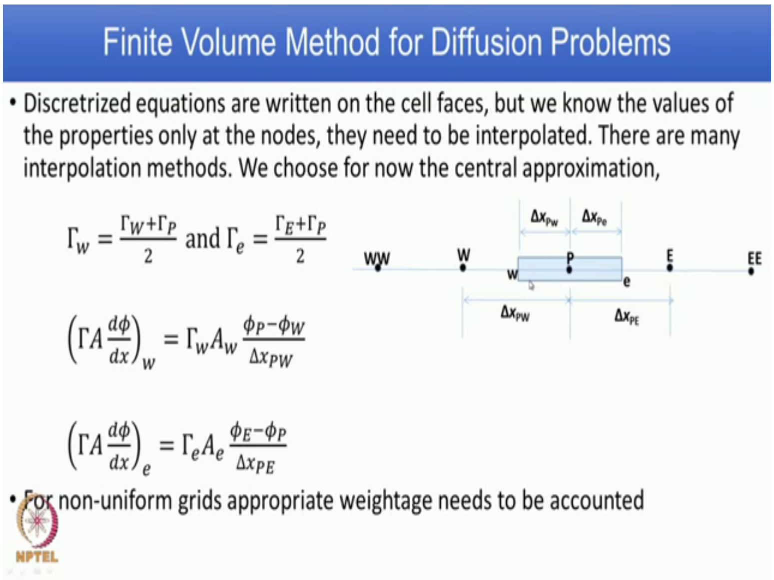
(Refer Slide time: 08:38)



So, this is the equation integrated the mesh and corresponding details are reproduced here, P is the node of interest and we go from P to the left on the west node; on the right on to the east node. We apply this integration for this mesh that is shown and we get here gamma d phi by d x evaluated at east face that is here minus gamma a d phi by d x evaluated at west face that is here on the left side because it is one dimension. We are using d instead of the doe and source term is evaluated as average over the surface over the volume, so it is s bar delta v. Now this is generalized form. So, in one dimension, you actually do not have V, but this is generalized form. A cross sectional area of the control volume and S bar is the average value of the source term.

Now let us see how to evaluate each of these terms for the mesh that is shown here. For example, gamma d phi by d x at east face that is on this side similarly gamma d phi by d x at the west face that is on this side; suppose it happen to solve let this then you get phi evaluated at node P similarly at node at E, EE and so on, similarly on the other side w and WW and so on. So, phi is variable that is evaluated at the node location, whereas d phi by d x is a gradient of phi which is required at the spaces, so in this case required at the east face or at the west face.

(Refer Slide time: 10:50)

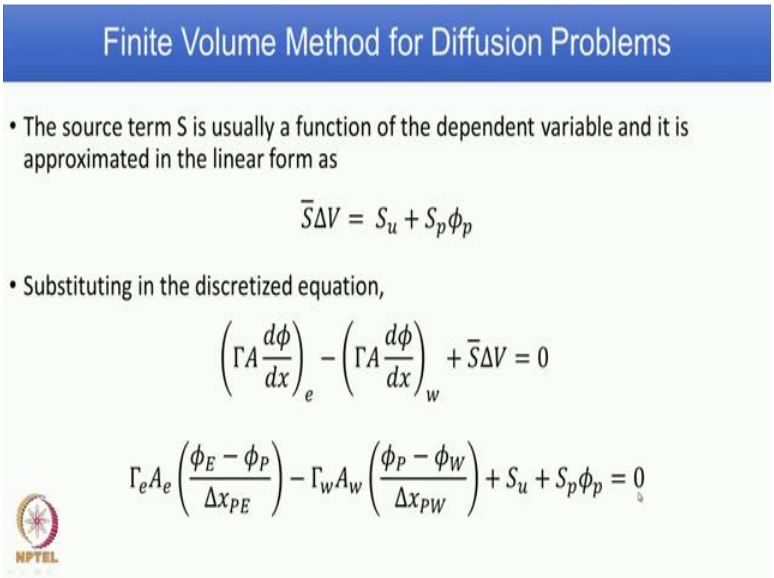


So, discretized equations are written on the cell faces that is what we have just seen, but we know the values of the property only at the nodes, so P, e, w, EE and WW and so on. However they need to be interpolated to get values on the faces. There are many interpolation methods available. We will choose initially central an approximation that is gamma at w face here is evaluated from, gamma available at P node and gamma available at W node as a average, so that is why it is gamma w plus gamma P by 2. Similarly gamma at the east face is evaluated from gamma, available at east as well as at P node as the central approximation gamma E plus gamma P by 2.

Now this gamma is diffusion coefficient I mentioned before. In general, it may be a fixed, other word it may be a constant, it may not vary for some cases. Later you will learn gamma also as a function of the temperature or as the function of the flow. In the case of two phase flows it may vary from face to face. So, here we are actually talking about the general procedure. The next term that is discretization equation is gamma A d phi by d x at w. So, we write this as gamma w A w and this d phi by d x at west face. So, we know the function value of P node, we know the function value at w node then use approximation between these two nodes and we get phi P minus phi w delta x P w shown here that is the distance between node P and node w that is what shown here. We write similarly for other face gamma A d phi by d x at east face as gamma EE again phi e minus phi P by delta x evaluated between the nodes P and e that is what is shown here as delta x P e. Again the question is about A, if it is the uniform mesh then A will be the same that is whether it is east face or west face. We are writing here generalized procedure that is why we still use the subscript e and w.

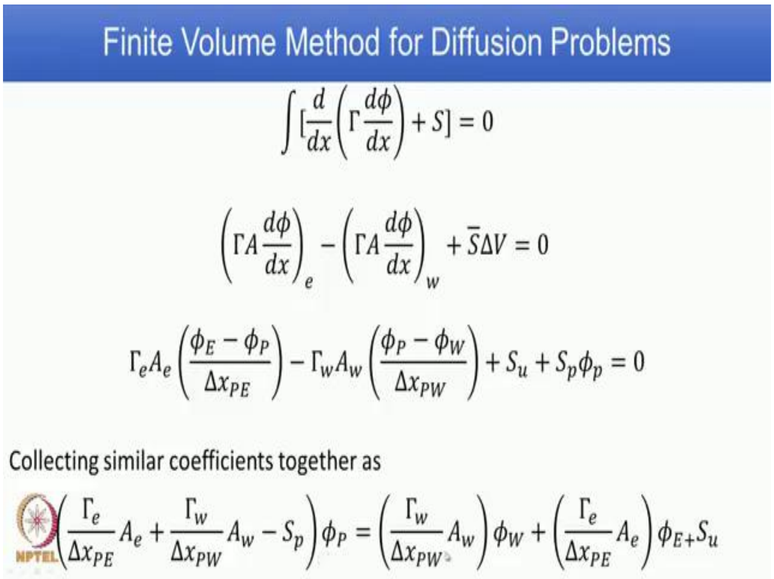
So, for a situation when the control volume shape is changing from one face to another face between node to node then this A can be used accordingly. This is again we are assuming a uniforms mesh arrangement that means, distance between P and w, distance between w and WW. Similarly on the other side, distance between P and e or distance between e and EE are same, in such case the weightage are equal from both the side for contribution at particular face. For example, for phi at P and phi at W will contribute same for evaluating phi at control surface W, similarly then other side. If you are having non-uniform mesh arrangement and then you have to take weightage accordingly and we will see that a little later.

(Refer Slide time: 14:42)



Now source term is usually a function of dependent variable; in this case it is phi itself. And it is approximated in the linear form as source term average multiply by delta V equal to S u plus S p plus phi p; and s u is the is a constant value and this is a linear variation. We have seen individually details, now we have to substitute in the discretized equation; this was the discretized equation we have seen how to evaluate each of this for each face. So, we will substitute, we get gamma e A e phi E minus phi P delta x P E; similarly, other term then for source term. So the equation now is the discretized equation of the diffusion equation by following finite volume procedure.

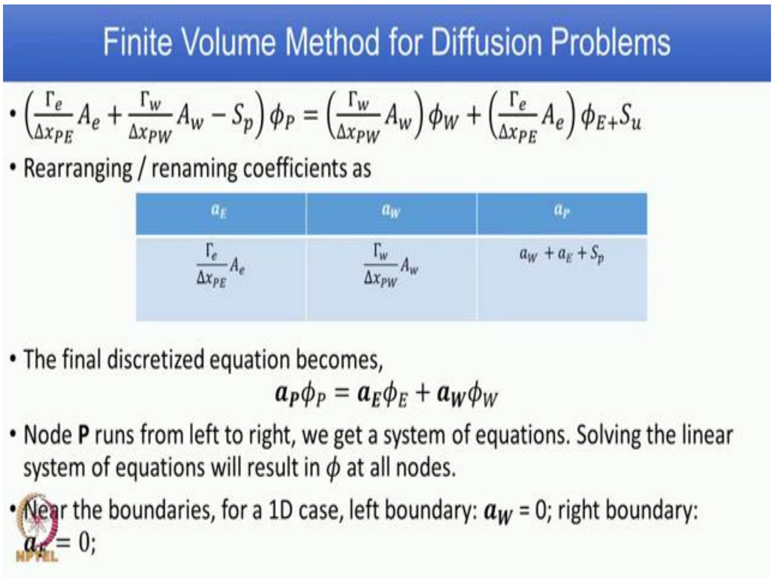
(Refer Slide time: 15:47)



We put all the things together; this was the original diffusion equation. And we get discretized equation, these are evaluated for the node that we have defined and we get final discretized equation in this form. We can observe in this equation phi P is a variable interest, P is the node of interest and you want to evaluate phi at the node of interest P, and you find the term appearing first term appearing in the second term also appear in the last term that is for the source term. This is a node this is a variable at the node of interest. So, we need to collect that coefficients, so collecting similar coefficient together as, so in this case phi P then you write down all the terms corresponding to phi p together. So, you get gamma e A e by delta x P E that is what is coming here as the first term. Similarly this term is here and then phi is also there and this is the quantity of interest so you are retain that on the left hand side, and that is why some minus sign and plus sign gets changed when you move from left to right then remaining terms are in this equation phi E and phi W again a group those coefficient properly, so phi W is written here and phi E is written here. Constant for source term the S U is written separately.

Now, if you look at the equation for the moment you imagine terms in the bracket as one coefficient. So, some coefficient into phi p equal to some coefficient phi w plus some coefficient phi E plus S u. Now the mesh that I have shown p is the node of the interest it is at particular instance or one particular location in the face you can imagine phi is running from left to right. In the case of 1D that is all the notes of interest or going from left boundary to right boundary then this equation is repeated for all the nodes from left boundary to right boundary know if you can understand this is the point of interest and phi w and phi e or known either from previous situation or the current situation that is why left hand side is unknown and right hand side is known quantity.

(Refer Slide time: 18:34)

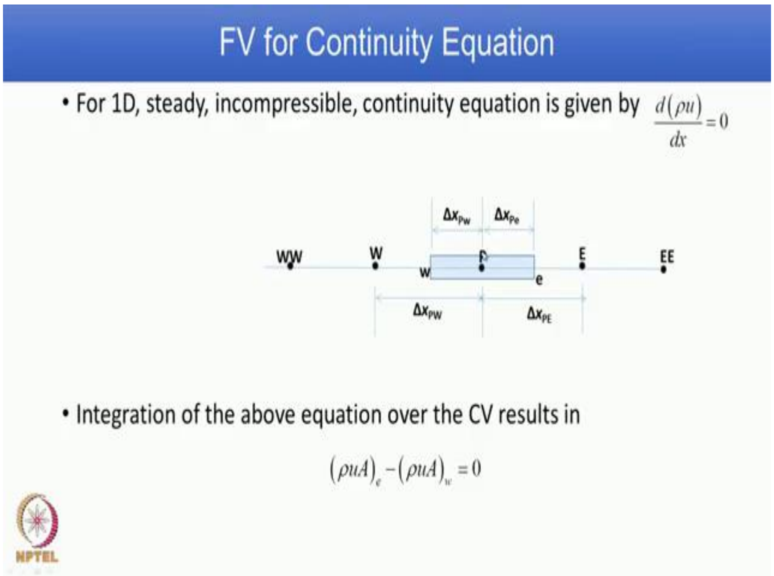


Rewrite that equation again as I mention all the bracket terms can be written as one value and that is shown here as A e A w A p and A e is actually the term corresponding to east Node gamma E by del x p e EE. Similarly a w is the coefficient corresponding to the west node that is shown here. Now you can observe that this term the first term for phi p is same as term for east node. If you look at this gamma e by d x p e e e is the first term for phi p that is same as gamma e d x p e a e coefficient term for east node. So, that we already define as A e, similarly the second term in the phi p is corresponds to Coefficient for phi w and that is what shown here. So, coefficient for phi p can be return in terms of this coefficient and you see here a w plus a e the only term is not represented the other term is s p and that you can add separately here. So, this is actually advantage because you evaluate a e and a w and that is good and that can be used even in a P also.

So, in terms of these define newly define coefficient the discretization equation is written as a P phi p equal to a e phi e KW and phi w that is. So, simple and as I mentioned before node p with the point of interest it is just a representation. So, it for a one d it will run from left boundary which is already as the boundary condition node immediate to the left boundary all the way up to node just before the right boundary. So, p will run from the left node left most node to the rightmost node then this discretized equation will become system of equations.

And you can represent in the form of a matrix, we have to have some procedure for inverting that matrices once inviting the matrices then all phi p are coming to be known such a procedure is solving the linear equation system and will have a separate class on matrices inversion procedure. So, near boundaries, in case of 1D case, for example the coefficient a w is actually west to the node of interest p. So, near boundary p will be one node very next to the left boundary; and for that node, there is no west node, because the west surface is the left boundary itself, in such case the coefficient a w is made to 0. Similarly on the right side, node very next to the boundary, there is no east node for that boundary, because east surface is the boundary condition itself, the location where the boundary condition is enforced. So, there is no east node, hence for the right boundary node near the right boundary A e is made to 0. You can imagine how it will be for the case of 2D and 3D, we will explain that in the subsequent slide.

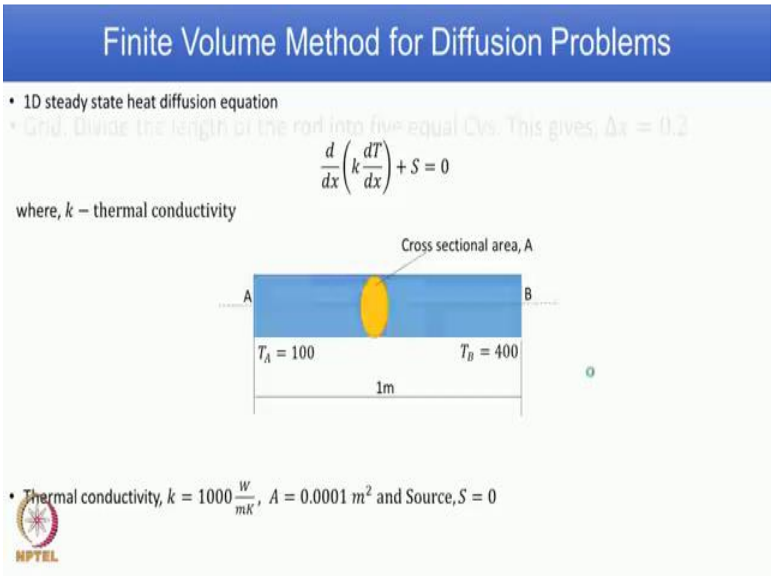
(Refer Slide time: 22:49)



We have Navier stock equation and definitely continuity equation question what we are saying is just one term in the Neumann shock equation that is diffusion term will see how to do for continuity equation finite volume formulation for continuity equation let us take one d study incompressible situation and continuity equation, please return because it one d we can write as d by d d by d x of doe u equal to zero quotes it incompressible. So, the row will not be there becomes only D U by d x equal to zero.

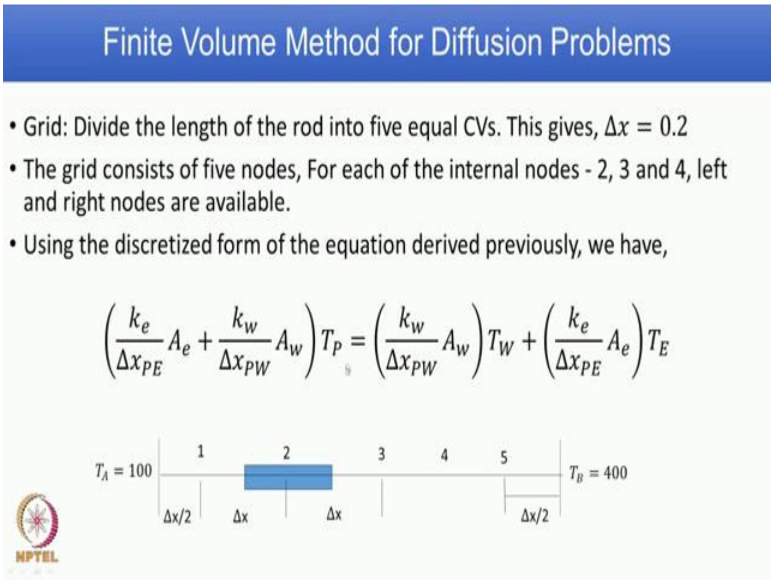
Again we are the same definition of node control volume distances define for 1D case. Now if you write integration of this equation for the control value formulation when you get row u a evaluated at east face minus row u a evaluated the west face, because it is continuity equal to zero, it is very simple. Now this row is again for in compressive flow not be there to come out and it the term will not be there. And if it is constant mesh then they a will not be there it just only u at east face minus u at the west face equal to zero of course, in 2D you have velocity in vertical direction that become v corresponding to top as soon as bottom.

(Refer Slide time: 24:22)



I get a feel of how this is actually implemented for that you take one on heat diffusion equation one dimensional steady state heat diffusion equation has shown here d by d x of k d t by dx plus x equal to zero where k is the thermal conductivity and for the problem that you are going to explain k is the 1000 watt meter pre Kelvin and area is 0.001 and 0.0001 metre square and their source term that is the very simple situation without anything it just heat diffusion problem figures shown here the cross section area that was shown here and boundary left boundary value given also temperature the left boundary as hundred degree Kelvin similarly boundary on the right side that is v t b given as a 400 degree Kelvin and the distance between left boundary to right boundary is 1 metre.

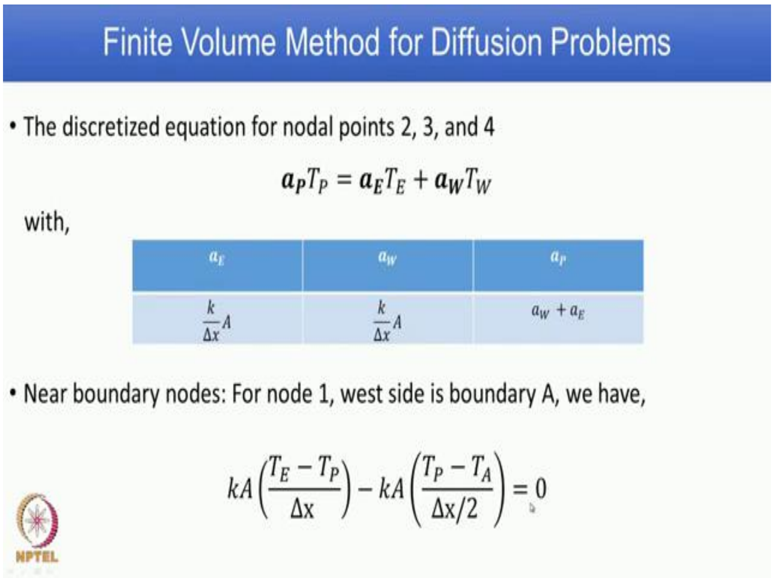
(Refer Slide time: 25:39)



Now, we divide the length of the rod into five equal control volume. So, one metre divided the given delta x point two and that is shown here. So, this is the left boundary t a 100 and this is right boundary t b 400 and define five equal control value five nodes 1,2,3,4,5 for representation we are showing one control volume which is in blue colour and distance of uniform between any two notes for delta x is same between node one and two, and it is same between node 3, node 2 and three similarly for other nodes near boundary. For example, left boundary distance between node one and the left boundary half because you are having uniform mesh it becomes delta x by 2.

Similarly, distance between node five in the right boundary is again because you are following uniform mesh it is become delta x by 2. So, grids consists of five nodes and you are able to distinguish node 2,3,4 are internal nodes; nodes are very near to the left boundary is node number one node very near to right boundary is node number 5 we have the discretized form of the equation only a diffusion equation and for the variable temperature, it is written here k e by del x p e a e plus k w by del x p w a w and we mention, there is no source term, so that term not appearing for the coefficient temperature p. Similarly term coefficient term for temperature evaluated to west node and Coefficient term for temperature variable evaluated at east node as I mentioned before this node p will run from left to right in this case it will run from 1 to 5, so but node one and node two are near the boundary; for the moment, I will consider only two three four internal node. So, this equation is written for each internal node, for example, if you write the equation for two then t w actually become t 1 and t become t 3, now repeat the procedure for 3 and 4.

(Refer Slide time: 28:27)



The discretized equation for nodal points 2, 3, 4 and you get the generalized form a E a w a p as mention before this k by delta x a k delta x a and this the summation of other two coefficient. In this case, diffusion coefficient is constant A is the 1D is also constant and delta x is also same hence it is happen very, very simple case near boundary loops that is for node one the west side boundary a and we have this equation modify has k A T e minus T p by delta x because for the node one each side is available. Hence, we write full term where as on the west side only half is available that is delta x by 2 on west side you have a boundary condition enforce that is effect here as temperature a so that is also used t p minus t A by delta x by 2 and k A is the diffusion coefficient set to 0.

In today's class, I will explain how to do for a diffusion equation finite volume producer for diffusion equation we understand terminologies used in finite volume mesh and we able to get approximation on east face and west face. We are able to understand how to deal for a boundary nodes for term goes to zero for the left side and the right side, and get some more understanding of formulation we started with one example problem discretized geometry five nodal points equal distance. We could do internal nodes without any difficulties a special treatment for boundary condition location node. And we explain for the left boundary node that is node one there, the control surface to the left of the node one is a temperature condition at given and we see how was the finite volume formulation gets reduced for the left boundary.

Next class we will continue this problem, and get idea of how to do for the right boundary set up the complete equation including the boundary nodes, and how to solve the final one full matrices, and we compare with the exact solution will also see how to extend this procedure for 2D as soon as 3 D, but this I come today's class.

Thank you.