Statics and Dynamics
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Lecture - 06

So, we will today start disusing have first example problem, and we will learn the use of two things. We will learn the use of; what is called a free body diagram, and second we will understand what a rigid body is. So, let us look at a very simple example.

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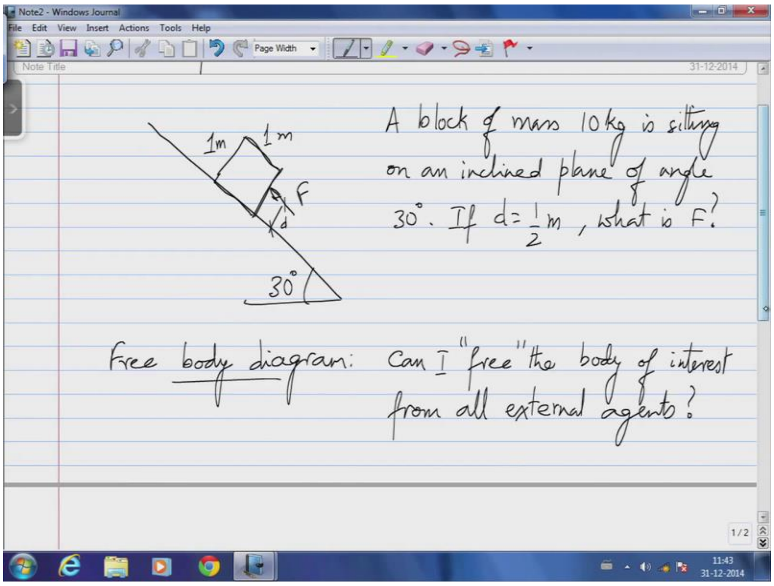

I have a block sitting on an inclined plane; a square block of side one meter. A force is excited on this block, at a distance d from the base of the block. We want to find the force required to whole this block in equilibrium. So, a block of mass ten kg is sitting on an inclined plane of angle 30 degrees. So, this angle here is 30 degrees. If I have a force F being excited at a distance, if d equal to half a meter what is F for equilibrium. So, we are going to, try to go through and understand how to solve this problem. The first concept that we have to understand is, what is called a free body diagram. The concept is quite simple. Can I free the body of interest? I put free in quotes, from all external agents. If I am able to do this, then I can only look at the body of interest.

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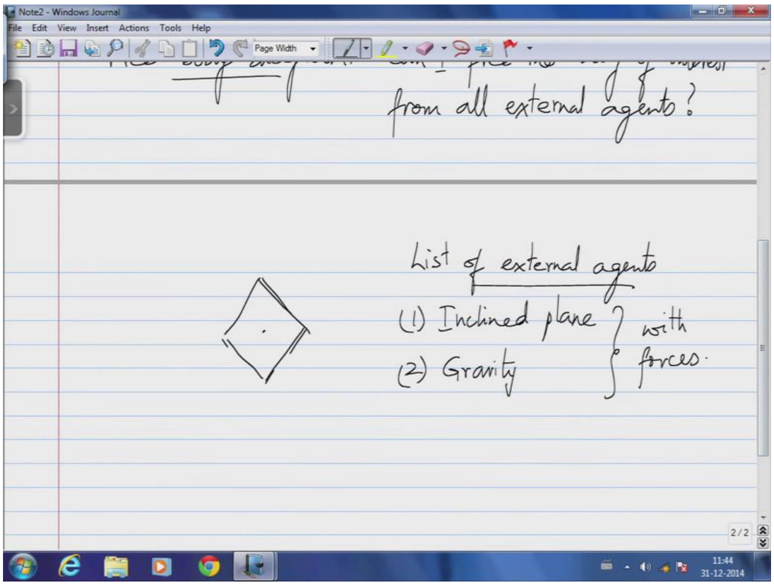

So, in this case the body of interest is this square block. Let us make a list of external agents that I have to free this body of. The first external agent is the inclined place itself. The second external agent that is going to influence this body, is gravity, or acceleration that is caused by earth iteration of this body. So, if I now take, I want to replace the act of these two external agents with forces. So, the concept of a free body diagram, essentially says I can free the body of all external agents, and replace the action of those external agents on this body as forces. And the moment I do that, I can apply Newton Law's of motion to this body alone, without leading to consider other external agent, such as the inclined plane itself. So, let us go through and draw the free body diagram. I am going to mark the center of mass.

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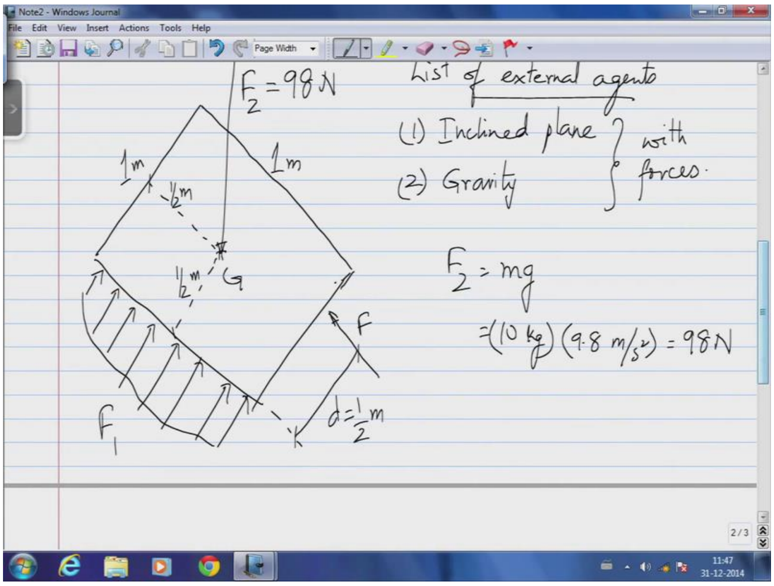

Let me magnify this slightly, so I can indicate what I need to indicate. The first action is that of the inclined plane, and what does the inclined plane do, if I take the body out, what the inclined plane was doing to this body, is essentially exerting a force underneath here. I do not know how it was exerting a force and there was kind of a force that, was being exerted by the inclined plane on this body. So, this is what I call F 1, F 1 as I have drawn it is a distribution of forces. There is a force that I need to compute f, and that is acting a distance d equal to half a meter. And the other force I will indicate as a F 2 which is due to gravity, is going to act vertically, and that is ten times g newton's; 10 is the mass.

So, the force F 2 is m g, which is ten kilograms multiplied with g which is 9.8 meter per second squared. So, this comes out to be 98 Newton's. So, I will replace this with a number that have I just computed 98 Newton's. The block is the one meter on the side, likewise said this is square block, which means the position of the center of mass above, is half a meter, and it is also half a meter from the left edge. So, for a uniform block, the forces are, the center of mass is located at the geometric centroids of this square which is half a meter from each of this sides, if I now take the sum of forces acting on this to be 0.

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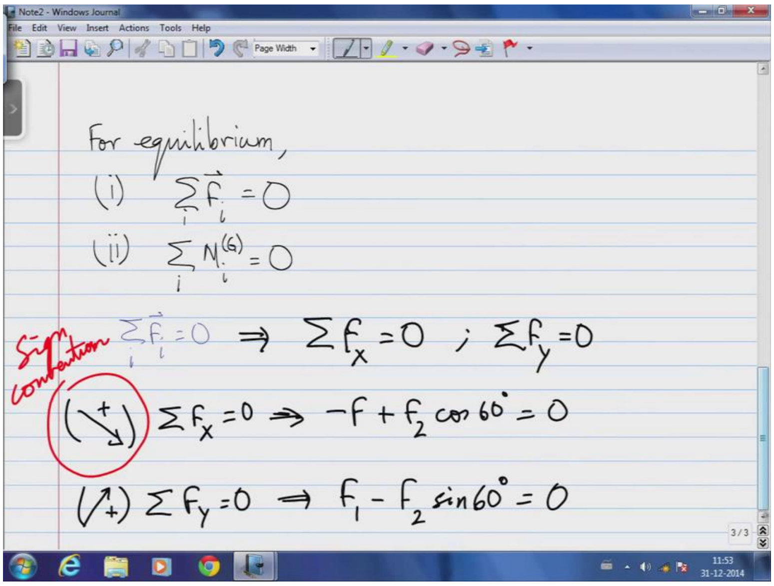

So, what I require for equilibrium, sum of all forces vector sum of all forces is equal to 0 at least that is the first condition. And the second condition, is that the summation of all the moments above g is equal to 0o. So, let us compute the summation of all the forces, but before I can compute the summation of all the forces.

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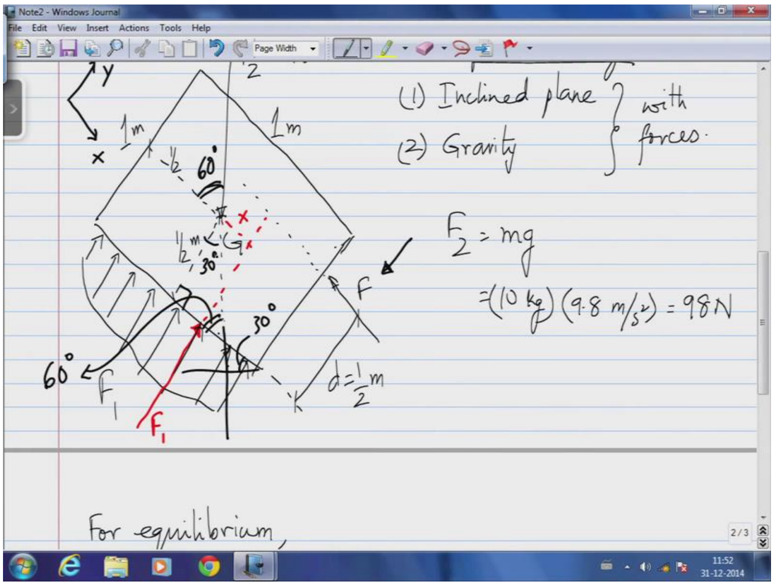

F has a unique line of action, F 2 has a unique line of action, but F 1 is a bit tricky. F 1 does not have a unique line of action which also means that, I need to compute the moment of a distributed force. So, this is kind of distributed force, but what I can do, is I can replace this with a force that acts at a single point on the bottom of this block, as a effective force, and let say that acts at distance x, to the right of the center of mass g. So, the first model that I have introduced is that I have taken what is actually a distributed force underneath the block.

The true F 1 and replaced it with the point force, with a single force that acts through a point on the bottom of the block, as the effective are the resultant force of the set of forces acting on the bottom side. So, let us see we are now ready to apply our loss of equilibrium, what we find is. Now let us take the summation over all the forces F I equal to 0. I can split this in to two components, that summation vectors sum of all the x components of the forces is 0, and the summation of all the y components of the forces is 0. So, for the sake of convenience I am going to define x has been along the inclined plane, and y has been perpendicular to the inclined plane. So, if you notice I did not define the coordinate system first.

I am defining yet at a point where I have to do the force computation, because I can look at the forces, after I have drawn the free body diagram, and choose a coordinate system that would require me to resolve the least number of forces along as components. Say for example, when I choose this x and y as I have drawn here, the force F1 indicated in red here, a is along the y direction, positive y direction is a matter of fact. The force F is along the negative x direction. So, they each do not contribute either y or an x component respectively. It is only the weight F 2 that I have to now resolve into its components, and the x and y direction. And in order to do that, in this triangle, this angle here is 30 degrees which means this is 60, and this is also 60 degrees, which puts this at 30 degrees, puts this as 60 degrees.

So, if I now take the summation of all the forces acting on this body in x direction, I have, I am going to take downward positive. I am going to write this as my sign convention all the time; summation x equal to 0. Now I am adding only the scalar components, this implies minus F plus F 2 cosine 60 degrees equal to 0 and if I take the forces in the positive direction of y being positive. So, this symbol here indicates this convention here indicates my sign convention. This implies that F 1 is in the positive y direction minus F 2 sign 60 equal to 0. So, let us go through complete this.

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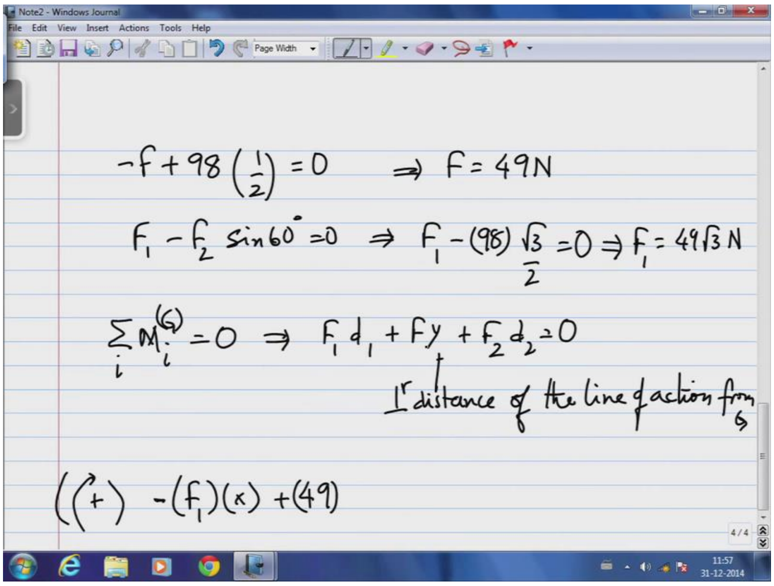

So, the first part says minus F plus F 2 has a magnitude of 98 eight Newton's. Cosine a 60 degrees is one half equal to 0, and F 1 minus F 2 sin 60 equal to 0. This implies F 1 is a force that I do not know the magnitude of minus 98 times sin 60 is root 3 over 2 equal to 0. So, from the first equation, I know that F equal to 49 Newton's, and F 1 equal to 49 times square root of 3 Newton's. Now I have not yet completed the problem, because for equilibrium I have the second condition that the moments have to all add up to 0.

Now how would I determine that If d is half a meter, I want to find this effective line of action x; shown here. The effective line of action is displaced from the center of mass, to accommodate the fact that the sums of the moments have to add up to 0. So, let us now apply that last condition, that summation due to all the forces, about g equal to 0, which implies F 1 d 1 plus F times d plus F 2 times d 2 equal to 0. This is in a slightly generalize way. I am going to use this specific notation. Now I am going to take clockwise positive this is my sign convention. If I choose angle, does not matter what sign convention is use, as long as you remain consistence.

So, right now I am going to assume that all clockwise moments are positive. Now the moment due to F 1 at least the way my I have drawn in this free body diagram F 1, is to the right of g, which means it is going to produce and counter clockwise moments. So, say F 1 times x within negative sign, because x positive means counter clockwise plus F has been determine to be 49 Newton's. Now this d is essentially the perpendicular distance of the line of action from g. It is not the d that we have shown in the figure. So, let us just to avoid confusion we will replace this with new symbol y.

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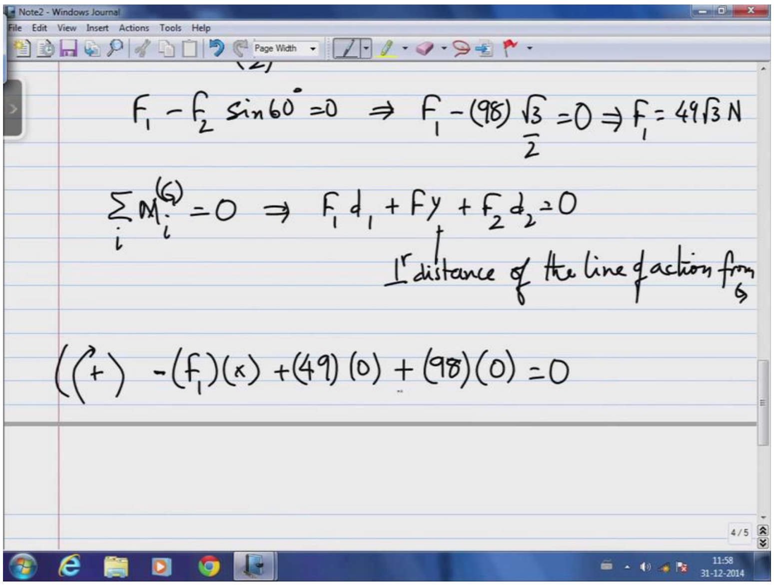

y for this particular instances 0, because the force F passes through the line of action, because the force is half a meter which implies y, which is the distance of the force to the line of action itself is 0, plus F 2 which is the weight of the body, which is 98 Newton's, also passes through the center of mass; therefore, the perpendicular distance to it is line of action is also 0. So, for equilibrium, we require that F 1 times x with the negative sign plus 49 time 0 plus 98 time 0 equal to 0. F 1 we just found is a non negative number, or a non-0 number which implies x equal to 0.

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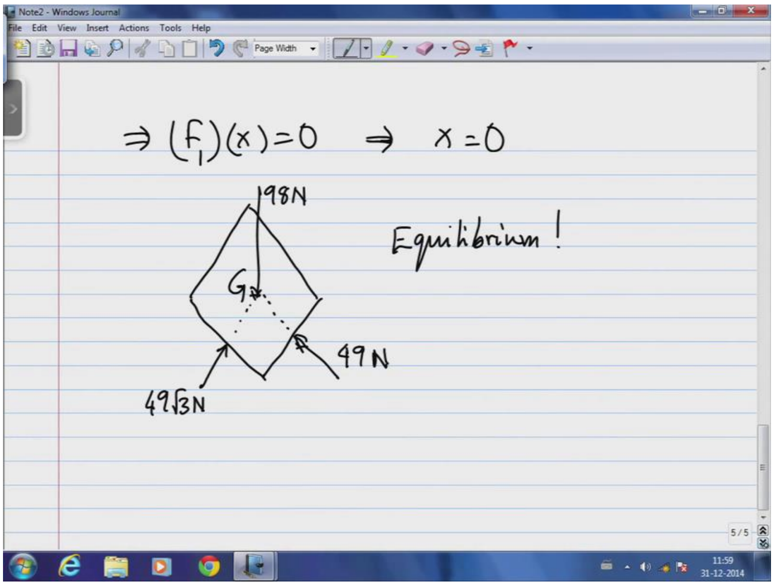

So, we are now determine this unknown quantity x. The unknown quantity F 1, and as well as, the unknown force F require to whole this body in equilibrium. So, let us draw if free body diagram of this square block once more, with the correct magnitude of the various forces. So, these are the three forces acting on this body. They are all passing the lines of actions of the three forces, or all passing through the center of mass g, which means, neither of the three forces is cosine the moment about g. So, essentially this system is equilibrium, if the forces are of these magnitudes.

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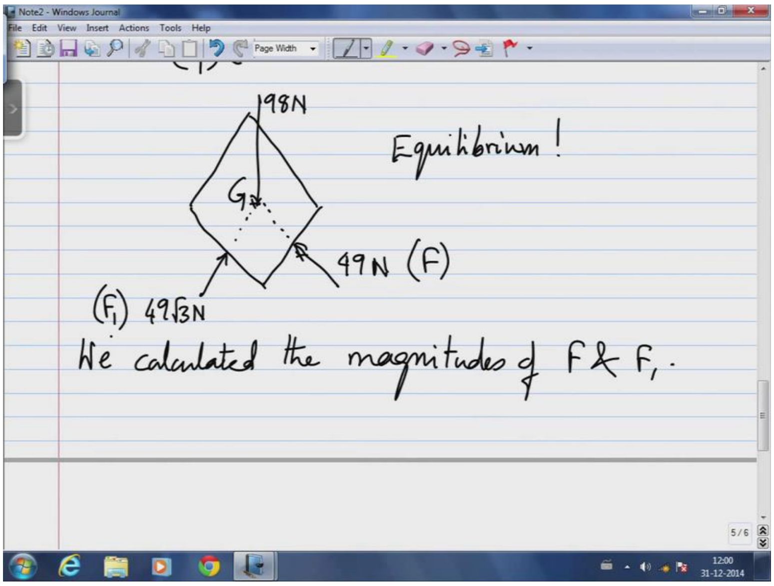

Now in this process we identified or calculated two forces. Let us differentiate between those two. We calculated the magnitude of both F and F 1. So, this is f, and this is F 1, the difference between. Let as understand the difference between F and F 1. What causes is F 1. F 1 is a force that is caused by the fact that, I have an inclined plane under the this block. Now why would the inclined plane push this block up with the force F 1, acting through the center of mass in this particular instance. It is simply comes from the fact that the inclined plane is rigid, the block is also rigid, and if I place this block on top of a inclined plane, the block does not penetrate into the inclined plane, neither does the inclined plane push the block by itself.

So, the force F 1 is actually being generated, is self generated from the constraint that the block cannot go into the inclined plane, and neither can inclined plane naturally push the block above it, and that they would two remain in context. So, it is essentially a force arising at the contact due to the constraint of no penetration. So, this constraint originated force F 1, masquerades as an unknown quantity in our free body diagram, because of the fact that the freed the body of the inclined plane as an agent. The moment you free the body of inclined plane, I have to account for the fact that this body was not allowed to, for into the inclined plane and that is replaced by a an effective force F 1 acting perpendicular to the point of contact, perpendicular at the point of contact.

So, F 1 is a force that originates out of this constraint. As oppose to that F is the force, is the unknown force we wish to calculate to keep this body in equilibrium. So, how much force do I have to exert to keep this body in equilibrium, at 10 kg mass sitting on a 30 degree inclined plane, has to have a force of 49 Newton’s pushing it through the center of mass, to keep the body in equilibrium, and that naturally generates an additional force of 49 times root 3 Newton’s, acting through the centre of mass, as a result of the inclined plane pushing the block. So, these two forces are fundamentally different; one is caused by the constraint itself, the F 1 force. F is the force that we control,

F is the force that we wish to control to keep the body in equilibrium. So, we learnt the idea of the free body diagram today. And on top of that we learn the application of, the two loss of equilibrium for finite sized bodies, the fact that the forces have to add up the 0, and moments about the center of mass also have to add up to 0. In addition we learnt the fact that forces can come from two different agents, two different kinds of physical processes; one, an external agent pushing it, and the second from a constraint. So, we will continue this discussion with another example problem in the next class.