Statics and Dynamics
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Lecture - 34

Welcome back. We are going to discuss another problem today, related to the use of both work energy formulation as well as our laws of dynamics, rigid body dynamics to understand the idea of normal reaction in the context of free body motion.

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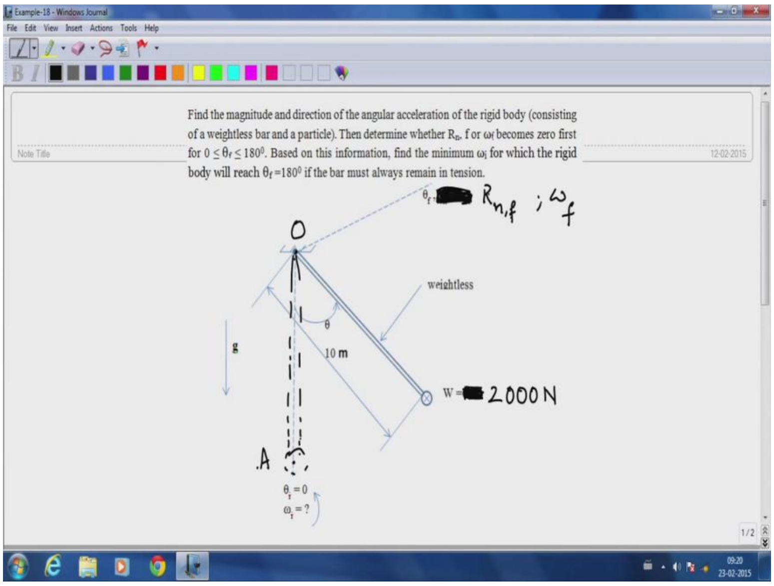

So, let us look at the example here, I have a bar here, it is a weightless bar; that is pivoted at some point O and at the end of the 10 meter along weightless bar, this is a little weight. We will consider this as a point mass for now and this mass is about 2000 Newton’s in weight. Initially, it is in a gravitational fields, so if nothing was done to it, it would be at rest kind of in a vertical position like this.

So, this is my initial condition at A, if I give it an initial angular velocity omega i and do nothing else, we are asked to find the point at which it would come to rest. So, let say this bar would move up and come to rest at some theta f. Now, if I give it as a sufficiently high angular velocity, this is a common observation that this bar would actually do a full circle and so it would go all the way up to the top and come back.

So, we ask ourselves two questions, one, this bar is a weightless, but rigid bar. So, it is able to sustain both a tensile force in it as well as a composing force in it. So, we ask ourselves the question, if this was a rigid bar, what angular velocity omega I do I need to make it do a full circle? The second question we can ask ourselves is, if this was not a rigid bar, but a weightless steel, so the difference between a rigid bar and a string is that, the string cannot sustain compression, it can only sustain tension.

So, if this was not a rigid bar, but a string, what angular velocity would I need to give to this string plus mass mechanism in order for it to do a full circle. These are the two questions we ask ourselves, I want you to pass the video and think about this, read the problem statement. So, you understand it try to rationalize in the own minds, which one of these two angular velocity should be greater, the one with the string or with the one with the rigid bar. That is the question to think about, pass the video, think about it.

We have done, start the video back again and you will see the mathematics behind these two calculations and the basic differences between these two calculations. Now, I do not want to take up a problem like this, without a physical situation that would describe this. So, if you think of this for a moment, let say you all anybody that is been to a circus, you see this, you know the circus act, where is a doom.

And this a motor cycle rider that starts initially at the bottom, kind of doing rounds at the bottom of the little steel doom kind of a structure and the pinnacle of the act is where the person does a full circle all the way around. So, in other words, he is able to do a full vertical circle, where at the top of the circle, he is practically inverted that on the motor cycle.

So, we ask ourselves the question, what initial velocity or initial angular velocity with respect to the center of the globe, should this person in motor cycle have, in order for that person to all ways remain in contact with the globe, even at the top. So, a model of this problem is where we imagine a little string tied between the center of the globe and the point, where the motor cycle is touching the globe and so that string if you will can never come under compression. That means, if that string is unable to sustain the weight of the motor cyclist; that motor cyclist is going to detach from the globe.

So, this is a nice model for us to understand, I use of this problem, so let us get back to the problem now. We have the bar initially in a vertical position, it is being given an initial angular velocity omega I.

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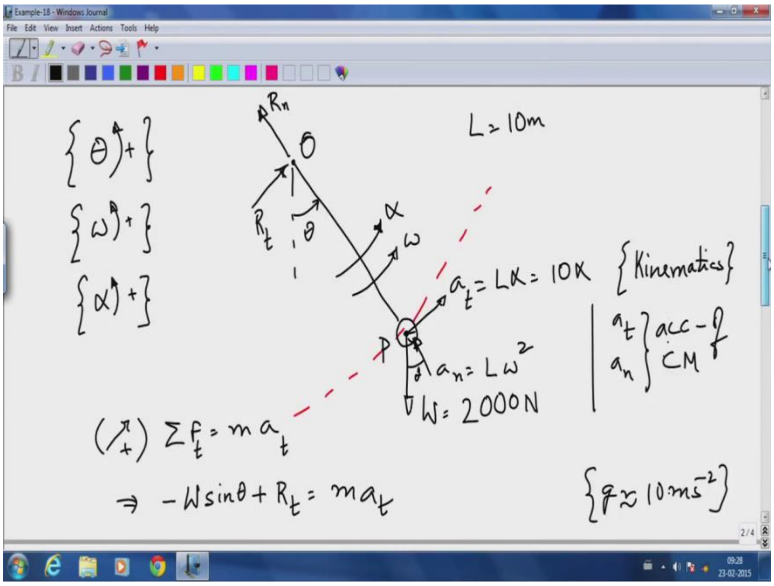

So, let us now draw a free body diagram of this at any other locations. So, this is my O, and this is the body I call this P, this is theta, theta positive, theta this way is positive, this is our sign convention, which also automatically means omega counter clockwise is positive and alpha, angular acceleration counter clockwise is positive. So, let say this is moving with an angular acceleration alpha, this rigid bar O P with the mass at the end.

The only forces acting on this bar or the weight which in this problem is 2000 Newton's, roughly modeled as the weight of the motor cyclist plus the motor cycle. Now, we also have other forces that occur at O and in a general way, I could have an R sub n, which is the tangential reaction plus n R sub t, R sub n, which is the normal reaction and R sub t, which is the tangential reaction.

So, I do not see any other forces acting on this bar O P with the mass at the end that should pretty much explain that should be all the forces. Only other thing we want to include is that, the linear acceleration of this at the center of mass of this bar O P, because the bar itself is weightless. The center of mass of this bar O P lies at the P, at the point P and that equals L times alpha, where L is the length of the bar, which is 10 meters, which in this cases 10 alpha.

So, the linear acceleration and the angular acceleration are related to the kinematic relationship. So, this is something we are used to seeing in the other problems as well, this is basically coming from kinematics. So, now, we are ready to apply the first law, first Euler's law to this free body diagram. So, let say our taking all forces up along a positive, sum of all forces is mass times the acceleration in that direction. Now, you have to be a little carful with this, the reason is, if this bar is rotating at some angular velocity omega, then the acceleration of P has two components to it.

If we go back to our kinematics, this L times alpha is simply only the tangential acceleration, this is also a normal acceleration, which has magnitude L omega squared. So, the normal acceleration of the point P, because it is tracing an arc, it is not moving on a straight line. So, we would not encounter this, if the body was moving on a straight inclined plane, you would only encounter the normal acceleration of the point, when the body is moving in a non straight path.

So, let us take some of all tangential forces is mass times tangential acceleration. So, what does that give us minus, this is theta, then this is also theta. So, minus W sin theta plus R sub t equals mass times tangential acceleration.

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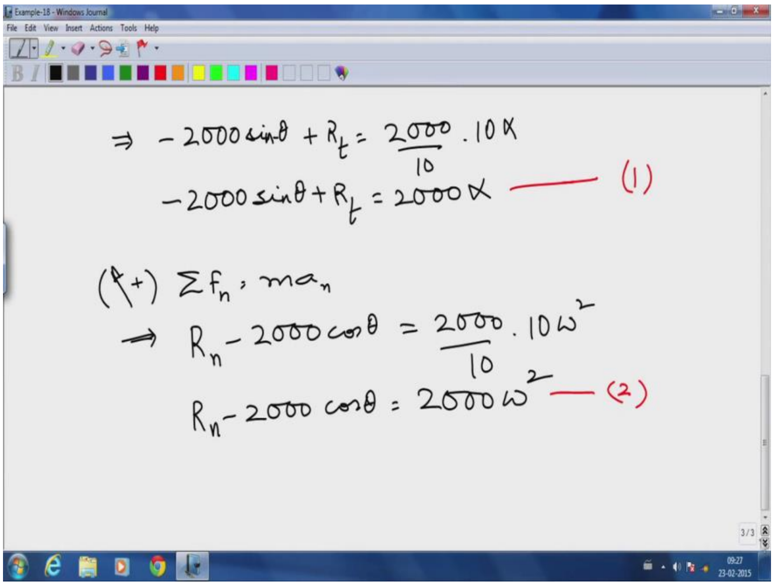

So, let us put some numbers. So, this says minus 2000 sin theta. So, I want to leave this let some generic angle theta plus R sub t is 2000 divided by 10, I am going to like always takes g is 10 meters per seconds squared times 10 alpha is sub t, 10 alpha. So, I have my first equation minus 2000 sin theta plus R sub t equals 2000 alpha. This is what I will end up calling my equation 1.

The second equation I have is that all forces normal or along the bar is also equal to the mass times acceleration of the center of mass. So, a t and a n are the accelerations of the center of mass. So, this implies, so I have R n acting in that direction minus 2000 cosine theta, because the weight of the particle acts opposite to the chosen sin convention that equals 2000 divided by 10. This is the mass of the particle times 10 omega squared. So, alpha and omega are in general functions of theta. So, let me complete this equation, this is what I am going to call equation 2.

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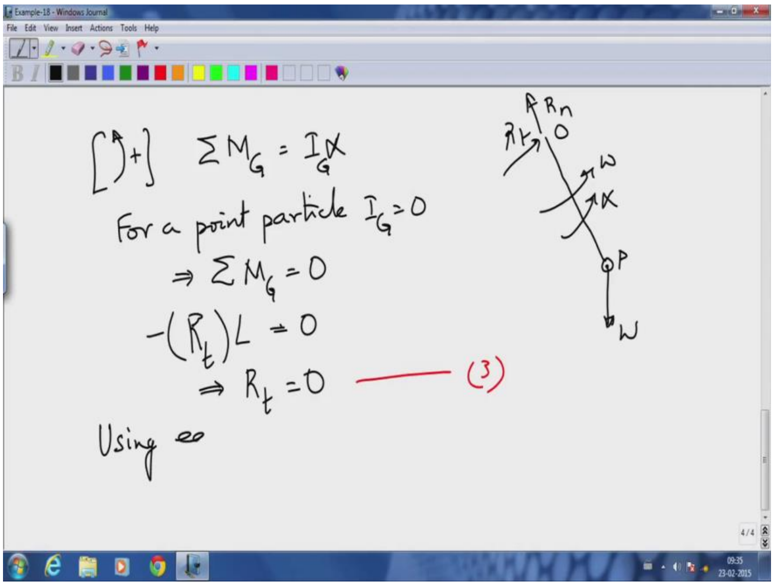

The last equation, let we complete this calculation comes from the fact, that if I take counter clockwise positive, sum of all moments about the center of mass equals I times alpha. So, if I go back for a moment ((Refer Time: 12:01)), this is my center of mass like we said, let us g and there is no other weight in the problem and the center of mass and P happens to be a point particle. We are modeling the motor cyclist plus the rider as a point Particle.

Motor cycle plus the rider as a point particle therefore, it is moment of inertia about the center of mass itself is 0. So, let us write this.

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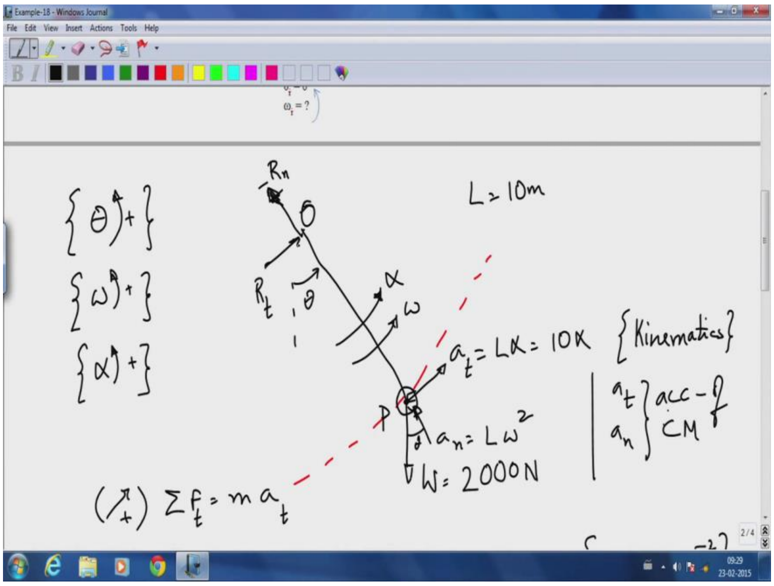

So, the sum of all moments, we look at the only force that causes a moment is R sub t ((Refer Time: 12:37)), R sub t causes a moment. So, R sub t, the diagram once more here. So, R 7 R sub t; that is my point O, this will point P is a weight W is alpha, omega. So, I G, which is the center of the moment of inertia about the center of mass of the particle and for a point particle I G is 0, because it has no desirable radius. So, which automatically means that sigma of M G equal to 0, the only force that causes moment about the point P is R sub t.

So, what we find is R sub t times L, if you want to be correct, it has to be a negative sign, because R t produces a clockwise moment and we have chosen counter clockwise moments positive and that is because alpha is counter clockwise positive. So, notice how the sign convention propagates all the way to through to all the equations, this equal to 0, which automatically implies that the tangential reaction at O is 0.

Now, if I G was not 0; that means, if let say instead of P, I have a finite cylinder or some body that had a moment of inertia, about it is own center of mass, R t is not necessarily 0. So, this is my equation number 3.

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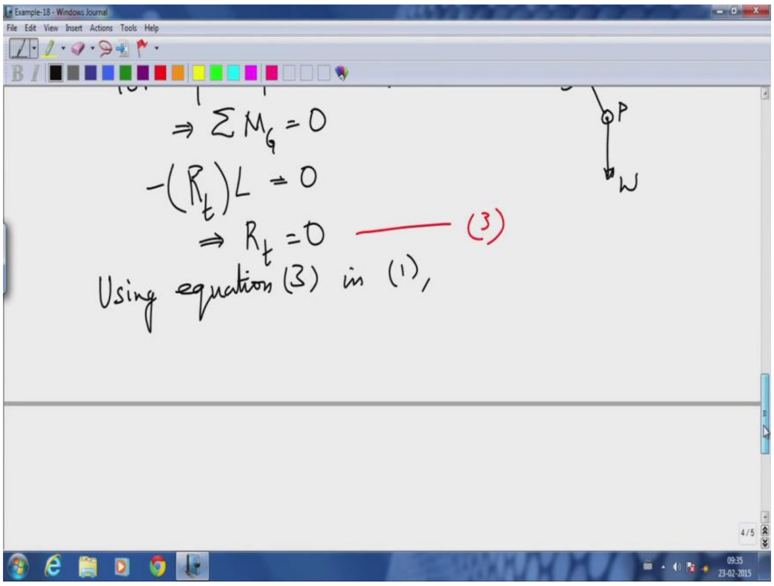

So, using equation 3 in 1, let see if we can simplify this ((Refer Time: 14:48)), one is the fact that minus 2000 sin theta plus R t is 2000 alpha.

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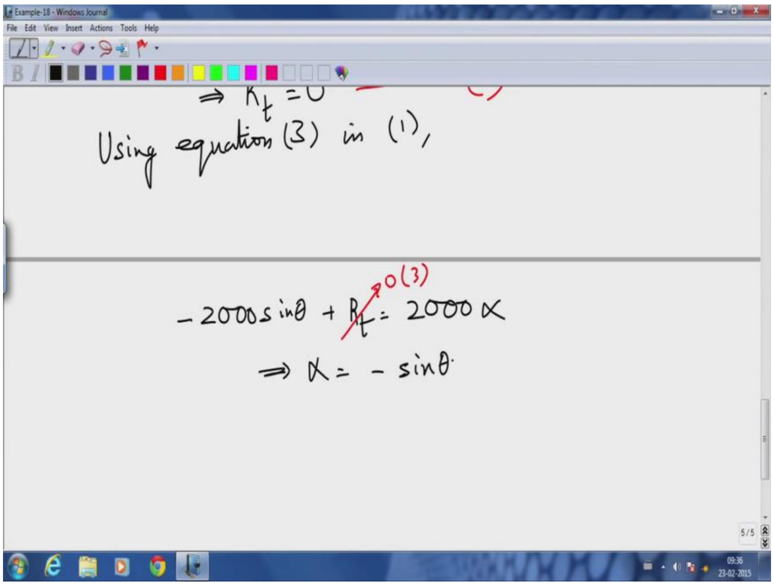

So, minus 2000 sin theta plus R t is 2000 alpha which automatically implies, because R t is 0 as for equation 3, automatically implies alpha equals minus sin theta.

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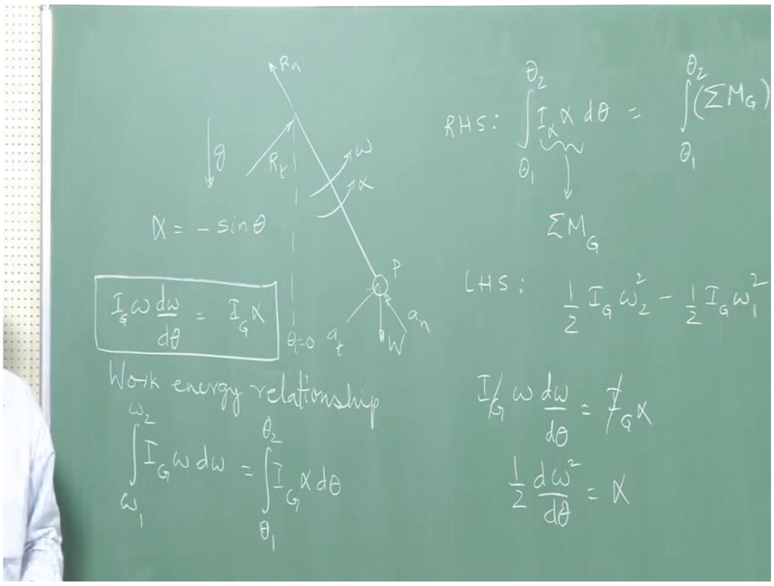

So, we found that alpha equal to minus sin theta. So, that is kind of, that is what we found from the laws of Mechanics. So, now, if I apply work energy concept to this, essentially I omega d omega d theta. So, if I know, if I want to write omega in terms of theta, omega is a function of theta, this is an alternate way of stating our work energy relationship. To bring it into a more familiar form I G W d omega from some omega 1 to omega 2 equals I G alpha d theta from some theta 1 to theta 2.

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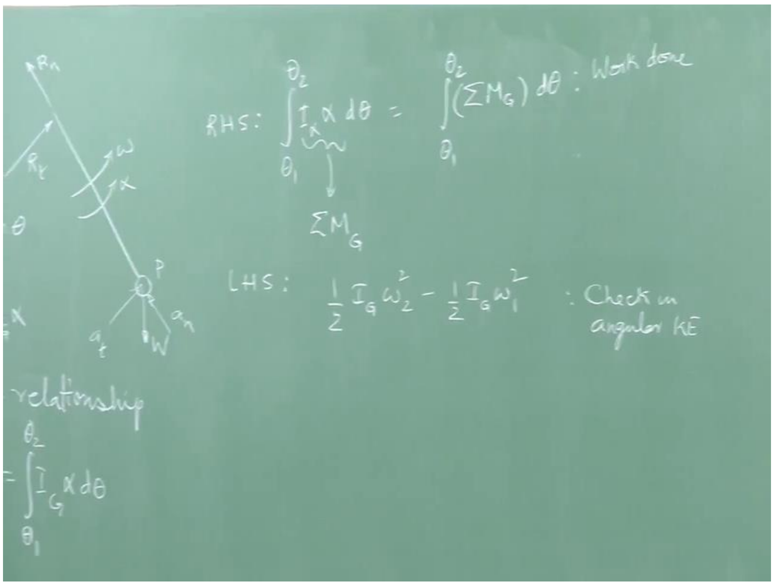

If I am going from some initial state to a final state, then the work done in the angular sense, so this part here, I can replace this part with the sum of all moment about G. So, this part is essentially the work done in an angular sense. This is my right hand side, the left hand side, if I simplifies that, would essentially give me half I G omega 2 squared minus half I G omega 1 squared. And we all recognize that this is the change in the kinetic energy, change in angular kinetic energy.

So, we essentially what we wrote down in the linear form as work down on the body is equal to the net change in the kinetic energy of the body is also true, when you start thinking of this in a angular sense. So, we go back to this form here, because it is essentially a differential form of this integral relationship, the differential form is valid at every point in the process.

So, at going from some angular position theta to another angular position theta plus d theta, a small incremental theta would allow me… This kind of a differential form will allow me to apply work energy relationship over that differential angular process.

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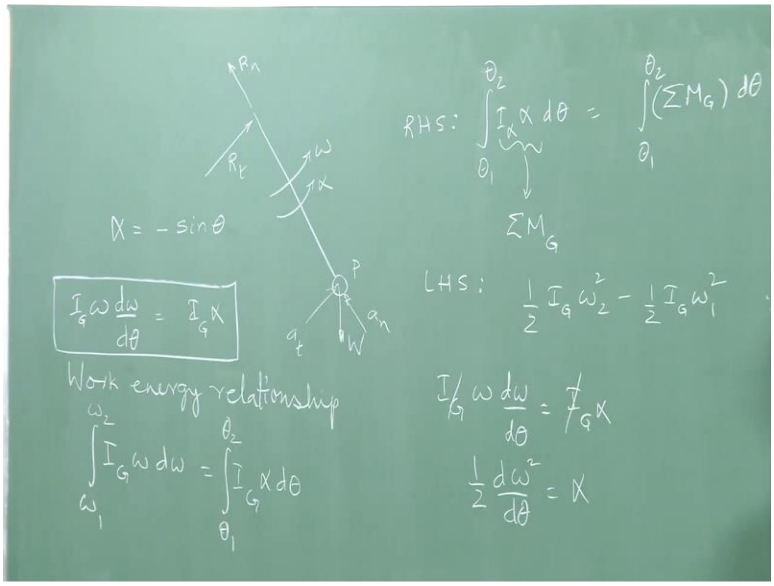

So, if I do that, I G omega d omega d theta equals I G alpha and I G can be canceled out. So, essentially this is half of d omega squared d theta equals alpha. So, if I go back and see, what we had before alpha is minus sin theta.

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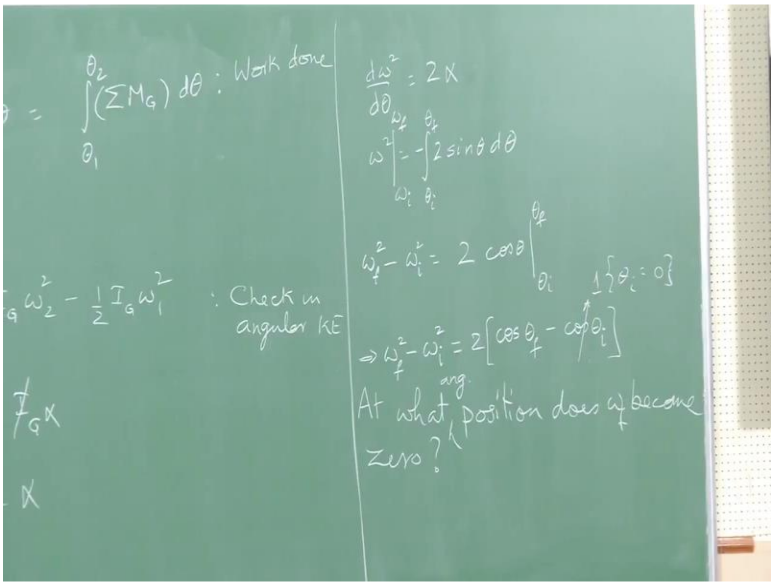

So, d omegas squared d theta equals 2 alpha omega squared is omega squared from some initial omega I to a final omega f. So, now, integrated this is going from some initial theta I to a final theta f minus 2 sin theta d theta. So, this is omega f squared minus omega I squared is equal to the integral of sin theta is minus cos theta. So, that gives this may exists to cos theta and this evaluated in the limits theta I and theta f.

So, I write this one final time omega f squared minus omega I squared is twice cos theta f minus cos theta. Now, in this particular problem, we are starting out ((Refer Time: 20:15)) with theta I being a 0. So, the body is a completely in a vertical position oriented with respect to gravity. So, theta I is 0 and we want to ask ourselves the first question, the first question is at what angular position with omega f become 0 for a given omega I.

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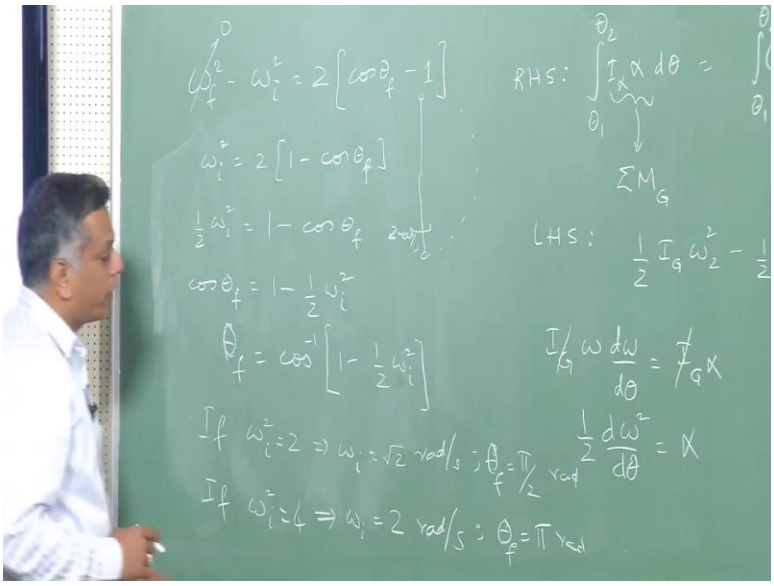

So, let us write the question at what position at what angular position thus omega f becomes 0? And if I ask that question erase the free body diagram here, what we find is that omega f squared minus some omega f squared equals 2 times cosine of theta f minus 1, because theta I is 0, this becomes 1 ((Refer Time: 21:34)), going to the fact that theta I is 0. So, for a given omega I, what is theta f? So, I can write this as omega I squared equals 2 times 1 minus cosine theta f, all if I rearrange this a little bit half of omega I squared.

So, if omega I is 0, just to kind of look at sock out some trivial cases, if omega I is 0, the bar is going over, theta f would also be cosine inverse of 1, which makes theta f also 0, as omega I increases for very small omega I, theta f would be slightly positive. So, as theta f increases as omega I increases theta f increases. So, just to get at if omega I squared is exactly 2, which implies omega I square root 2 radians per second. Then, omega I squared is 2, so this becomes 1, 1 minus 1 is 0, cosine inverse of gives me theta f as pi over 2.

So, if omega I squared is square root of 2 that gives me theta f is pi over 2. Now, the next case is if omega I squared is such that, what is inside this radical is actually minus 1; that is this part is not 1, but 2. So, if half omega I squared is 2; that means, if omega I squared is 4, which implies omega I is 2 radians per second. What do I have, when theta f, the final position one omega I omega f becomes 0 is pi.

So, if I give and angular velocity of 2 radians per second; that means, this bar is going to go up to the top most position pi, theta f equals pi radians. So, all of this, I am mean, we are not going to, are in the units of angular units of radians. So, if this goes up to the top then theta f is pi. Now, technically this bar started in this downwards vertical position and if I get this 2 radians per second, this bar would travels an arc and come to rest at that point.

If would come to rest at the top of that point, which means that any if I give it to angular velocity slightly greater than this, slightly greater than 2 radians per second, this is going to complete of full circle Now, let us go back and see, what it means, if omega I squared is slightly greater than 4 radians 4, what you find is that the argument inside this cosine inverse is no longer less, no longer greater than minus 1, it is actually greater than minus it is minus 1.1 or 1.2.

So, if omega is square is greater than 4, this part can never be, you can never find a cosine inverse of the argument there for real theta. That means, the bar would never come to rest, there is no such thing as a theta f at which omega f is 0, we will just continue to rotate forever. So, the minimum angular velocity omega I needed to meet the bar go of it circle is 2 radians per second.

Notice that, this does not depend on the mass of the bar; it does not depend on the actual mass of the particle that you are considering. So, the next question we want to ask is, what is the nature of the forces inside this bar as if I get only 2 radians per second in this bar ever experience a compressive force. So, let us ask if that question and answer it.

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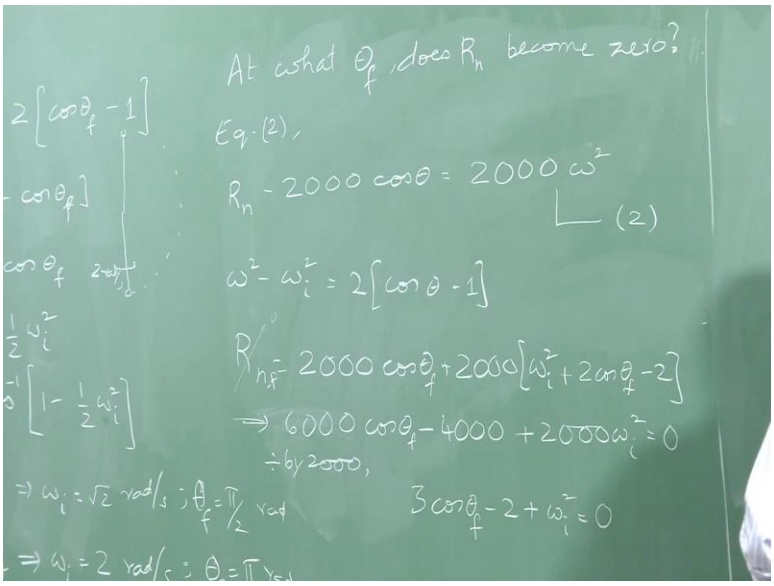

So, I will write the question first at what theta f, thus R n become 0 and if you go back to our equation 2, it says R n minus 2000 cosine theta equals 2000 omega squared, omega be meaning the instantaneous omega. So, from here, omega squared minus omega I squared equals 2 times cosine theta minus 1. So, at any theta, this gives me the corresponding omega, this comes from a work energy relationship.

If I now substitute this in this equation 2, R n equals 2000 cosine theta plus 2000 times omega I squared; I am going to move this over to the other side plus 2 cosine theta minus 2. So, this is R n, now if you go back to our free body diagram we had chosen R n in a tensile sense to be positive. If you notice we had R n pointing away from O being positive. So, R n positive implies the bar is under tension, R n negative implies the bar is now gone under compression.

So, let see at what theta would R n becomes for a given omega I, so omega I is given at what theta would R n becomes 0 for a given omega I. So, omega I is given at what theta would R n become 0. So, I set this to 0 in able to calculate what theta I is, what theta f is. So, if I go head simplify this equation at 2000 cosine theta plus another 4000 cosine theta, so this gives me 6000 cosine theta minus 4000 plus 2000 omega I squared equal to 0. So, I is divide by 2000, this is 3 cos theta minus 2 plus omega I square equal to 0.

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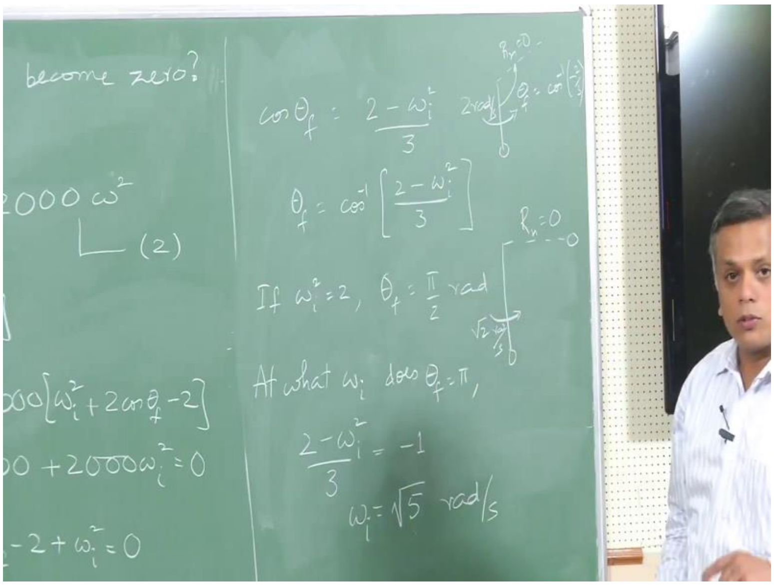

So, R n is 0, if R n f is 0 that happens at this theta f. So, if I rearrange the few of these times that gives me cosine theta f equals 2 minus omega I squared divided by 3 or theta f is cosine inverse of 2 minus omega I square divided by 3. So, if I ask the question for a given omega I squared, let say, if omega I squared is 2, which is what we form was sufficient to bring the bar to rest at pi over 2 radians, which is 90 degrees. If omega squared equals 2, then theta f equal to pi over 2, that means, when the bar if I give this bar square root to radians per second.

When, the bar becomes horizontal, R m is 0 in that bar. So, if I give it slightly greater angular velocity, it is going to go pass that, that R n would become negative. So, if I do not want for a given omega I, I now have a way of finding the theta f at which R n becomes 0. So, if I want to turn around and ask the question at what omega I would theta f become originate positive, even all the way till to theta f equal to pi.

So, at what omega I, this theta f equal to pi, this says that even at the top of the circle the bar is only under tension. So, if I want to find this, theta f is pi; that means, 2 minus omega square over 3 has to equal minus 1, which means omega is square root of 5 radians per second. So, I have to give this initial angular velocity to this square root of 5 radians per second for the bar to remain under tension to out all the way to pi.

So, let us go back, compare these two numbers for the bar to come to rest at the top of the circle ((Refer Time: 34:37)). So, if for it to just reach the top of the circle, if I given initial velocity of 2 radians per second that is enough. So, if I give it to in initial angular velocity of 2 radians per second, the bar would hit to the top and any, if I give it a infinite similar small greater amount them 2, it would complete the full circle. That is all I need for it to do the full circle.

Whereas, if I want the bar to remain under tension all the way to the top 2 radians per second is not enough, I have to give it in a angular velocity of square root of 5 radians per second, square root of 5 is much greater than 2 in the sense of, it is greater than 2. And it is require at least square root of 5 radians per second for the bar to reach the top and the normal reaction in the bar to be positive. So, if I give an angular initial velocity greater than square root of 5 radians per second, then I am assured that d bar would always remain under tension.

Now, if I go back and look at what happens, if I only give 2 radians per second, for only give 2 radians per second, then omega I squared is 4, 2 minus 4 is minus 2. So, at some angle, if this initial velocity is only 2 radians per second; that means, there is some angle, some theta f, which is cosine inverse of minus 2 over 3 at which R n becomes 0. R n is 0, the bar is continued to move, because I forgive it 2 radians per second, the bar would go all the way to the top.

So, for this remainder f this angular motion, the bar is under compression. So, fewer a motor cycle rider, this is not accepted Now, let us make show the completely understand this one last time, if I give it 2 radians per second, I way get to the top, but the bar would have to sustain a compressive force from this angular position to the angular position theta f equal to pi. If I give it square root of 5 radians per second, the bar would get to the top easily, because 2 radians per second was enough to get it to the top.

The bar would get to the top easily and the normal reaction in the bar would become 0 only at the top. So, if I give it slightly more than square root of 5 radians per second, the bar would complete a full circular motion with the bar being intention all the time, we will continue with discussion in a other example problem.