Statics and Dynamics
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Lecture – 04

So, we will today start discussing our first example problem. And we will learn the use of two things. We will learn the use of what is called a free body diagram, and second we will understand what a rigid body is. So, let us look at a very simple example.

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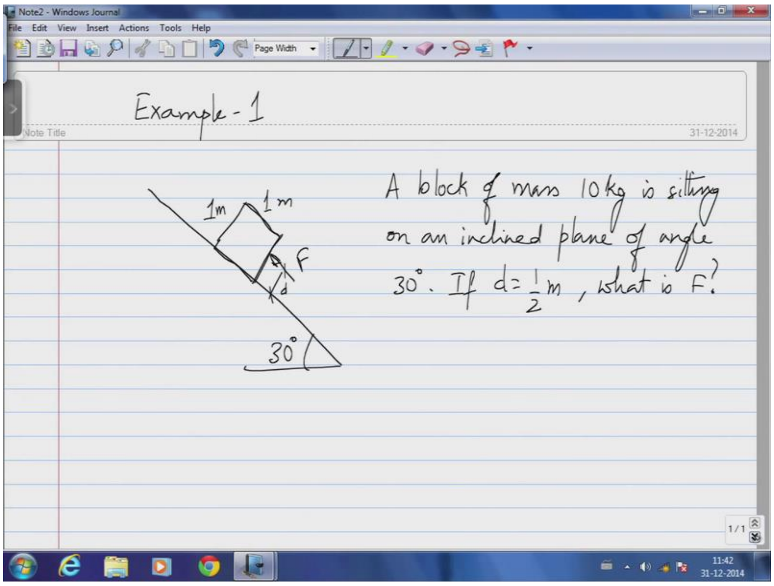

I have a block sitting on an inclined plane, a square block of side one meter. A force is exerted on this block at a distance d from the base of the block. We want to find the force required to hold this block in equilibrium. So, a block of mass ten kg is sitting on an inclined plane of angle 30 degrees. So, this angle here is 30 degrees. If I have a force F being exerted at a distant d, if d equal to half a meter, what is F for equilibrium? So, we are going to try to go through and understand how to solve this problem.

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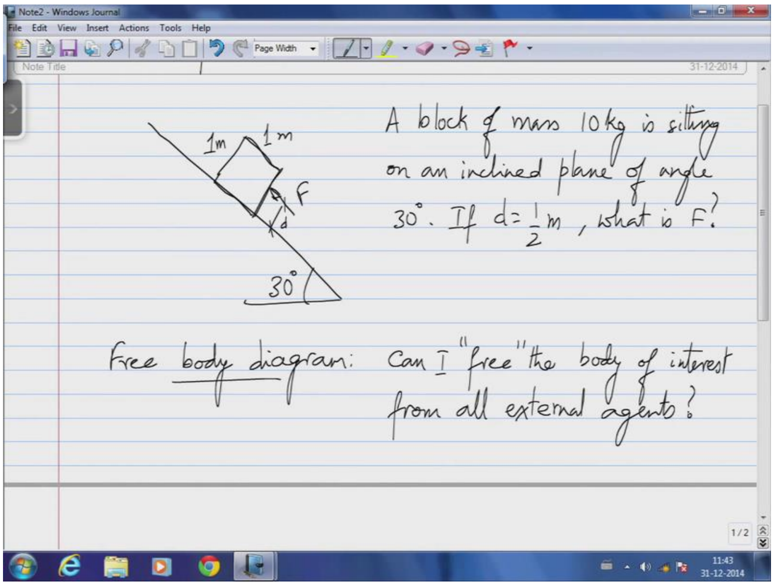

The first concept that we have to understand is what is called a free body diagram. The concept is quite simple. Can I "free" the body of interest, I put free in quotes, from all external agents? If I am able to do this, then I can only look at the body of interest.

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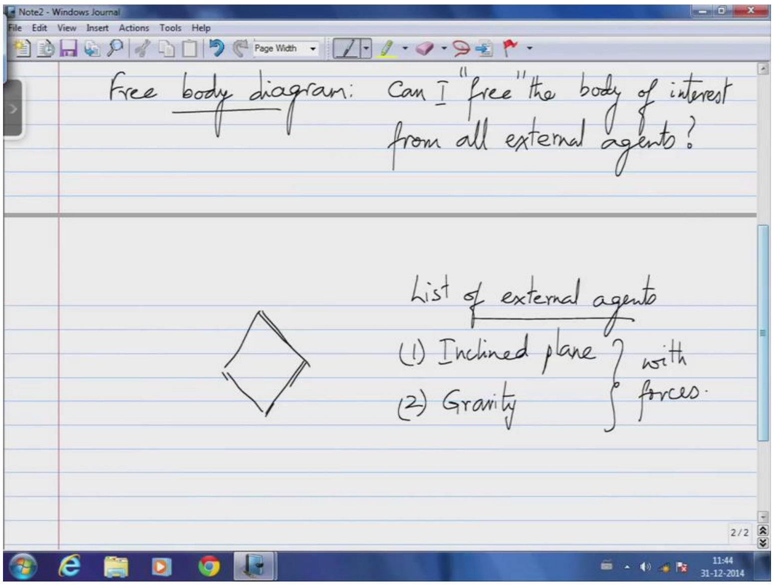

So, in this case the body of interest is this square block. Let us make a list of external agents that I have to free this body off. The first external agent is the inclined plane itself. The second external agent that is going to influence this body is gravity or acceleration that is caused by Earth's attraction of this body. So, if I now take; I want to replace the act of these two external agents with forces. So, the concept of a free body diagram essentially says I can free the body of all external agents and replace the action of those external agents on this body as forces. And the moment I do that I can apply Newton's laws of motion to this body alone without needing to consider other external agent such as the inclined plane itself. So, let us go through and draw the free body diagram. I am going to mark the center of mass.

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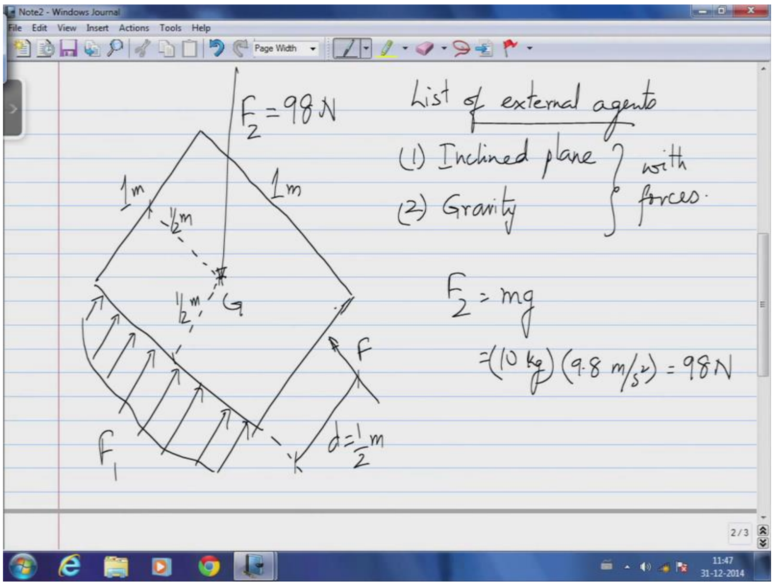

Let me magnify this slightly. So, I can indicate what I need to indicate. There is; the first action is that of the inclined plane. And what does the inclined plane do? If I take the body out, what the inclined plane was doing to this body is essentially exerting a force underneath here. I do not know how it was exerting a force. But, there is a kind of a force that was being exerted by the inclined plane on this body. So, this is what I will call F 1. F 1 as I have drawn it, it is the distribution of forces. There is the force that I need to compute F and that is acting a distance d equal to half a meter. And the other force I will indicate as F 2, which is due to gravity is going to act vertically and that is ten times g Newtons. Ten is the mass. So, the force F 2 is mg, which is ten kilograms, multiplied with g, which is nine point eight meters per seconds square. So, this comes out to be 98 Newtons. And replace this with a number that I have just computed; 98 Newtons. The block is one meter on the side. Like I said which is a square block; which means the position of this center of mass above is half a meter. And it is also half a meter from the left edge. So, for a uniform block the forces are the center of masses is located at the geometric centroid of this square, which is half a meter from each of the sides.

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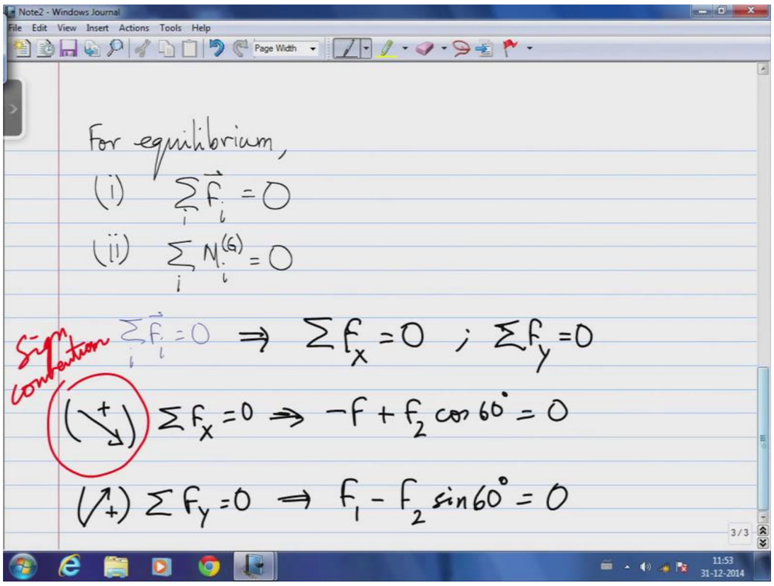

If I now take the sum of forces acting on this to be 0. So, what I require for equilibrium, sum of all forces, vector sum of all forces is equal to 0 at least. That is the first condition. And the second condition is that the summation of all the moments about G is equal to 0. So, let us compute the summation of all the forces.

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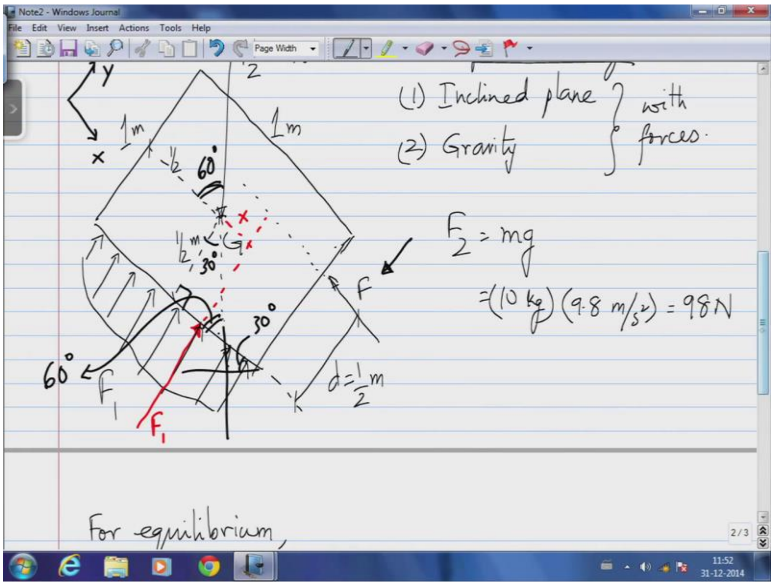

But you know before I can compute the summation of all the forces, F has a unique line of action; F 2 has a unique line of action. But F 1 is a vector key. F 1 does not have a unique line of action, which also means that I need to compute the moment of a distributed force. So, this is kind of a distributed force. But, what I can do is I can replace this with a force that acts at a single point on the bottom of this block as an effective force. And let us say that acts at a distance x to the right of the center of mass g.

So, the first model that I have introduced is that I have taken what is actually a distributed force underneath the block, the true F 1, and replaced it with a point force with a single force that acts through a point on the bottom of the block as they are effective or the resulting force of the set of forces acting on the bottom side.

So, let see if we are now ready to apply our laws of equilibrium. What we find is now let us take the summation over all the forces. F i equal to 0. I can split this into two components. That summation, a vector sum of all the x components of the forces is 0 and the summation of all the y components of the forces is 0.

So, for the sake of convenience I am going to define x has been along the inclined plane and y has been perpendicular to the inclined plane. So if you notice, I did not define my coordinate system first. I am defining it at a point where I have to do the force computation; because I can look at the forces, after I have drawn the free body diagram and choose a coordinate system that would require me to resolve the least number of forces along as components. Say for example, if I choose this x and y as I have drawn here. The force F 1 indicated in red here is along the y direction, positive y direction as a matter of fact. The force F is along the negative x direction. So, the each do not contribute either a y or a x component respectively. It is only the weight F 2 that I have to now resolve into its components and the x and y direction. And in order to do that if in this triangle, this angle here is 30 degrees; which means this is 60 and this is also 60 degrees which puts this at 30 degrees, which puts this at 60 degrees.

So, if I now take the summation of all the forces acting on this body in the x direction, I have, I am going to take downward positive. I am going to write this as my sign convention all the time. Summation x equal to 0. Now, I am adding only the scalar components. This implies minus F plus F 2 cosine 60 degrees equal to 0.

And if I take the forces in the positive direction of y, it been positive. So, this symbol here indicates; this convention here indicates this is my sign convention. This implies that F 1 is in the positive y direction minus F 2 sin 60 equal to 0. So, let us go through, complete this.

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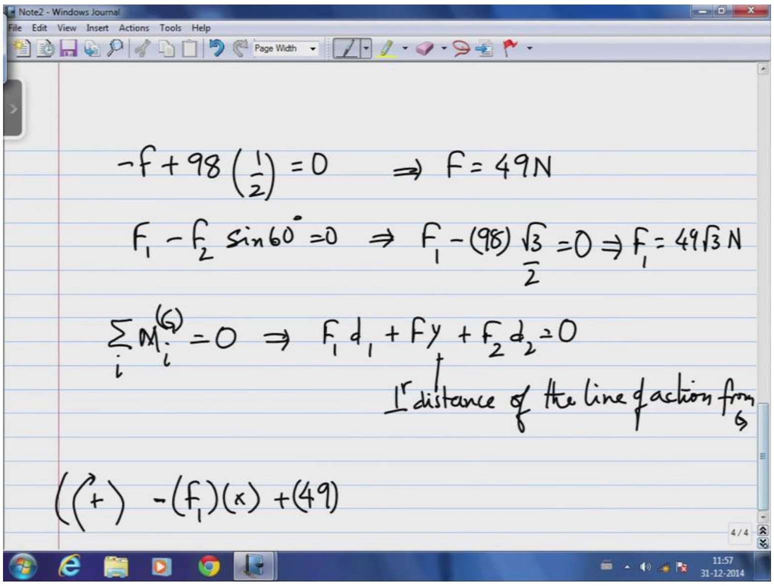

So, the first part says minus F plus F 2 has a magnitude of 98 Newtons. Cosine of 60 degrees is one half equal to 0. And F 1 minus F 2 sin 60 equal to 0. this implies F 1 is a force that I do not know the magnitude of; minus 98 times sin 60 is root three over two equal to 0. So, from the first equation I know that F equal to 49 Newtons and F 1 equal to 49 times square root of three Newtons.

Now, I have not yet completed the problem; because for equilibrium, I have the second condition that the moments have to all add up to 0. Now, how would I determine that? If d is half a meter, I want to find this effective line of action x shown here. The effective line of action is displaced from the center of mass to accommodate the fact that the sum of the moments have to add up to 0.

So, let us now apply the last condition that summation due to all the forces about G equal to 0; in which implies F 1 d one plus F times d plus F 2 times d two equal to 0. This is in a slightly generalized way. I am going to use the specific notation. Now, I am going to take clock wise positive. This is my sign convention. If I choose, and it does not matter what sign convention you use as long as you remain consistent.

So, right now I am going to assume that all the clock wise moments are positive. Now, the moment due to F 1; at least the way by I have drawn in this free body diagram, F 1 is to the right of G; which means it is going to produce a counter clock wise moments. So, let us say F 1 times x with a negative sign, because x positive means counter clock wise, plus F has been determined to be 49 Newtons. Now, this d is essentially the perpendicular distance of the line of action from G. It is not the d that we have shown in the figure. So, let us just to avoid confusion, we will avoid, we will replace this with a new symbol y.

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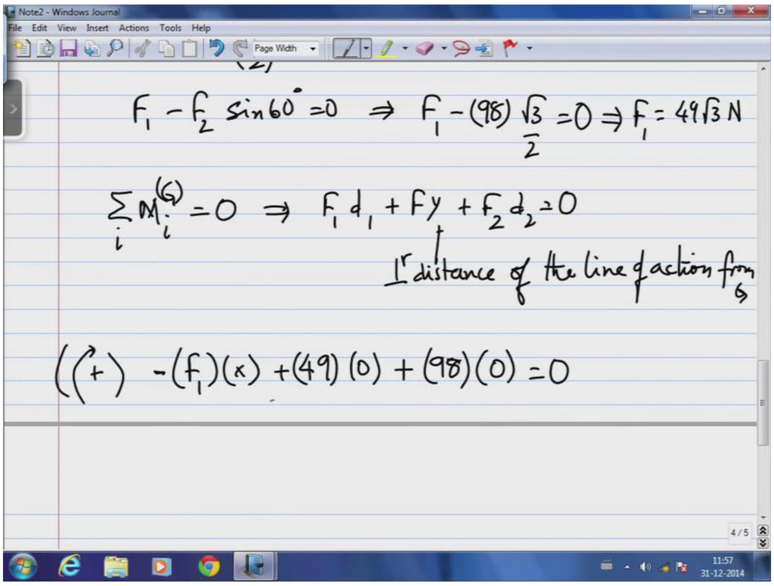

y for this particular instance is 0 because the force F passes through the line of action because the force is half a meter which implies y, which is the distance of the force to the line of actions itself is 0; plus F 2 which is the weight of the body which is 98 Newtons also passes through the center of mass. Therefore, the perpendicular distance to its line of action is also 0. So, for equilibrium we require that F 1 times x with the negative sign plus 49 times 0 plus 98 time 0 equal to 0.

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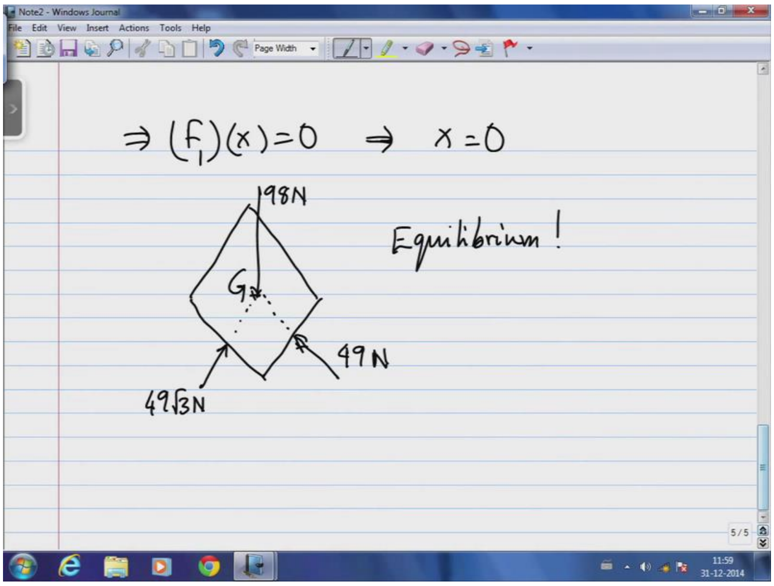

F one we just found is a non-negative number or a non-zero number, which implies x equal to 0. So, we now determined this unknown quantity x, the unknown quantity F 1 and as well as the unknown force F required to hold this body in equilibrium. So, let us draw a free body diagram of this square block once more with the correct magnitudes of the various forces. So, these are the three forces acting on this body. They are all passing; the lines of action of the three forces are all passing through the center of mass G; which means neither of the three forces is causing a moment about G. So, essentially this system is in equilibrium, if the forces are of these magnitudes.

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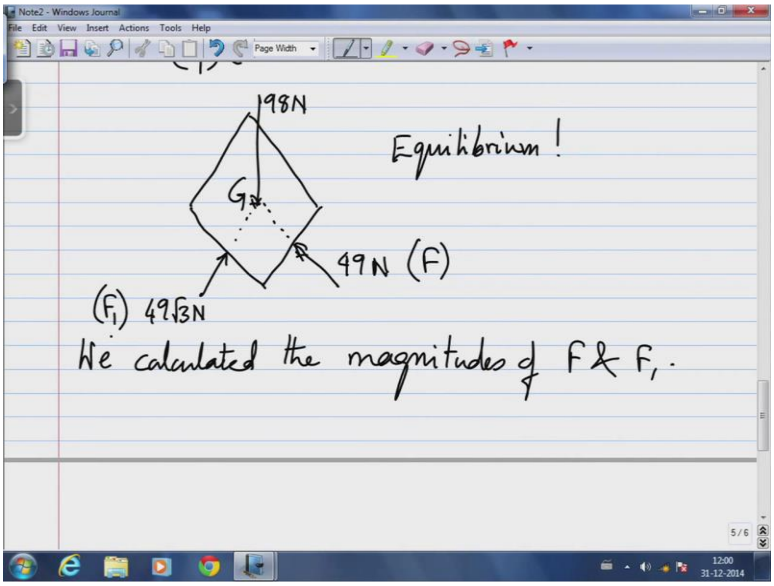

Now, in this process we identified or calculated two forces. Let us differentiate between those two. We identified, we calculated the magnitudes of both F and F 1. So, this is F and this is F 1. Let us understand the difference between F and F 1. What causes F 1? F 1 is a force that is caused by the fact that I have an inclined plane underneath this block.

Now, why would the inclined plane push this block up with the force of one acting through the center of mass in this particular instance? It simply comes from the fact that the inclined plane is rigid, the block is also rigid. And that if I place this block on top of an inclined plane, the block does not penetrate into the inclined plane neither does the inclined plane push the block by itself. So, the force F 1 is actually being generated. It is self-generated from the constraint that the block cannot go into the inclined plane and neither can inclined plane naturally push the block above it and that they would too remain in contact. So, it is essentially a force arising at the contact due to the constraint of no penetration.

So, this constraint originated force F 1 masquerades as an unknown quantity in our free body diagram; because of the fact that free the body of the inclined plane as an agent. the moment you free the body of the inclined plane, I have to account for the fact that this body was not allowed to fall into the inclined plane, and that is replaced by an effective force F 1 acting perpendicular to the point of contact; perpendicular at the point of contact.

So, F 1 is a force that originates out of this constraint; as opposed to that F is the unknown force we wish to calculate to keep this body in equilibrium. So, how much force do I have to exert to keep this body in equilibrium at ten kg mass sitting on a 30 degree inclined plane has to have a force of 49 Newtons pushing it through the center of mass to keep the body in equilibrium. And that naturally generates an additional force of 49 times square root three Newtons acting through the center of mass, as a result of the inclined plane pushing the block. So, these two forces are fundamentally different. One is caused by the constraints itself, the F 1 force; F is the force that we wish to control to keep the body in equilibrium.

So, we learned the idea of a free body diagram today. And on top of that we learned the application of the two laws of equilibrium for finite sized bodies. The fact that the forces have to add up to 0 and the moments about the center of mass also have to add up the 0. In addition, we learnt the fact that forces can come from two different agents. Two different kinds of physical process is: one, an external agent pushing it and the second from a constraint. So, we will continue this discussion with an other example problem in the next class.