Statics and Dynamics
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Lecture – 33

We go and look at the different way of solving problems involving dynamical systems today called work energy methodology. The way work energy methodology works, is it still start with Newton's second law, F equals m a, and unlike what we did in the last time. We looked at a set of problems involving, what is called the impulse momentum approach.

In the impulse momentum approach, we take Newton second law and do a time integration from some initial time t 1 to a final time t 2. What we find is that the impulse which is the integral of the force over the duration of the time is equal to the actual change in the momentum. So, we going to do same thing, except instead of integrating in time, we want to look at the integration space.

So, for example, if I have a force, whose variation I know spatially, not necessarily with time. How would I take advantage of the known variation of force with respect to the spatial coordinate and understand the dynamics of a body?

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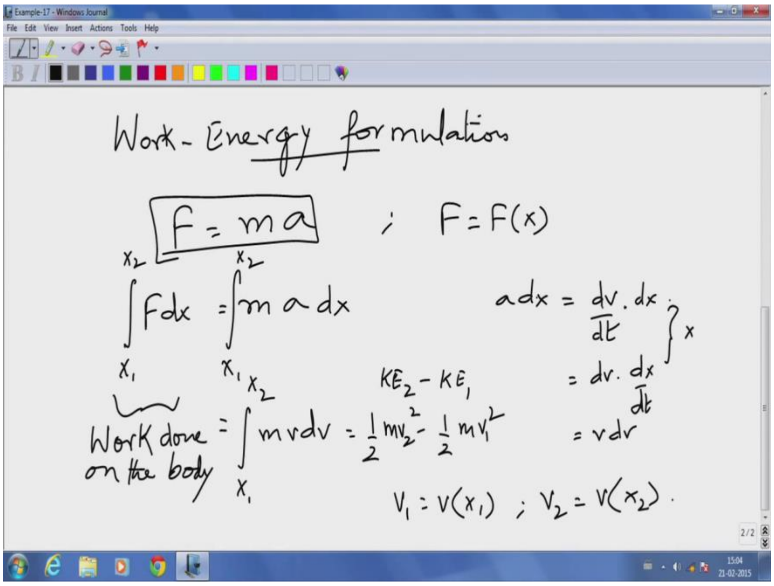

So, with this is going to be called work energy formulation, the basic principle is as follows. We still start with Newton's second law and let say the general case F is some function of a spatial coordinate. For that situation, I do an integration of this from some initial location of the body x 1 to a final location of the body x 2 and do the same thing on the right hand side.

Now, let us look at the right hand side first, acceleration a d x can be written as d v d t time d x, which is nothing but, the rate of change of velocity integrated over some infinite distance d x Now, this is not exactly correct, but it is a convenient way to understand the basic transformation. So, from here, we can write this as d v times d x d t.

Now, the logic here is as follows, if v d v is essentially d x d t times d v, d v being the rate of the net change in the velocity over some change in the independent coordinate. So, if I replace this a d x with v d v, what you end of finding is that, this is equal to half m v 2 squared minus half m v 1 squared, where v 1 is the velocity at x 1 and v 2 is the velocity at x 2. So, the right hand side of this equation is simply the actual change in the kinetic energy, if v 2 is half and v 2 squared is K E 2 minus K E 1.

The left hand side on the other hand is essentially the integral of a spatially varying force with respect to x from some initial location x 1 to x 2. And this is what we are used to understanding at the work done on the body. So, Newton's second law in a weaker form written in a spatial coordinate essentially amongst to saying that the work done on a body. The clean two spatial locations x 1 to x 2 is exactly equal to the net change in the kinetic energy of the body within those two states.

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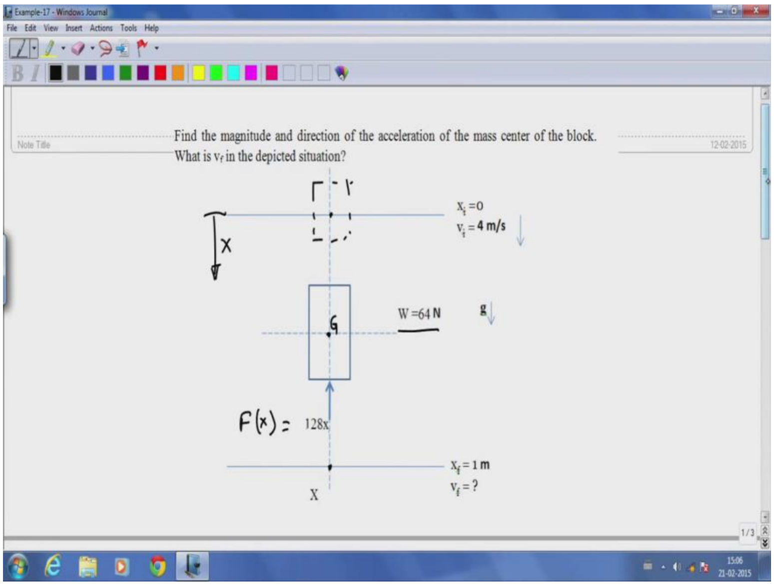

So, we are going to put this to some text here, let us take an example problem. We have a block, whose weight is 64 Newton's; that is acting; that is following in a gravitational field. It is initial velocity at some x i equal to 0 is known to be 4 meters per second. And at some final location, when the center of mass of this body is now at this location x f equals 1 meter, we are asked to find the actual velocity of that body.

Now, gravitational force is going to accelerate this body from x i equal to 0 to x i equal to 1. We will also have another force F of x whose magnitude goes as 128 times x, 128 times, x being the actual location of this body. So, meaning when the body is at this point, there is no force acting on this body and ask the body moves downward as x increases. The magnitude the force increases and it is direction is always opposing the motion of this body. So, this is the very simple model of it body falling on a little spring or a cushion, we are asked to find the velocity, when the body is at 1 meters.

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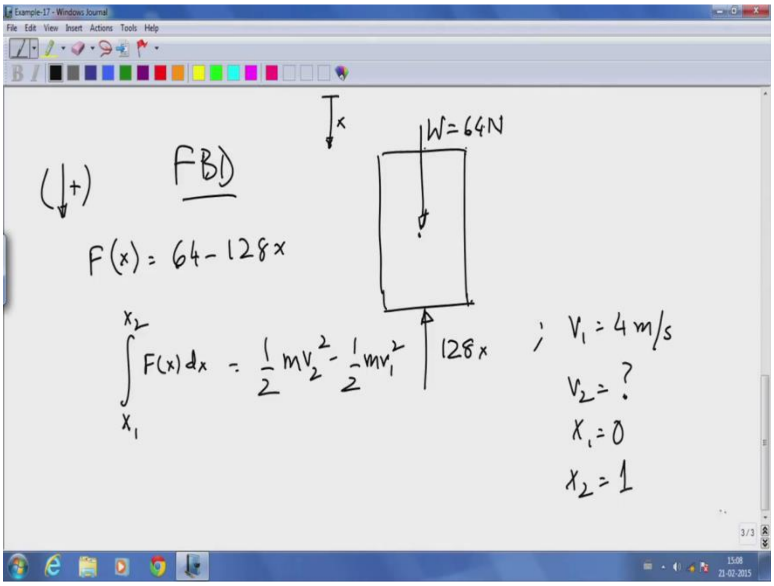

So, let us draw a free body diagram of this body first. The weight is acting on this body; where it is magnitude 64 Newton’s. At any x location, the force acting on this is 128 x and it is opposite. There are no other forces acting on this body. So, at any given instance of time or at any given x location, the sum total force acting on this body is I am going to take downwards positive as my convention is 64 minus 128 x. This is the net force acting on this body at any instant of time, at any x location, at any position of this body.

The only know the force as a function of position on the body, position of the body. So, now I do know that between x 1 to x 2 this F of x d x is equal to half m v 2 squared minus half m v 1 squared. We have told v 1 is 4 meters per second, v 2 is an unknown, we are asked to find, what v 2 would be. So, let do this integration between x 1 equal to 0 and x 2 equal to 1.

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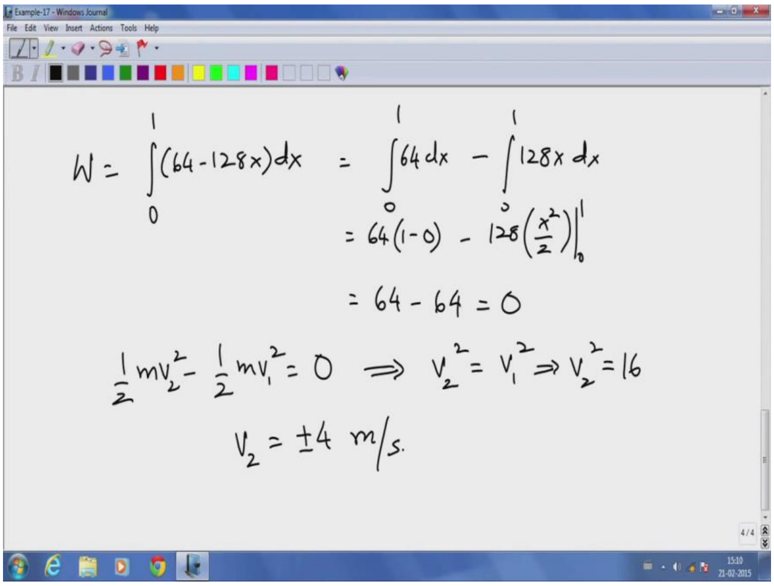

So, integral 0 to 1 64 minus 128 x d x, the actual work done is equal to that. Let us compute that first, this is integral 0 to 1 64 d x minus integral 0 to 1 128 x d. This is simply 64 times 1 minus 0 minus 128 times x squared over 2 evaluated in the limits 0 to 1. This is 64 minus 64, which is 0. So, the actual work done on this body is 0, which also should mean half m v 2 squared minus half m v 1 squared is equal to 0. May we do not know v 2, but we do know v 1, which implies v 2 squared equals v 1 squared. If v 2 is unknown, v 1 is 4, what we do know is v 2 is plus or minus 4 meters per second?

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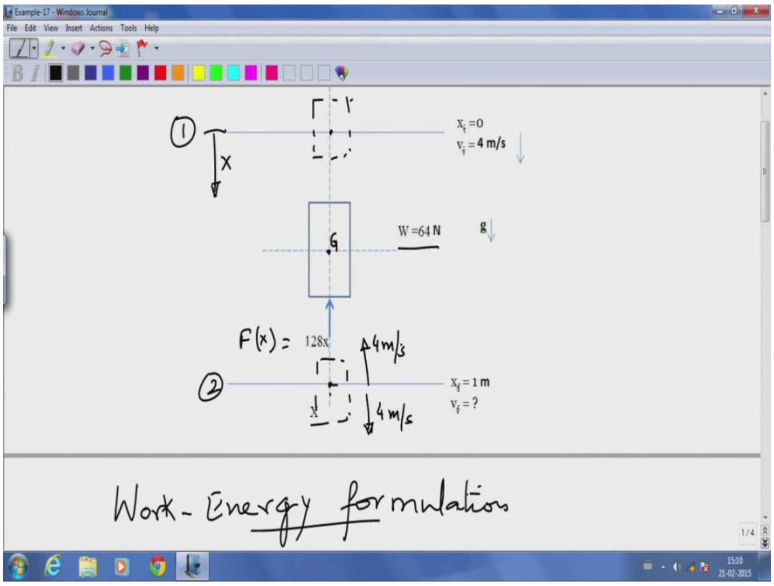

So, the two possibilities for the actual velocities at this location, I call this location 2 and relation to this being location 1. The body is here, when the body is here, it is either leaving down at 4 meters per second or it could be leaving up at 4 meter per seconds. So, let us in order to really understand, what is happening here, let us do this in a slightly more generalized way and that will help us understand this.

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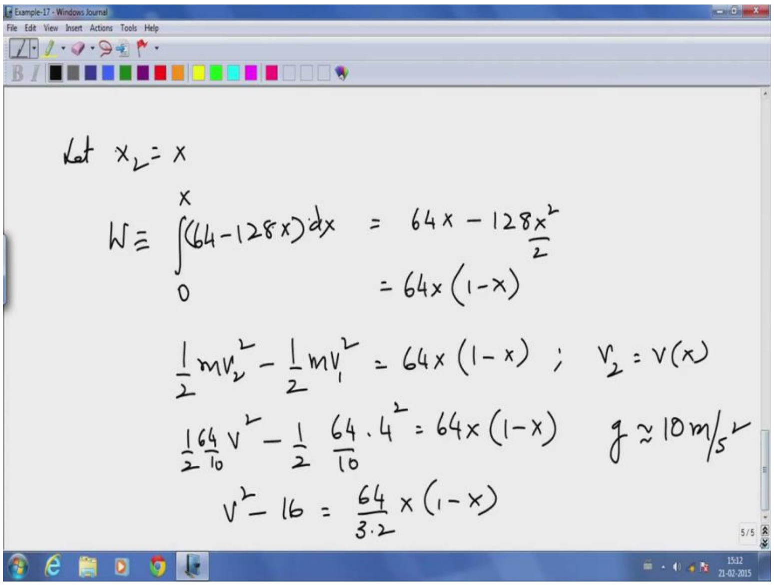

I want to do this without setting a number for x 2, I am going to do this in a symbolic fashion, I am going to let x 2 be any location x. So, W is integral 0 to x, 64 minus 128 x, d x; it is go through to integrate that. So, that gives me 64 x minus 128 x squared over 2, which is 64 x times 1 minus x. So, half m v 2 squared minus half m v 1 squared equals 64 x times 1 minus x; that is what we just found.

So, v 2 being v at any location x, so I will write it as v squared minus half times 64 divided by 10, I am going to take g to be 10 meters per seconds squared, I am not going to complicate the numbers, v 1 is 4 squared at 64 x times 1 minus x. So, I am going to replace m here as well with the same 64 over 10. So, what I do know is that, v squared minus 16 equals 64 divided by 3.2 x into 1 minus x. For v squared minus 16 equals 20 x into 1 minus x, v squared equals 16 plus 20 x minus 20 x squared.

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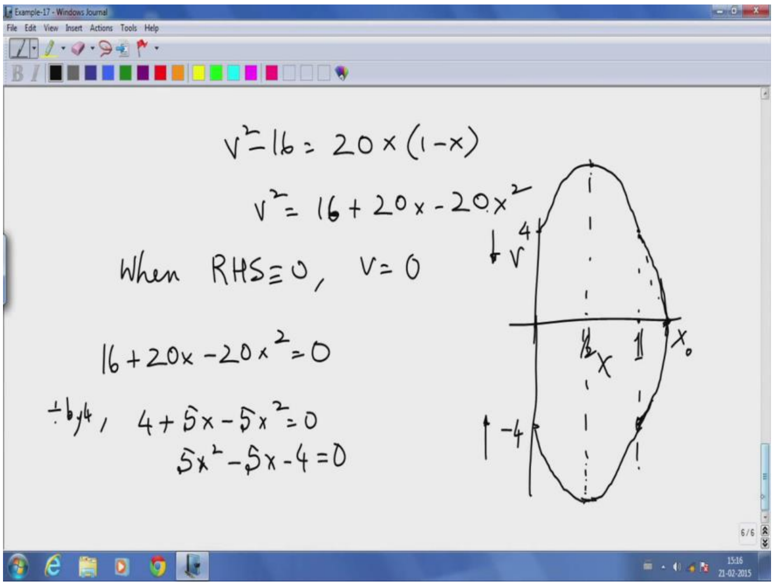

So, if I plot v as a function of x, you will notice that, when the right hand side goes to 0, v equals 0 and when the right hand side is 1 or 0, v equals plus or minus 4. So, essentially what we find is, that if I plot the velocity of function of x, it takes on the number 4 at x equal to 0 and at x equal to 1. So, initially the velocity increases, and then decreases, so you can notice that, the velocity here would be maximum, when this right hand side is, when the derivative of this right hand side is a 0. So, that could be roughly read about x equal to half.

Now, passed this point, it becomes 0, and then further increases that again. So, essentially the force that is opposing the motion of this body in a downward direction, which is of magnitude 128 times x. Has two possibilities, so this is not exactly correct. So, when in the downward the velocity essentially increases still about half and then decreases from to 1. And at some other location it goes to 0 at which point the velocity changes sign and start to rebound backup again.

So, you would get occur that looks like this we completed, it would be symmetric about the horizontal axis. So, at this point, it would come back to the minus 4, meaning, if this was 4 in the downward direction, this is minus 4, meaning it is block would be upward direction. At some location down, at some location past the 1 meter mark, the body with come to rest, and then rebind back up again, 1 meter bounds when it passes the 1 meter mark, it would be meaning with the same velocity of minus 4.

And then at the same half meter mark, it would have the maximum negative velocity, meaning maximum upward moving velocity. And, then come back up to a minus 4 velocity, come back up to 4 meter per second; that in the upward direction when it comes to x equals to 0. The actual location of this, I will call it smart can be found the taking the route of this equations 16 plus 20 x minus 20 x. So, let us do that. So, if I divide this by 4, we get is to be 4 plus 5 x minus 5 x squared equal to 0.

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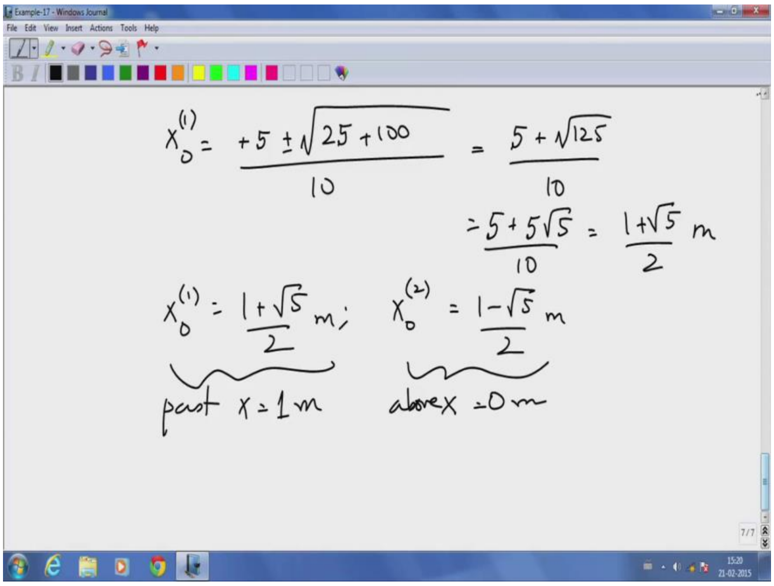

So, the x naught is minus plus 5 plus or minus root of 25 plus 100 divided by 10. So, 1 plus square root of 5 divide by 2 is one possibility, call this x naught1, x naught1 is 1 plus square root of 5 by 2. The other possibility is 1 minus square root of 5 divided by 2 meters. If you look at these to numbers, this is past x equal to 1 meter; this is actually negative meaning above x equals 0 meters.

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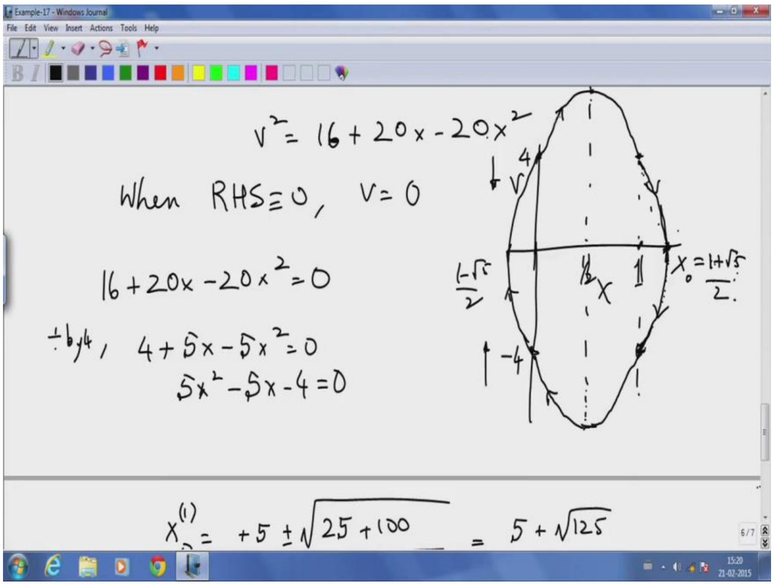

So, you can think of this again as rebounding to some other point 1 minus square root of 5 over 2 minus, and then cycle continues again. So, the body initially dropped on this little phone, it reaches the maximum velocity at x equals half a meter, back to 4 meter per second in the downward direction at 1 meter, because this is the time, when this ring all the phone beginning to decelerate the body. And, then it reaches the different point x naught given by 1 plus square root of 5 1 minus, where it is completely brought to rest.

And now, begins to accelerate in the upward direction, so the velocity becomes the negative. So, if you ask the question, what is the velocity of this body at 1 meter, it could be either in the downward direction or in the upward direction. And, therefore you unable to completely resolve the direction of the motion some work energy kind of in approach. So, essentially in still happen some ambiguity on the direction of the motion, if you chose this also problem using what can I energy approach.

Especially, because in this particular case, the force field is given by 128 x, which is conserve force field. As you will see this body a continuous to executive this kind of periodic motion forever, so it reaches again the point 1 minus square root of 5 divided by 2 has the highest point. And, then it begins to again decelerate due to gravity decelerate or accelerate due to gravity in a downward sense in the motion repeats.

So, this kind of a problem, where you know the force acting on a body as a function of x, requires work energy formulation. If you choose to convert to this to time coordinate and use impulse momentum, it would either be not possible or very difficult. I will take about that, if we have time towards the next lecture as to what condition under inversion is possible.

That for now, this is the very useful way to understand the overall change in the kinetic energy the body will leaving some ambiguity in directions of motions, which is going to be the case now these kind of problem. We will continue this discussion in the next example problem.